Journal of Glaciology, Vol. 30, No. 105, 1984

REVIEW

KOLUMBAN HUTTER. *Theoretical glaciology: material science of ice and the mechanics of glaciers and ice sheets*. Dordrecht, etc., D. Reidel Publishing Co.; Tokyo, Terra Scientific Publishing Co., [1983]. xxxii, 510 p. (Mathematical Approaches to Geophysics.) Guilders 240, $104.

This is a most impressive book. Most shortcomings cited in this review reflect on Hutter's decision to limit the scope of his study, not the study itself, which is extremely thorough and well-presented. His writing is lucid despite the fact that English is not his native tongue.

The book is organized into seven chapters. Ch. 1 presents the fundamentals of continuum mechanics, with detailed treatments of the balance equations, the response of the material to environmental conditions, an application of the entropy principle to stress and temperature fields, and phase changes as one important material response. Ch. 2 examines several constitutive equations as they relate to the mechanical properties imposed by the hexagonal symmetry of randomly oriented polycrystalline ice, and phase transformations that involve brine in sea ice. These considerations constitute Part I of the book, which unites continuum mechanics and materials science in a framework within which glaciers can be studied.

Part II provides numerous glaciological applications of the principles developed in Part I. Ch. 3 gives the basic practical application; the flow of ice masses under the force of gravity, masses that range from small valley glaciers to continental ice sheets. Two-dimensional flow is examined in terms of stress and temperature field equations, and the stress and thermal boundary conditions that prevail when the bed and surface slopes are nearly identical. Ch. 4 examines the velocity field of these slabs as a consequence of temperature-stress feedback. It begins by stating the basic boundary-value problem, reduces it to linear form, and then provides solutions of zero and first order, with numerical results for steady-state solutions and an examination of surface waves on glaciers. These chapters present what might be called classical glaciology.

The remainder of Part II concentrates on new developments, many by Hutter himself. Ch. 5 treats cases where the surface and bed slopes are substantially unequal; what Hutter calls the shallow-ice approximation. The basal shear stress formulation is modified for this situation, and steady-state solutions are presented for ice flow and surface profiles that result from variable bed topography. Ice divides of ice sheets are examined and an appendix examines higher-order stress solutions in the shallow-ice approximation. Ch. 6 analyzes the response of valley glaciers and continental ice sheets to seasonal and climatic change, including the motion of kinematic waves and other surface waves on glaciers. Ch. 7 concludes the book by expanding the two-dimensional treatments for ice sheets to three dimensions, and by focusing on the effect of varying cross-sections of valley glaciers. Variational principles are introduced to treat local phenomena such as ice falls and calving ice cliffs. Some finite-element solutions to these problems are examined.

The book was written between 1979 and 1981, although publications as recent as 1982 are referenced occasionally, especially Hutter's own work. It is therefore surprising that some major contributions to ice-sheet modeling are not examined. These include the advance and retreat of ice sheets in response to Milankovitch variations in insolation modeled by Budd and Smith in 1979, and the 18k BP global reconstructions of former ice sheets and disintegration of the West Antarctic ice sheet modeled by CLIMAP in 1981. Each of these works addressed important glaciological themes in Hutter's book. The work by Budd and Smith is, in my opinion, epochal. However, it is based on the pioneering three-dimensional model published by Mahaffy in 1976, which Hutter (p. 298) dismisses because it gave "little (consideration) to momentum balance or constitutive relations" favored by Hutter. The real question (how serious is this omission?) is left unanswered.

The CLIMAP model was the first to reconstruct flow lines of former ice sheets using glacial geology as basic input to produce domes and saddles along ice divides, ice streams and ice lobes along ice margins, and regions having freezing, frozen, melting, and melted beds in between. This neglect may, however, be a consequence of Hutter's decision to focus on internal creep and ignore basal sliding. This decision is unfortunate because it precludes any treatment of ice-stream dynamics (including glacial surges), perhaps the most exciting arena of contemporary glaciological research, and of Thomas's marine ice-sheet instability mechanism that was used in the CLIMAP ice-sheet disintegration model to study the stability of the West Antarctic ice sheet, a question which Weertman has called, "Glaciology's grand unsolved problem".

In view of these omissions, Hutter's book is more a study of ice dynamics than of glaciology in the full sense. His approach is primarily that of an engineer, not a scientist. The mathematical developments are readily applied to structural mechanics, the flexural strength of plates, and related engineering problems. The continuum mechanics and balance equations in ch. 1 are part of the core curriculum of mechanical engineering. The constitutive equations in ch. 2 focus on rheological models based on springs and dashpots in series and parallel linkages, another staple of the mechanical engineer. Dislocation theory, which describes the real deformation processes in crystalline materials and which concerns the physicist, is scarcely mentioned. Yet a great body of glaciological literature deals with studies of dislocations, both in the laboratory and theoretically. Hutter's definitions of primary, secondary, and tertiary creep (p. 76) are subtly different from what a scientist would use, but they are acceptable engineering definitions.

Hutter makes some misleading statements. On p. 82: "At very high stresses of more than 7 bars, the creep rate accelerates and approaches a different steady state, the tertiary creep." Tertiary creep is the accelerating stage, not the steady-state stage, in Hutter's Figure 2.12. Another example is, "Polycrystalline ice is an aggregate of randomly-oriented grains of single ice crystals" (p. 60), and "At large stresses and for very slow processes, as for instance quasi-static load histories and deformations of glaciers under their own weight, polycrystalline ice must be treated as an isotropic plastic body" (p. 81). Polycrystalline glacial ice has preferred grain fabrics much more commonly than random fabrics, and any number of analytical and numerical solutions for glacial flow assume neither plastic nor isotropic deformation.

Occasional misleading statements are merely lapses in what is an overall impressive work. I was particularly mindful of Hutter's treatment of viscoelasticity. One consequence of using dashpot simulations is Hutter's preference for the viscous end of the viscoplastic creep spectrum, especially at low stresses and strain-rates. Although many classical solutions to glaciological problems employ the plastic end of the spectrum, Hutter states that "polycrystalline ice may be considered to be a linear viscoelastic body at stresses below one bar" (p. 63) "because no real material can respond infinitely fast to applied strain rates" (p. 84). My interest in the possibility of thermal convection at depth in polar ice sheets has focused on the transition from elastic deformation to primary creep deformation, and it seems to be quite important whether this earliest creep is linear or not. I would very much like Hutter to bring his perspective to this question. Thomas has shown that secondary creep in floating ice shelves is non-linear below stresses of one bar, but the non-linearity of primary creep has only been established at higher stresses in the careful work by Duval. An initially infinite strain-rate, of course, virtually guarantees the initiation of convection in ice sheets because it produces an infinite Rayleigh number. Maintaining convection is another question. Resolution of this problem for ice sheets has implications for mantle convection and plate tectonics.

The mathematical ice-flow model presented in ch. 3 includes a fascinating treatment of ice-water interaction within the ice and at the bed. The model ice sheet has cold ice and temperate ice zones, with the frozen boundary following the ice-rock interface and then moving up to the ice surface. As shown in Hutter's Fig 3.1, the temperate ice zone ends as a floating ice shelf. Temperate ice shelves are non-existent, but no matter, the meaty part of this model deals with the concept of sliding friction in a temperate ice layer on a rough bed. The possible sliding laws include double-valued sliding velocities and surge-like velocities for a given basal shear traction. Future applications of these solutions to ice-stream dynamics and glacial surges are obvious (they include the question of creep instability and the cusp catastrophe phenomenon), and we can only hope that Hutter will develop these kinds of solutions in more detail. Hutter's treatment of ice shelves avoids the really interesting cases of pinning points at ice rises, interactions with ice streams, grounding-line instability, and iceberg calving dynamics.

Ch. 4 presents Hutter's solutions to the classical problem of an ice slab moving over an undulating bed. He develops analytical solutions for a linear (Newtonian) rheology and numerical solutions for a non-linear (power-law) rheology. This is basically an analysis of longitudinal stress gradients, which affect the temperature distribution as well. Hutter presents numerical results for steady-state flow and discusses time-dependent accumulation. It leads naturally into an analysis of surface-wave stability and, ultimately, to his shallow-ice approximation in ch. 5. My only comment here would be that finite viscosities must be introduced arbitrarily to eliminate mathematical singularities in the non-linear solutions. Here we see a practical reason why Hutter preferred to believe in ch. 2 that Newtonian creep existed at low stresses. It adds respectability to this dubious procedure in ch. 5; however it is not supported by laboratory creep data at moderate stresses nor by dislocation theory. Hutter prefers linear dashpots to dislocations.

The shallow-ice approximation is Hutter's way of obtaining stresses, velocities, temperatures, and thicknesses along the flow lines of an ice sheet. Ice thickness is assumed to change slowly along the flow lines, so stress, velocity, and temperature can be averaged over horizontal distances of several thicknesses. This treatment has important applications for ice-sheet modeling, but a sliding law needs to be incorporated. In many regions of the Greenland and Antarctic ice sheets, the bed is quite rugged on a big scale, and the shallow-ice approximation would not apply. In view of this, I would say that it is useful mainly as an instructional tool in its present stage of development.

Ch. 6 is an exhaustive treatment of kinematic waves on glaciers, including the question of whether linear equations are justified to obtain solutions or if kinematic wave theory even explains surface waves. This is an aspect of glaciology that holds little interest for me so I did not read ch. 6 carefully (I make my own waves).

Ch. 7 has a thorough treatment of the effect that channel cross-section has on the flow of valley glaciers, but little of this is really new. Hutter's discussion of finite-element solutions for selected glaciological problems is interesting but, again, is not new to those who have been using this modeling method. I would have liked to see much more here, because I am convinced that by the end of this century finite-element modeling will completely dominate glaciology. A section on variational principles applied to localized features was, for me, the best part of this chapter and this approach will ultimately be followed to deal with the problem of rugged subglacial landscapes in modeling ice-sheet dynamics. The finite-element method will have to be employed, of course.

In summary, I would say that Hutter's main contribution with this book is to develop some quite detailed solutions to glaciological problems within a consistent mathematical framework that weds continuum mechanics to materials science. Since this was the announced goal in the introduction, I would say that Hutter has succeeded in his task. In the process, Hutter has laid bare hidden assumptions in earlier, less rigorous treatments of these problems. I can only hope that Hutter will now direct his critical eye to the other major cause of ice motion: basal sliding.

One word of caution: this book contains a number of errors in the mathematics and, although these errors do not seem to invalidate the mathematical treatments, the reader should carefully work through every derivation before applying the final equations.

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