14

The electromagnetic and weak interactions of quarks

In the Standard Model it is the quarks' colour that is the source of their strong interaction. In this chapter we shall consider only the electromagnetic and weak interactions of quarks, and colour will not enter. The theory will be constructed in close analogy with the electroweak theory for leptons set out in Chapter 12. The theory for quarks is not as well founded in experiment as the theory for leptons. This is because quarks cannot be isolated from hadrons. Experiments can only be performed on composite quark systems, and the basic Lagrangian density is obscured at low energies by the strong interactions. At higher energies, and especially through the hadronic decays of the Z bosons, the electroweak physics of the isolated quarks can to some extent be discerned. In Chapter 15 some of the relevant experimental data on these decays will be described.

14.1 Construction of the Lagrangian density

At low energies, the model has to describe decays like

$$\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$$

or, at quark level,

$$\mathrm{d} \to \mathrm{u} + \mathrm{e}^- + \bar{\nu}_\mathrm{e}.$$

This decay is mediated by the W boson. Comparing it with muon decay,

$$\mu^- \to \nu_\mu + \mathrm{e}^- + \bar{\nu}_\mathrm{e},$$

which is also mediated by the W boson, suggests that the left-handed components u_L and d_L of the quark fields should be put together in an $SU(2)$ doublet,

$$L = \begin{pmatrix} u_\mathrm{L} \\ d_\mathrm{L} \end{pmatrix}, \qquad (14.1)$$

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while u_R and d_R are, like ν_R and e_R, unchanged by $SU(2)$ transformations. We shall see that this simple assignment would be correct if Nature had provided us with only one type of up quark, and only one type of down quark.

With such an assignment there is no freedom in the construction of the weak interaction. There is only one way to make the dynamical part of the quark Lagrangian density gauge invariant. The coupling to the field $\mathbf{W}_\mu$ is uniquely determined by $SU(2)$ symmetry and the coupling to the field B_μ is fixed by the quark electric charges: $2e/3$ on the u quark, $-e/3$ on the d quark. Hence

$$\begin{aligned}\mathcal{L}_{\rm dyn} = {} & \mathrm{L}^\dagger\tilde{\sigma}^\mu \mathrm{i}[\partial_\mu + (\mathrm{i}g_2/2)\mathbf{W}_\mu + (\mathrm{i}g_1/6)B_\mu]\mathbf{L} \\ & + \mu_{\rm R}^\dagger\sigma^\mu \mathrm{i}[d_\mu + (2\mathrm{i}g_1/3)B_\mu]u_{\rm R} \\ & + d_{\rm R}^\dagger\sigma^\mu \mathrm{i}[\partial_\mu - (ig_1/3)B_\mu]d_{\rm R},\end{aligned} \tag{14.2}$$

where $g_2 \sin\theta_{\rm w} = g_1 \cos\theta_{\rm w} = e.$

To conform with the transformation laws (11.4b) and (11.6) on the gauge fields, the $U(1) \times SU(2)$ transformation of the quark fields must be

$$\begin{aligned}\mathbf{L} &\to \mathbf{L}' = \mathrm{e}^{-\mathrm{i}\theta(x)/3}\mathbf{UL}, \\ u_{\rm R} &\to u_{\rm R}' = \mathrm{e}^{-4\mathrm{i}\theta(x)/3}u_{\rm R}, \\ d_{\rm R} &\to d_{\rm R}' = \mathrm{e}^{2\mathrm{i}\theta(x)/3}d_{\rm R}.\end{aligned} \tag{14.3}$$

Using (11.17) and (11.29), $\mathcal{L}_{\rm dyn}$ can be written in terms of the fields $W_\mu^\pm$, Z_μ and A_μ and becomes

$$\begin{aligned}\mathcal{L}_{\rm dyn} = {} & \mathrm{L}^\dagger\tilde{\sigma}^\mu i\begin{pmatrix} \partial_\mu + \frac{2\mathrm{i}e}{3}A_\mu + \frac{\mathrm{i}e}{3\sin 2\theta_{\rm w}}(1+2\cos 2\theta_{\rm w})Z_\mu, & \frac{\mathrm{i}e}{\sqrt{2}\sin\theta_{\rm w}}W_\mu^+ \\ \frac{\mathrm{i}e}{\sqrt{2}\sin\theta_{\rm w}}W_\mu^-, & \partial_\mu - \frac{\mathrm{i}e}{3}A_\mu - \frac{\mathrm{i}e}{3\sin 2\theta_{\rm w}}(2+\cos 2\theta_{\rm w})Z_\mu \end{pmatrix}\mathrm{L} \\ & + u_{\rm R}^\dagger\sigma^\mu \mathrm{i}\left[\partial_\mu + \frac{2\mathrm{i}e}{3}A_\mu - \frac{2\mathrm{i}e}{3}\tan\theta_{\rm w}Z_\mu\right]u_{\rm R} \\ & + d_{\rm R}^\dagger\sigma^\mu i\left[\partial_\mu - \frac{\mathrm{i}e}{3}A_\mu + \frac{\mathrm{i}e}{3}\tan\theta_{\rm w}Z_\mu\right]d_{\rm R}.\end{aligned} \tag{14.4}$$

However, the Standard Model postulates three families, or generations, of quarks. We therefore introduce *three* left-handed $SU(2)$ doublets:

$$\begin{pmatrix} u_{\rm L1} \\ d_{\rm L1} \end{pmatrix}, \begin{pmatrix} u_{\rm L2} \\ d_{\rm L2} \end{pmatrix}, \begin{pmatrix} u_{\rm L3} \\ d_{\rm L3} \end{pmatrix},$$

and six right-handed singlets: $u_{\rm R1}, d_{\rm R1}; u_{\rm R2}, d_{\rm R2}; u_{\rm R3}, d_{\rm R3}$. For a more compact notation we shall denote these by

$$\mathrm{L}_k = \begin{pmatrix} u_{\rm L k} \\ d_{\rm L k} \end{pmatrix}, \; u_{{\rm R}k}, \; d_{{\rm R}k} \text{ with } k = 1, 2, 3.$$

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As in the lepton case, we take the dynamical part of the total quark Lagrangian as a sum:

$$\mathcal{L}_{\mathrm{dyn}}(\text{quark}) = \sum_{k=1}^{3} \mathcal{L}_{\mathrm{dyn}}(\mathrm{u_k}, \mathrm{d_k}). \qquad (14.5)$$

14.2 Quark masses and the Kobayashi–Maskawa mixing matrix

To retain renormalisability we must retain gauge symmetry, and give mass to the quarks by coupling to the Higgs field as in Chapter 12 where we gave mass to the leptons. For the $\mathrm{d_k}$ quarks this is straightforward. The most general form we might consider that preserves the gauge symmetries is

$$\mathcal{L}_{\mathrm{Higgs}}(\mathrm{d}) = -\sum \left[G^{\mathrm{d}}_{ij}\left(\mathbf{L}^{\dagger}_i \mathbf{\Phi}\right) d_{\mathrm{R}j} + G^{\mathrm{d}*}_{ij} d^{\dagger}_{\mathrm{R}j} (\mathbf{\Phi}^{\dagger}\mathbf{L}_i)\right], \qquad (14.6)$$

as we discussed in the lepton case in Section 12.6. After the symmetry breaking of the Higgs field Φ, this gives the mass term for the d-type quarks:

$$\mathcal{L}_{\mathrm{mass}}(\mathrm{d}) = -\phi_0 \sum \left[G^{\mathrm{d}}_{ij} d^{\dagger}_{\mathrm{L}i} d_{\mathrm{R}j} + G^{\mathrm{d}*}_{ij} d^{\dagger}_{\mathrm{R}j} d_{\mathrm{L}i}\right]. \qquad (14.7)$$

A *priori*, G^{d}_{ij} is an arbitrary 3×3 complex matrix. As we remarked in Section 12.6, such a matrix can always be put into real diagonal form with the help of two unitary matrices, so that we can write

$$\phi_0 \mathbf{G}^{\mathrm{d}} = \mathbf{D}^{\dagger}_{\mathbf{L}} \mathbf{m}^{d} \mathbf{D}_{\mathbf{R}},$$

where $\mathbf{m}^{d}$ is a real diagonal matrix, and $\mathbf{D_L}$, $\mathbf{D_R}$ are unitary matrices. If the diagonal elements are distinct, as appears experimentally to be the case, $\mathbf{D_L}$, $\mathbf{D_R}$ are unique, except that both may be multiplied on the left by the same phase-factor matrix

$$\begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & e^{i\alpha_3} \end{pmatrix}. \qquad (14.8)$$

In the Standard Model as set out in Chapter 12, the neutrinos were taken to have zero mass. However, for the u-type quarks, which are here making up a left-handed doublet, we need a mass term. For this purpose we introduce the 2×2 matrix in $SU(2)$ space

$$\varepsilon = \begin{pmatrix} \varepsilon_{AA} & \varepsilon_{AB} \\ \varepsilon_{BA} & \varepsilon_{BB} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

A suitable $SU(2)$ invariant expression which we can construct from the doublets $\mathbf{\Phi}$ and $\mathbf{L}_i$ is $\left(\mathbf{\Phi}^{\mathrm{T}} \varepsilon\, \mathbf{L}_i\right)$, where $\mathbf{\Phi}^{\mathrm{T}} = (\mathbf{\Phi}_{\mathrm{A}}, \mathbf{\Phi}_{\mathrm{B}})$ is the transpose of $\mathbf{\Phi}$ (Problem 14.3).

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We then take

$$\mathcal{L}_{\text{Higgs}}(u) = -\sum_{ij} \left[G^{\text{u}}_{ij} \left(\mathbf{L}^{\dagger}_i \, \varepsilon \, \boldsymbol{\Phi}^* \right) u_{\text{R}j} - G^{\text{u}*}_{ij} \, u^{\dagger}_{\text{R}j} (\boldsymbol{\Phi}^{\text{T}} \, \varepsilon \, \mathbf{L}_i) \right] \tag{14.9}$$

where G^{u}_{ij} is another complex 3×3 matrix. On symmetry breaking, this gives the u-quarks mass term

$$\mathcal{L}_{\text{mass}}(u) = -\phi_0 \sum \left[G^{\text{u}}_{ij} u^{\dagger}_{\text{L}i} u_{\text{R}j} + G^{\text{u}*}_{ij} u^{\dagger}_{\text{R}j} u_{\text{L}i} \right], \tag{14.10}$$

which is, as we might expect, similar to (14.7), and likewise preserves the gauge symmetries. It can be brought into real diagonal form in a similar way:

$$\phi_0 \mathbf{G}^{\text{u}} = \mathbf{U}_{\text{L}}{}^{\dagger} \mathbf{m}^{\text{u}} \mathbf{U}_{\text{R}},$$

where $\mathbf{U}_{\text{L}}$ and $\mathbf{U}_{\text{R}}$ are unitary matrices, and $\mathbf{m}^u$ is diagonal.

$\mathbf{U}_{\text{L}}$ and $\mathbf{U}_{\text{R}}$ may be both multiplied on the left by a phase factor matrix, say

$$\begin{pmatrix} e^{\text{i}\beta_1} & 0 & 0 \\ 0 & e^{\text{i}\beta_2} & 0 \\ 0 & 0 & e^{\text{i}\beta_3} \end{pmatrix}.$$

The theory is most directly described in terms of the 'true' quark fields, for which the mass matrices are diagonal, so that we define the six quark fields:

$$\begin{aligned} d'_{\text{L}i} &= D_{\text{L}ij} d_{\text{L}j}, & d'_{\text{R}i} &= D_{\text{R}ij} d_{\text{R}j}, \\ u'_{\text{L}i} &= U_{\text{L}ij} u_{\text{L}j}, & u'_{\text{R}i} &= U_{\text{R}ij} u_{\text{R}j}. \end{aligned} \tag{14.11}$$

The quark mass contribution to $\mathcal{L}$ becomes:

$$\mathcal{L}_{\text{mass}}(\text{quarks}) = -\sum_{i=1}^{3} \left[m^{\text{d}}_i \left(d'^{\dagger}_{\text{L}i} d'_{\text{R}i} + d'^{\dagger}_{\text{R}i} d'_{\text{L}i} \right) + m^{\text{u}}_i \left(u'^{\dagger}_{\text{L}i} u'_{\text{R}i} + u'^{\dagger}_{\text{R}i} u'_{\text{L}i} \right) \right]. \tag{14.12a}$$

We identify the Dirac spinors

$$\begin{pmatrix} u'_{\text{L}1} \\ u'_{\text{R}1} \end{pmatrix}, \begin{pmatrix} u'_{\text{L}2} \\ u'_{\text{R}2} \end{pmatrix}, \begin{pmatrix} u'_{\text{L}3} \\ u'_{\text{R}3} \end{pmatrix}$$

with the u, c and t quarks, respectively, and the Dirac spinors

$$\begin{pmatrix} d'_{\text{L}1} \\ d'_{\text{R}1} \end{pmatrix}, \begin{pmatrix} d'_{\text{L}2} \\ d'_{\text{R}2} \end{pmatrix}, \begin{pmatrix} d'_{\text{L}3} \\ d'_{\text{R}3} \end{pmatrix}$$

with the d, s and b quarks, so that we might rewrite (14.12a) as

$$\begin{aligned} \mathcal{L}_{\text{mass}}(\text{quarks}) = & -\left[m^d \left(d^{\dagger}_{\text{L}} d_{\text{R}} + d^{\dagger}_{\text{R}} d_{\text{L}} \right) + m^u \left(u^{\dagger}_{\text{L}} u_{\text{R}} + u^{\dagger}_{\text{R}} u_{\text{L}} \right) \right] \\ & -\left[m^s \left(s^{\dagger}_{\text{L}} s_{\text{R}} + s^{\dagger}_{\text{R}} s_{\text{L}} \right) + m^c \left(c^{\dagger}_{\text{L}} c_{\text{R}} + c^{\dagger}_{\text{R}} c_{\text{L}} \right) \right] \\ & -\left[m^b \left(b^{\dagger}_{\text{L}} b_{\text{R}} + b^{\dagger}_{\text{R}} b_{\text{L}} \right) + m^t \left(t^{\dagger}_{\text{L}} t_{\text{R}} + t^{\dagger}_{\text{R}} t_{\text{L}} \right) \right]. \end{aligned} \tag{14.12b}$$

The terms in (14.12b) correspond to six Dirac fermions.

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We have dropped the primes, and for the remainder of the book u_k and d_k, for $k = 1, 2, 3$, will denote true quark fields.

In the $\mathcal{L}_{\text{dyn}}$ given by (14.2) and (14.5), the 'diagonal' terms do not mix u-type and d-type quarks and are invariant under the unitary transformations (14.11). However, the terms that arise from the off-diagonal elements of the matrix $\mathbf{W}_\mu$, mix u and d quarks through their coupling to the $W^\pm$ boson fields, and these terms are profoundly changed.

The diagonal terms give $\mathcal{L}_{q\text{Dirac}}$ and $\mathcal{L}_{qz}$ that parallel the expressions (12.12) and (12.23) of the lepton theory of Chapter 12. The complete electroweak Lagrangian density for the quarks is

$$\mathcal{L}_q = \mathcal{L}_{q\text{Dirac}} + \mathcal{L}_{q\text{z}} + \mathcal{L}_{q\text{w}} + \mathcal{L}_{q\text{H}}$$

where

$$\begin{aligned}\mathcal{L}_{q\text{Dirac}} = \sum_i &\left[u^\dagger_{\text{L}i}\tilde{\sigma}^\mu \text{i}\{\partial_\mu + \text{i}(2e/3)A_\mu\}u_{\text{L}i} + u^\dagger_{\text{R}i}\sigma^\mu \text{i}\{\partial_\mu + \text{i}(2e/3)A_\mu\}u_{\text{R}i}\right] \\ &+ \left[d^\dagger_{\text{L}i}\tilde{\sigma}^\mu \text{i}\{\partial_\mu - \text{i}(e/3)A_\mu\}d_{\text{L}i} + d^\dagger_{\text{R}i}\sigma^\mu \text{i}\{\partial_\mu - \text{i}(e/3)A_\mu\}d_{\text{R}i}\right] + \mathcal{L}_{q\text{mass}}\end{aligned} \tag{14.13}$$

$$\begin{aligned}\mathcal{L}_{qz} = \sum_i \Bigg[&-u^\dagger_{\text{L}i}\tilde{\sigma}^\mu u_{\text{L}i}\left(\frac{e}{\sin(2\theta_\text{w})}\right) Z_\mu(1 - (4/3)\sin^2\theta_\text{w}) \\ &+ u^\dagger_{\text{R}i}\sigma^\mu u_{\text{R}i}\left(\frac{e}{\sin(2\theta_\text{w})}\right) Z_\mu \frac{4}{3}\sin^2\theta_\text{w} \\ &+ d^\dagger_{\text{L}i}\tilde{\sigma}^\mu d_{\text{L}i}\left(\frac{e}{\sin(2\theta_\text{w})}\right) Z_\mu(1 - (2/3)\sin^2\theta_\text{w}) \\ &- d^\dagger_{\text{R}i}\sigma^\mu d_{\text{R}i}\left(\frac{e}{\sin(2\theta_\text{w})}\right) Z_\mu \frac{2}{3}\sin^2\theta_\text{w}\Bigg].\end{aligned} \tag{14.14}$$

In the $\mathcal{L}_{q\text{w}}$, part of the Lagrangian density, the terms

$$-\frac{e}{\sqrt{2}\sin\theta_\text{w}}\sum_i \left[u^\dagger_{\text{L}i}\tilde{\sigma}^\mu d_{\text{L}i}W^+_\mu + d^\dagger_{\text{L}i}\tilde{\sigma}^\mu u_{\text{L}i}W^-_\mu\right],$$

when written in terms of the 'true' quark fields given by (14.11), become

$$\begin{aligned}\mathcal{L}_{q\text{w}} = -\frac{e}{\sqrt{2}\sin\theta_\text{w}}&\left(u^\dagger_\text{L}, c^\dagger_\text{L}, t^\dagger_\text{L}\right)\begin{pmatrix} V_{\text{ud}} & V_{\text{us}} & V_{\text{ub}} \\ V_{\text{cd}} & V_{\text{cs}} & V_{\text{cb}} \\ V_{\text{td}} & V_{\text{ts}} & V_{\text{tb}}\end{pmatrix}\begin{pmatrix}\tilde{\sigma}^\mu d_\text{L} \\ \tilde{\sigma}^\mu s_\text{L} \\ \tilde{\sigma}^\mu b_\text{L}\end{pmatrix} W^+_\mu \\ &+ \text{Hermitian conjugate},\end{aligned} \tag{14.15}$$

where $\mathbf{V} = \mathbf{U}_\text{L}\mathbf{D}^\dagger_\text{L}$.

Since the product of two unitary matrices is unitary, $\mathbf{V}$ is a 3×3 unitary matrix. The elements of $\mathbf{V}$ are not determined within the theory. It is in this matrix that another four of the parameters of the Standard Model reside. An

$n \times n$ unitary matrix is specified by n^2 parameters (Appendix A), so we apparently have nine parameters to be measured experimentally. However, five of these can be absorbed into the non-physical phases of the quark fields, through the phase-factor matrices associated with $\mathbf{D}_{\mathrm{L}}$ (see (14.8)) and $\mathbf{U}_{\mathrm{L}}$. (There are five, rather than six, non-physical phases since only phase differences appear in $\mathbf{V}$. For example $V_{\mathrm{ud}} = \exp[\mathrm{i}(\beta_{\mathrm{u}} - \alpha_{\mathrm{d}})]\, V^0_{\mathrm{ud}}$.)

When the quark phase factors have been extracted, the resulting matrix $\mathbf{V}^0$ is dependent on four physical parameters. It is called the *Kobayashi–Maskawa* (*KM*) *matrix* (Kobayashi and Maskawa, 1973).

14.3 The parameterisation of the KM matrix

A 3×3 rotation matrix is also a unitary matrix. A more general unitary matrix can be constructed as a product of rotation matrices and unitary matrices made up of phase factors. There is no unique parameterisation of the KM matrix by this method. That advocated by the Particle Data Group is

$$\begin{aligned}\mathbf{V} = &\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} \mathrm{e}^{-\mathrm{i}\delta/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{\mathrm{i}\delta/2} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} \\ 0 & 1 & 0 \\ -s_{13} & 0 & c_{13} \end{pmatrix} \\ &\times \begin{pmatrix} \mathrm{e}^{\mathrm{i}\delta/2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \mathrm{e}^{-\mathrm{i}\delta/2} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ = &\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-\mathrm{i}\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta} & c_{23}c_{13} \end{pmatrix}\end{aligned} \tag{14.16}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$. The four parameters are the three rotation angles $\theta_{12}, \theta_{23}, \theta_{13}$, and the phase δ.

Evidently, if $s_{13} = 0$ or $\sin\delta = 0$ then $\mathbf{V}$ is real. Less evidently, if $s_{12} = 0$ then $\mathbf{V}$ is made real by redefining the quark fields

$$\mathrm{e}^{\mathrm{i}\delta}u_1 \to u_1, \quad \mathrm{e}^{\mathrm{i}\delta}d_1 \to d_1,$$

and if $s_{23} = 0$ then $\mathbf{V}$ is made real by redefining

$$\mathrm{e}^{-\mathrm{i}\delta}u_3 \to u_3, \quad \mathrm{e}^{-\mathrm{i}\delta}d_3 \to d_3,$$

as the reader may verify.

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A general redefinition of the quark phases,

$$d_i \to e^{i\alpha_i} d_i,\ u_i \to e^{i\beta_i} u_i,$$

will change the matrix elements of **V** by

$$V_{ij} \to e^{i(\alpha_i - \beta_j)} V_{ij}. \tag{14.17}$$

Using this freedom, the three rotation angles can be chosen all to lie in the first quadrant.

Jarlskog (1985) gives an important necessary and sufficient condition for determining whether, given a unitary matrix **V**, it is possible to make it real by such changes. She considers the imaginary part of any one of the nine products, $V_{ij} V_{kl} V_{kl}^* V_{il}^*$ with $i \neq k$ and $j \neq l$, for example

$$\mathrm{Im}\left(V_{11} V_{22} V_{21}^* V_{12}^*\right) = J \text{ say}. \tag{14.18}$$

J is invariant under a general phase change (14.17), so that if J is not zero then it cannot be made so, and hence **V** cannot be made real. All nine quantities are equal to $\pm J$. In the parameterisation of equation (14.16),

$$J = c_{12} c_{13}^2 c_{23} s_{12} s_{13} s_{23} \sin\delta. \tag{14.19}$$

(The conditions already obtained for the reality of the KM matrix are contained in the condition $J = 0$.)

Having fixed the KM matrix there remains only one global $U(1)$ symmetry which leaves it unchanged. All six quark fields, left and right, can be multiplied by the same phase factor. As a consequence, only the total quark number current and hence the total quark number is conserved. At the macroscopic level this is observed as baryon number conservation.

14.4 *CP* symmetry and the KM matrix

We shall now show that, if the KM matrix cannot be made real by a redefinition of the quark phases, the Standard Model does not have *CP* (change conjugation, parity) symmetry.

We saw in Section 12.5 that the Weinberg–Salam electroweak theory is invariant under the *CP* operation. Similarly, *CP* is a symmetry of every term in the Standard Model of the weak and electromagnetic interactions of quarks, except for those terms that give the interaction between the quarks and the W bosons. These are the terms that involve the KM matrix.

The *CP* transforms of the *W* fields are defined in equation (12.32):

$$W_0^{+CP} = -W_0^-,\ \ W_i^{+CP} = W_i^-,$$

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and the quark fields transform like all fermion fields:

$$q_{\mathrm{L}}^{CP} = -i\sigma^2 q_{\mathrm{L}}^*, \; q_{\mathrm{R}}^{CP} = i\sigma^2 q_{\mathrm{R}}^*.$$

To show how *CP* symmetry is violated, we consider the terms (14.15), which we write as

$$(-e/\sqrt{2}\sin\theta_{\mathrm{w}}) \sum_{i,j} \left[u_{\mathrm{L}i}^\dagger \tilde{\sigma}^\mu V_{ij} d_{\mathrm{L}j} W_\mu^+ + d_{\mathrm{L}j}^\dagger \tilde{\sigma}^\mu V_{ij}^* u_{\mathrm{L}i} W_\mu^- \right]$$
$$(i = \mathrm{u, c, t};\; j = \mathrm{d, s, b})$$

Replacing the fields by their CP transforms gives

$$(-e/\sqrt{2}\sin\theta_{\mathrm{w}}) \sum_{ij} \left[-u_{\mathrm{L}i}^{\mathrm{T}} (\tilde{\sigma}^\mu)^{\mathrm{T}} V_{ij} d_{\mathrm{L}j}^* W_\mu^- - d_{\mathrm{L}j}^{\mathrm{T}} (\tilde{\sigma}^\mu)^{\mathrm{T}} V_{ij}^* u_{\mathrm{L}i}^* W_\mu^+ \right]$$

where, as in Section 12.5, we have used the results

$$(\sigma^2)^2 = 1, \quad \sigma^2 \sigma^i \sigma^2 = -(\sigma^i)^{\mathrm{T}}.$$

On transposing this expression with respect to the spinor indices we introduce a minus sign from the anticommuting fermion fields, and obtain the CP transformed expression

$$(-e/\sqrt{2}\sin\theta_{\mathrm{w}}) \sum_{i,j} \left[d_{\mathrm{L}j}^\dagger \tilde{\sigma}^\mu V_{ij} u_{\mathrm{L}i} W_\mu^- + d_{\mathrm{L}i}^\dagger \tilde{\sigma}^\mu V_{ij}^* d_{\mathrm{L}j} W_\mu^+ \right].$$

This is the same as the original term if and only if V_{ij} is real for all i, j.

Experimental evidence for the breakdown of *CP* symmetry first became apparent in 1964, in the decay of the K^0 ($\mathrm{d\bar{s}}$) meson. We shall discuss this decay and its implications in Chapter 18, where we consider what is known experimentally about the parameters of the KM matrix. It is an interesting fact that *CP*-violating effects in the Standard Model are proportional to *J*.

14.5 The weak interaction in the low energy limit

Combining the results of Chapter 12 (equation (12.18)) with those of the present chapter (equation (14.15)), we have the complete interaction of the W bosons with all the fermions, both leptons and quarks, of the Standard Model:

$$\mathcal{L}_{\mathrm{Wint}} = (-e/\sqrt{2}\sin\theta_{\mathrm{w}}) \left[j^{\mu\dagger} W_\mu^+ + j^\mu W_\mu^- \right]$$

where

$$j^\mu = \sum_{\mathrm{leptons}} e_{\mathrm{L}l}^\dagger \tilde{\sigma}^\mu \nu_{\mathrm{L}l} + \sum_{ij} d_{\mathrm{L}j}^\dagger \tilde{\sigma}^\mu u_{\mathrm{L}i} V_{ij}^* \; (i = \mathrm{u, c, t};\; j = \mathrm{d, s, b}). \tag{14.20}$$

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Note that we have suppressed colour indices in this chapter. The labels *i,j* on the quark spinors in (14.15) carry with them implied colour indices which are also summed over.

By eliminating the *W* field as in Section 12.2, we obtain the low energy effective interaction

$$\mathcal{L}_{\mathrm{Weff}} = -2\sqrt{2} G_F j_\mu^\dagger j^\mu. \tag{14.21}$$

For example, the part of this effective interaction which is basically responsible for all nuclear β decays involves the electron field and the u and d quarks ($i = j = 1$):

$$\mathcal{L}_{\mathrm{eff}} = -2\sqrt{2} G_{\mathrm{F}} \left[g_{\mu\nu} v_{\mathrm{eL}}^\dagger \tilde{\sigma}^\mu e_{\mathrm{L}} d_{\mathrm{L}}^\dagger \tilde{\sigma}^\nu u_{\mathrm{L}} V_{\mathrm{ud}}^* \right] + \text{Hermitian conjugate} \tag{14.22}$$

That part of the effective interaction responsible for the decay $\mathrm{K}^0 \to \pi^+\pi^-$ $\left(\bar{s} \to \mathrm{u} + \bar{\mathrm{u}} + \bar{\mathrm{d}}\right)$ is

$$\mathcal{L}_{\mathrm{eff}} = -2\sqrt{2} G_{\mathrm{F}} \left[g_{\mu\nu} s_{\mathrm{L}}^\dagger \tilde{\sigma}^\mu u_{\mathrm{L}} u_{\mathrm{L}}^\dagger \tilde{\sigma}^\nu d_{\mathrm{L}}\, V_{\mathrm{us}}^* V_{\mathrm{ud}} \right]. \tag{14.23}$$

We have also the complete interaction of the Z boson with all the fermions. Combining (12.23) with (14.14) gives

$$\mathcal{L}_{\mathrm{Zint}} = \frac{-e}{\sin(2\theta_{\mathrm{w}})} (j_{\mathrm{neutral}})_\mu Z^\mu \tag{14.24}$$

where

$$\begin{aligned}(j_{\mathrm{neutral}})^\mu = & \sum_{\mathrm{leptons}} \left[\nu_{\mathrm{L}l}^\dagger \tilde{\sigma}^\mu \nu_{\mathrm{L}l} - \cos(2\theta_{\mathrm{w}}) e_{\mathrm{L}l}^\dagger \tilde{\sigma}^\mu e_{\mathrm{L}l} \right. \\ & \left. + 2\sin^2\theta_{\mathrm{w}} e_{\mathrm{R}}^\dagger \sigma^\mu e_R \right] \\ & + \sum_i \left[u_{\mathrm{L}i}^\dagger \tilde{\sigma}^\mu u_{\mathrm{L}i} \left(1 - \frac{4}{3}\sin^2\theta_{\mathrm{w}}\right) - u_{\mathrm{R}i}^\dagger \sigma^\mu u_{\mathrm{R}i} \left(\frac{4}{3}\sin^2\theta_{\mathrm{w}}\right) \right. \\ & \left. - d_{\mathrm{L}i}^\dagger \tilde{\sigma}^\mu d_{\mathrm{L}i} \left(1 - \frac{2}{3}\sin^2\theta_{\mathrm{w}}\right) + d_{\mathrm{R}i}^\dagger \sigma^\mu d_{\mathrm{R}i} \left(\frac{2}{3}\sin^2\theta_{\mathrm{w}}\right) \right].\end{aligned}$$

By eliminating the Z field, we obtain the low energy effective interaction

$$\mathcal{L}_{\mathrm{Zeff}} = -\left(G_{\mathrm{F}}/\sqrt{2}\right) (j_{\mathrm{neutral}})_\mu (j_{\mathrm{neutral}})^\mu. \tag{14.25}$$

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Problems

14.1 Verify that the transformations (14.3) along with (11.4b) and (11.6) leave $\mathcal{L}_{\text{dyn}}$ invariant.

14.2 Obtain $\mathcal{L}_{\text{qZ}}$, (equation (14.14)) from (14.4).

14.3 Show that $(\boldsymbol{\Phi}\, \varepsilon\, L)$ is an $SU(2)$ invariant. (Show that $U^{\mathrm{T}}\, \varepsilon\, U = \varepsilon\, \det(U)$)

14.4 Write down the interaction Lagrangian density between the quark fields and the Higgs field, which appears in (14.6) and (14.9).

Estimate the coupling constant c_{t} between the Higgs field and the top quark.

14.5 Which terms in (14.20) and (14.21) are responsible for the meson decays

$$\begin{aligned}
&\mathrm{K}^{+}\,(\mathrm{u}\bar{\mathrm{s}}) \rightarrow \mu^{+} + \nu_{\mu},\\
&\mathrm{D}^{+}\left(\mathrm{c}\bar{\mathrm{d}}\right) \rightarrow \overline{\mathrm{K}^{0}}\left(\bar{\mathrm{d}}\mathrm{s}\right) + \mathrm{e}^{+} + \nu_{\mathrm{e}},\\
&\mathrm{B}^{+}\left(\mathrm{u}\bar{\mathrm{b}}\right) \rightarrow \overline{\mathrm{D}^{0}}\,(\bar{\mathrm{c}}\mathrm{u}) + \pi^{+}\left(\mathrm{u}\bar{\mathrm{d}}\right)?
\end{aligned}$$

Sketch appropriate quark diagrams.

14.6 There are no ‘flavour changing neutral currents’, i.e. there are no terms in the neutral current of (14.24) that involve a change of quark flavour. Draw Feynman diagrams from higher orders of perturbation theory that simulate the flavour changing neutral current decays

$$\mathrm{b} \rightarrow \mathrm{s} + \gamma, \quad \mathrm{b} \rightarrow \mathrm{s} + \mathrm{e}^{+} + \mathrm{e}^{-}.$$