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**Letter to the Editor**

# Ion holes in dusty pair plasmas

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**Abstract.** The existence of Bernstein–Greene–Kruskal (BGK)-like trapped ion modes in dusty plasmas is investigated by means of the pseudo-potential method applied to the Vlasov–Poisson system. The nonlinear dispersion relation, determining the phase velocity, and the pseudo-potential, representing the wave form and hence its spectral decomposition, are derived and analysed with respect to the effect of dust. Dust is found to diminish the region in $\omega, k$-space, where periodic wave solutions of fast and slow mode character exist. Localized wave solutions in the form of solitary ion holes, owing their existence to the presence of dust, coexist as well, but turn into slow ion acoustic double layers in the limit of vanishing dust.

## 1. Introduction

One of the main challenges in plasma theory is the failure of linear transport models (or nonlinearly extended models relying in lowest order on linear wave theory such as weak or strong turbulence models) to describe wave-driven plasma dynamics and anomalous transport [1–5]. A striking example is the quasilinear theory for the bump-on-tail instability, which has been shown to be incomplete due to the neglect of mode coupling and trapping nonlinearity [6–8] and which hence misses an important ingredient in plasma dynamics, the formation of coherent kinetic structures. Another example is the destabilization of linearly stable, current-carrying plasmas through the spontaneous generation and growth of negative energy phase space holes, which cannot be explained by contemporary plasma wave theory due to the neglect of nonlinearity to lowest order (see [9–12] and references therein). This annoying fact, which is more problematic the lower the collisionality is [13], therefore calls for models in which no reference to linearized wave theory is made. This implies that for plasmas in the collisionless limit, nonlinearity has to be fully taken into account in the kinetic description and that plasmas of simplified composition, such as pair plasmas, are most suitable for making further progress in the understanding of plasmas. Some steps in this direction have already been made in [14–16]. In the first two papers, ultra-low-frequency electrostatic modes within the Vlasov–Poisson description have been found and analysed, which owe their existence to the trapping particles in the potential trough of the wave, namely the trapping of charged dust particles in the so-called trapped dust acoustic wave (TDAW) for ordinary plasmas in [14] and of ions in trapped ion waves (periodic phase space holes and double layers, the latter under symmetric ion conditions) for pair plasmas in [15]. In [16] the search for trapped ion modes in pair plasmas has been generalized by allowing different trapped ion conditions and the extension to finite amplitudes proving the existence of solitary phase space holes in pair plasmas. Comprehensive reviews of phase space holes and related structures, pointing out their proper analytic description, their observation under different plasma environments (such as extraterrestrial plasmas and particle accelerators), their generalization to magnetized, quantized and relativistic plasmas, their mutual interaction and impact on plasma (and fluid) dynamics, etc., can be found in [5, 17, 18].

In the present paper we enrich the model in [15] by including a dust component and analyse its effect on its particular trapped ion modes.

## 2. Densities

We look for equilibria of the Vlasov–Poisson system composed of the three plasma species: positive ions, negative ions and negatively charged dust particles and adopt the distribution ansatz of [19], based on Maxwellians in the unperturbed region. By a velocity integration we then obtain the density expression for each species, which in the limit of equal temperature and trapping conditions of the two ion species in the small amplitude limit $0 \leqslant \Phi \leqslant \Psi \ll 1$ becomes

$$
\begin{aligned}
n_+(\Phi) &= \nu_+(1+K_+)n_0(v_0, \beta, \Psi - \Phi), \\
n_-(\Phi) &= \nu_-(1+K_-)n_0(v_0, \beta, \Phi), \\
n_\mathrm{d}(\Phi) &= \nu_\mathrm{d}(1+K_\mathrm{d})n_0(v_0\sqrt{\theta_\mathrm{d} m_\mathrm{d}/m}, \beta_\mathrm{d}, Z_\mathrm{d}\theta_\mathrm{d}\Phi)
\end{aligned}
\tag{1}
$$

where $\nu$ and $K$ represent normalization constants, $Z_\mathrm{d}$ represents the charge of a dust grain, $\beta_\mathrm{d}$ is the trapping parameter for dust particles, $\theta_\mathrm{d} = T/T_\mathrm{d}$, $T$ being the temperature of both ions, and for normalization we use the positive ion quantities at $\Phi = \Psi$, that is, where the trapping of positive ions is absent. Later, dust trapping will be neglected by assuming phase velocities $v_0$ in the ion thermal range. In the above expressions the common function $n_0(v_0, \beta, \Phi)$ is defined by

$$
n_0(v_0, \beta, \Phi) = 1 + a\Phi - \tfrac{4}{3}b(\beta, v_0)\Phi^{\frac{3}{2}} + \cdots, \tag{2}
$$

with

$$
a = -\frac{1}{2}Z_\mathrm{r}'\left(\frac{v_0}{\sqrt{2}}\right), \tag{3a}
$$

$$
b(\beta, v_0) = \frac{1}{\sqrt{\pi}}\left(1 - \beta - v_0^2\right)\exp\left(-\frac{v_0^2}{2}\right), \tag{3b}
$$

where $Z_\mathrm{r}$ is the real part of the plasma dispersion function. Note that in view of $v_0 \simeq O(1)$ and of the large dust mass $m \ll m_\mathrm{d}$, the term $-\frac{1}{2}Z_\mathrm{r}'(v_0\sqrt{\theta_\mathrm{d} m_\mathrm{d}}/\sqrt{2m})$ is negligible in the third equation of (1), in addition to the dust trapping term, such that the dust particles can be treated as immobile further on.

## 3. Normalization

Charge neutrality in the completely unperturbed plasma, $\Psi \equiv 0$, noting that the $K$ are of the order of $O(\Psi)$, requires

$$
\nu_- + Z_\mathrm{d}\nu_\mathrm{d} = \nu_+ = 1. \tag{4a}
$$

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If we assume that $K_+ = 0$ describes a spatially localized state with asymptotic charge neutrality, that is, with $\Phi = \Psi$ at infinity, then the normalization condition of $O(\Psi)$ becomes

$$\nu_- K_- + (1 - \nu_-) K_{\rm d} = \nu_-(B - a)\Psi \tag{4b}$$

where we have defined

$$B = \tfrac{4}{3} b(\beta, v_0)\sqrt{\Psi}. \tag{5}$$

Spatially periodic waves are admitted by letting $K_+ = (k_0^2/2)\Psi$, where $k_0$ is related with the actual wavenumber $k$ (see [15]). For the densities we therefore obtain

$$n_+(\Phi) = 1 + \frac{k_0^2}{2}\Psi + a(\Psi - \Phi) - \frac{B}{\sqrt{\Psi}}(\Psi - \Phi)^{\frac{3}{2}} + \cdots, \tag{6a}$$

$$n_-(\Phi) = \nu_-\left[1 + K_- + a\Phi - \frac{B}{\sqrt{\Psi}}\Phi^{\frac{3}{2}} + \cdots\right]. \tag{6b}$$

The dust density is then given by the constant value $n_{\rm d} = (1 - \nu_-)(1 + K_{\rm d})/Z_{\rm d}$ where $K_-$ and $K_{\rm d}$ are related through (4b).

## 4. Nonlinear dispersion relation

From Poisson's equation, written as $\Phi''(x) = n_- + Z_{\rm d} n_{\rm d} - n_+ =: -V'(\Phi)$, by an integration we find the 'classical' or pseudo-potential $V(\Phi)$, assuming $V(0) = 0$,

$$-V(\Phi) = -\frac{k_0^2}{2}\Psi\Phi + (1 + \nu_-)a(\Phi^2/2 - \Psi\Phi) + \frac{B}{\sqrt{\Psi}}\left\{\nu_-\left(\Phi\Psi^{\frac{3}{2}} - \frac{2}{5}\Phi^{\frac{5}{2}}\right) + \frac{2}{5}\left[\Psi^{\frac{5}{2}} - (\Psi - \Phi)^{\frac{5}{2}}\right]\right\}. \tag{7}$$

Bounded, and hence physically meaningful solutions, are obtained by the condition $V(\Psi) = 0$, which, in view of the pseudo-energy relation $\Phi'(x)^2/2 + V(\Phi) = 0$, implies zero electric field at the potential maximum. It becomes

$$\frac{k_0^2}{2} - \frac{(1 + \nu_-)}{2}\frac{1}{2}Z_{\rm r}'\left(\frac{v_0}{\sqrt{2}}\right) = \frac{(2 + 3\nu_-)}{5}B. \tag{8}$$

This is the celebrated nonlinear dispersion relation as it determines the up to now unknown phase speed $v_0$ in terms of the other (free) parameters. Replacing $a$, given by (3a), in (7) by (8) and performing a renormalization, $\phi = \Phi/\Psi$ and $\mathscr{V}(\phi) = V(\Phi)/\Psi^2$, we obtain

$$-\mathscr{V}(\phi) = \frac{k_0^2}{2}\phi(1 - \phi) + \frac{B}{5}\{2[1 - \nu_-\phi^{\frac{5}{2}} - (1 - \phi)^{\frac{5}{2}}] - \phi[(4 + \nu_-) - (2 + 3\nu_-)\phi]\}. \tag{9}$$

This expression together with the 'classical' energy law $\phi'^2(x)/2 + \mathscr{V}(\phi) = 0$ determines by a quadrature the electrostatic potential $\phi(x)$ and, hence, its spatial wave form (or spectral decomposition). Equations (8) and (9) are our main results from which the effect of dust on the wave characteristics can be evaluated.

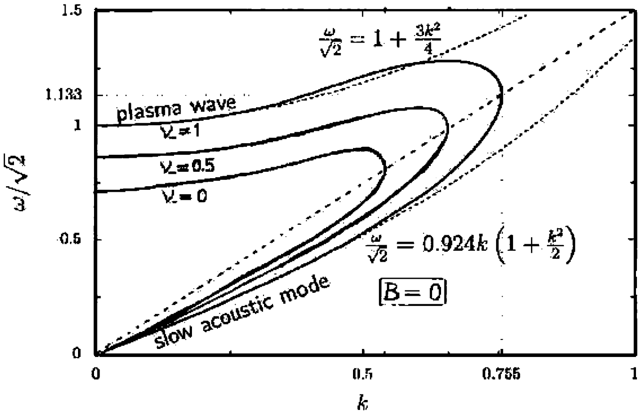


**Figure 1.** $\omega$ as function of $k$ for harmonic waves: $B = 0$.

## 5. Effect of dust on wave propagation

First, in the dust-free limit $\nu_- \to 1$, we recognize the corresponding equations, (7) and (9), respectively, of [15] and the results as exhibited in Figs 2–4 of that paper. For a given wavenumber $k$ (respectively, quasi-wavenumber $k_0$; the difference between them is explained in [15]) a fast and a slow wave solution $\omega_{\mathrm{f}}$ and $\omega_{\mathrm{s}}$, respectively, are found. In the harmonic limit, $B \to 0$; when, as seen from [15, (9)], a single sinusoidal wave exists, the fast wave is an ordinary ion plasma wave in the long wave limit. The slow mode, on the other hand, being of acoustic type, $\omega \sim k$, has nothing to do with an ordinary ion acoustic wave, as is often believed in the literature (the latter being strongly Landau damped in a pair plasma and cannot, therefore, exist), but is the so-called slow ion acoustic mode, a nonlinear mode, which owes its existence to the trapping of ions [20]. For $B \neq 0$, as seen from [15, Fig. 3], we find characteristic deformations of the nonlinear dispersion relation $\omega_0(k_0, B)$. Most significant in the case of $B > 0$ is a cut-off in the quasi-wavenumber $k_0(B)$ below which no solution exists, whereas for $B < 0$ the fast mode becomes acoustic-like in the long wavelength limit. Whereas $k_0 \neq 0$ represents a periodic 'cnoidal' wave solution, a name that is reminiscent of Jacobian elliptic functions, such as the cn-function, and is composed of an infinite wave spectrum [4], the solution for $k_0 = 0$ represents a localized double layer propagating on the slow branch only and is hence referred to the 'slow ion acoustic double layer' (SIADL) [21].

The effect of dust is now seen as follows. Fig. 1 shows the solution of the nonlinear dispersion relation (8) for three values of $\nu_-$ in the harmonic case $B = 0$, $\nu_- = 1$ representing no dust and $\nu_- = 0$ representing the absence of a negative ion component. An increase in dust density leads to a reduction in $\omega$–$k$ space in which the solution lies. The frequency of the ion plasma wave in the long wavelength becomes $\sqrt{1 + \nu_-}$ and, hence, diminishes with decreasing negative ions. For $\nu_- = 0$, it is given by unity, which physically means that only the single positive ion component takes part in the oscillations necessary for wave propagation resulting in the usual plasma frequency.

The influence of dust in the case of a cnoidal wave is seen in Fig. 2 for the special value of $B = 0.01$ and three values of $\nu_-$. Again, the region of possible solutions of (8) shrinks when the fraction of dust in the negatively charge background

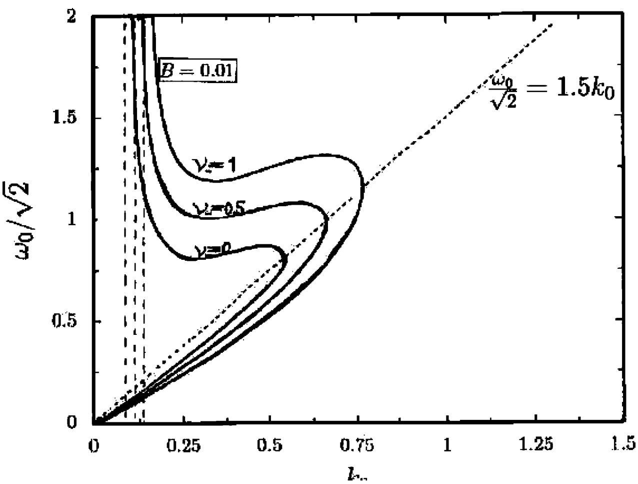


**Figure 2.** $\omega_0$ as a function of $k_0$ for periodic cnoidal waves for three values of $\nu_-$ and $B = 0.01$.

increases. Also a shift of the cut-off quasi-wavenumber in dependence of $\nu_-$ is seen, which is given by

$$k_0(B, \nu_-) = \sqrt{\frac{2(2+3\nu_-)B}{5}}.$$

The turning point, where fast and slow solutions coincide, is given by

$$k_0 = \sqrt{(1+\nu_-)0.285 + \frac{2(2+3\nu_-)B}{5}}.$$

For larger $k_0$ no solution exists. Of special interest are spatially localized solutions, which are obtained in the limit $k_0 \to 0$. From (9) it immediately follows that

$$\mathscr{V}(0) = \mathscr{V}(1) = 0 \quad \mathscr{V}'(0) = -B(1-\nu_-)/5 \quad \mathscr{V}'(1) = 0$$

$$\mathscr{V}''(0) = (7-12\nu_-)B/10 \quad \mathscr{V}''(1) = -(8-3\nu_-)B/10.$$

For a solution to exist $\mathscr{V} < 0$ must hold in $0 < \phi < 1$ and, hence, $B > 0$. This gives rise to a solitary ion hole solution for $\nu_- < 1$, which turns into a double layer solution in the dust-free limit. Through $\nu_-$, the symmetry in $\mathscr{V}(\phi)$ with respect to $\phi \to 1-\phi$ is broken. The solution to the corresponding nonlinear dispersion relation (8) shows that only the slow branch can admit solitary ion holes. Since the quantity $a = -\frac{1}{2}Z_r'(v_0/\sqrt{2})$ has a maximum value of one for $v_0 = 0$, an upper bound for $B$ exists, $B < 5(1+\nu_-)/2(2+3\nu_-)$, beyond which no solitary wave propagation is possible. The values $B > 0$ and $0 < v_0 < 1.307$, furthermore, imply through (3b) and (5) that the trapping parameter $\beta$ is strictly negative. The members of this three-parametric $(k_0, B, \nu_-)$ class of solutions are, therefore, phase space holes (solitary or a train of periodic holes) with a deficit of particles in the trapped ('resonant') particle region in velocity space.

## 6. Conclusions

An important issue in plasma physics is the experimental accessibility and observation of trapped ion modes especially in view of the fact that the charging of dust particles requires free electrons [22], which have been neglected in the present model. One way to address this is given by the experimental setup of [23–25]. In their device, the pair-plasma constituents are produced through the process of electron-impact ionization and electron attachment in the hollow electron-beam region in the outer region of the cylindrical vessel, which then penetrate by ambipolar diffusion across the magnetic field into the central part of the vessel in which electrons are absent due to the magnetic filtering. If, therefore, dust can be added in the outer region, it will be charged negatively by the free electrons existing there, participate in the diffusion process and establish a quasi-neutral dusty pair plasma in the central target chamber where collective processes are excited by biasing cylindrical and grid electrodes. Indeed, the first experiments performed with this device without dust [24, 25] yield electrostatic waves, which show all the signatures of trapped ion modes in the harmonic ($B=0$) and non-harmonic ($B<0$) wave limit, respectively. One can distinguish between the slow ion acoustic wave branch (note that the ordinary ion acoustic wave branch cannot be made responsible for this branch because of its non-existence in an electron-free pair plasma), the so-called 'intermediate frequency wave' branch and the ion plasma wave branch. Theoretically these modes are discriminated between by different values of $k_0$ (hence, $v_0$ and $\beta$) with $B = 0$ and ($B<0$), respectively, with $\nu_- = 1$ (see [15, Fig. 2]). Later, through a change of the excitation method [26], modes could be detected [26, Figs 6 and 11] that stretch over into the frequency domain in a narrow band of $k$ values very similar to the non-harmonic branches $B > 0$ of [15, Fig. 3], a property that is clearly not covered by an ordinary ion plasma mode. Therefore, if dust is added, one could detect not only the dust induced shift of the plasma frequency (and, hence, the portion of dust in the negative charge carriers) in the harmonic wave limit (Fig. 1), but also the dust-dependent shift of the cut-off wavenumber (Fig. 2) in the case of cnoidal waves, $B > 0$. In both cases, information about the trapping states could be extracted via (3b) and (5).

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