interpolation and Chapters 10 and 11 on spherical wavelet approximation. In between these there are Chapter 7, which treats numerical integration (including rules generated from integrating either polynomial or spline interpolants), Chapter 8 on singular integrals (used for generating certain types of theoretical approximation) and Chapter 9 on the Gabor and Toeplitz transforms (which have applications in signal processing).

After the two substantial chapters on wavelet approximation the authors complete the book by giving four more chapters containing vector and tensor counterparts of some of the scalar results obtained in Chapters 1–11.

This book fills a large gap in the literature by giving a modern treatment of spherical approximation techniques which starts from first principles but reaches modern developments. It will be of interest to researchers engaged in a wide range of practical problems arising from spherical geometries, as well as theoreticians who are interested in the challenging range of approximation theoretic problems (many of which are still open) arising from the special non-Euclidean structure of the spherical geometry.

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ROBERTS, P. C. *Multiplicities and Chern classes in local algebra* (Cambridge Tracts in Mathematics no. 133, Cambridge, 1998), xi + 303 pp., 0 521 47316 0 (hardback), £37.50 (US$59.95).

In 1957, Serre [**3**] introduced an algebraic measure of intersection multiplicity for two finitely generated modules M and N over a regular local ring $(A, \mathfrak{m})$. If the supports of M and N intersect in the maximal ideal $\mathfrak{m}$, then this multiplicity is given in the form of an Euler characteristic, with the length of $M \otimes_A N$ being adjusted by an alternating sum of torsion terms. In particular, if $M = A/\mathfrak{p}$ and $N = A/\mathfrak{q}$, with $\mathfrak{p}$ and $\mathfrak{q}$ corresponding to the vanishing ideals of subvarieties V and W, respectively, of a smooth variety, which meet in a point P, then this multiplicity gives the multiplicity of the intersection of V and W at P. Serre proved various natural properties of this intersection multiplicity in the case where A contained a field and posed some conjectures about the corresponding situation when A did not. These conjectures, which were homological in nature, were tackled by Peskine and Szpiro, by Hochster, by the author, and others, and soon gave rise to a host of new results and techniques in Commutative Algebra/Algebraic Geometry, and to further conjectures. New algebraic methods (use of the Frobenius, big Cohen–Macaulay modules, Tight Closure, etc.) were boosted by geometrical methods arising from work of Fulton and colleagues (see [**1**]) or from K-theory, and it is in the use of the latter, at the hands of the author and others, where some of the most striking advances have taken place in this general area of the so-called Homological Conjectures. The present book aims to survey these developments, concentrating mainly but not exclusively on the input from geometry, while emphasizing algebraic methods. As such, it is very welcome, especially, but not exclusively, to algebraists.

The book is in two parts. The main, second part is devoted to giving an account of the notion of so-called local Chern characters, first in the case of matrices and then in the more general case of complexes, and the associated notion of the Todd class. This is based on applying the theory of Chern classes of locally free sheaves (on quite general multi-graded rings) to the Grassmannian. The crucial properties of these local characters (an additivity and multiplicativity formula, a Riemann–Roch theorem and local Riemann–Roch formula, mostly all based on a splitting

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principle as in the case of Chern classes) are then applied to prove one of the basic Homological Conjectures, namely the New Intersection Theorem, in the case where A does not contain a field. Indeed a definition is supplied for intersection multiplicities in the non-regular case, but cautionary examples (due to Dutta, Hochster and McLaughlin, and to the author) are given to show that interesting pathologies can arise in general.

The first part gives the algebraic, geometrical and homological background required for a discussion of multiplicities, Intersection Theory and the Homological Conjectures, plus a brief discussion of Tight Closure and the technique of reduction to characteristic p. An added bonus is the inclusion of delightful excursions into the subjects of mixed, Buchsbaum–Rim and Hilbert–Kunz multiplicities, and of the asymptotic Dutta multiplicity. Indeed, in the penultimate chapter, this last multiplicity is shown to have a crucial connection with local Chern characters. (For an interesting preview of this connection, in nascent form, see [**4**].)

This book would form an excellent basis for a working seminar. It is full of elegant methods and arguments, and of interesting ideas and techniques. The discussion is grounded in useful calculations and examples, to which the general argument often devolves. All but the final chapter come supplied with highly relevant exercises. The structure of the book and placing of arguments have been very carefully considered, so that the necessary motivation, often in the form of a telling example, special case or comment, is given when needed. Only in the closing chapters concerning local Chern characters and their properties did I find lacking some comments to give *geometric* insight and motivation as to what was going on: these objects and their properties seemed (at this reading) to comprise a magic black box. (However, *algebraic* motivation and insight are supplied, and the applications in the final chapter amply justify the rather stiff dose of theory that precedes it.) There are a number of misprints, some trivial, some obvious and some to ensure that the reader stays alert. Here and there the argument could do with a little more focus, and some awkward switches in notation, etc., could be tidied up. But the author has performed a signal service in providing such a well-motivated survey of such a broad range of material, some of it quite technical, which leads the reader to the forefront of some of the deepest modern developments in Intersection Theory.

(In the meantime, the author's equally helpful account of Gabber's proof of Serre's non-negativity conjecture (based on de Jong's theory of 'regular alterations') has just appeared; see [**2**].)

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