11. ORBITAL CHARACTERISTICS OF COMETS PASSING THROUGH THE 1:1 COMMENSURABILITY WITH JUPITER

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Abstract. When, in consequence of a close approach, a comet of the Jupiter group changes its osculating semimajor axis from $a > 1$ to $a < 1$ ($a' = 1$ for Jupiter's orbit), or vice versa, then the normal case is that of an abrupt change from one side of Jupiter's orbit to the other one. Under special conditions, however, temporary capture into satellite or 'Trojan' status is possible. P/Slaughter-Burnham, the first known comet in temporary 1:1 resonance with Jupiter, sheds some light on the requirements for Trojan captures. In consideration of the recent finding that the Trojan 'cloud' around L_4 contains probably as many as 700 Trojans brighter than magnitude 20.9, it is suggested that at least some comets of the Jupiter group may have originated among these accumulations around L_4 and L_5.

The available evidence indicates that most comets of the Jupiter group are able, through close approaches to Jupiter, to change from outside into inside orbits, and vice versa, in the sense of changing their osculating semimajor axis a from $a > 1$ to $a < 1$, or from $a < 1$ to $a > 1$, respectively. For convenience, Jupiter's mean solar distance a' is chosen here as the unit of length. The actual cases on record in the literature suggest that during such passages through Jupiter's orbit the resulting changes in a tend to be quite large, regardless of the other orbital changes involved. This probably explains the fact that until recently no comets had been observed in temporary captures either as Jupiter satellites (for one or more complete revolutions) or 'Trojans'. Such captures require a to approach the narrow range $0.95 \leq a \leq 1.05$ (approximately) in a rather special or almost asymptotic manner, or from rather selective initial conditions, as indicated for example by the results of many numerical integrations for asteroids experiencing temporary captures as Jupiter satellites (Hunter, 1967). As to capture into temporary 'Trojan' status, Thüring's (1959) numerical integrations of certain evidently unstable librations about the equilateral points of the restricted three-body problem provided a first demonstration of the possibility of such particular transfers through the 1:1 commensurability of the heliocentric orbital periods involved, or through $a = a' = 1$.

While the basic long- and short-period families of libration orbits, in the rotating reference frame of the restricted problem, are stable against very small superposed oscillations, certain limits of stability exist for the more complicated (and in reality more frequent) librations which incorporate substantial long- *and* short-period components. Further complications are introduced by the ellipticity of Jupiter's orbit and by additional planetary perturbations. For unstable librations involving sufficiently large short-period oscillations superposed on any one of the well-known reference orbits of long period, it has been shown (Rabe, 1966) that such motions would indeed exhibit the spiraling features noted by Thüring (1959). The same result

Chebotarev et al. (eds.), The Motion, Evolution of Orbits, and Origin of Comets, 55–60.

is easily established for sufficiently large long-period fluctuations superposed on a reference libration of short period. As the total libration amplitude increases due to the spiraling effects, Jupiter is approached ever more closely, and all numerical integrations of such unstable librations support indeed the conclusion that the temporary Trojan status begins and ends with a particularly close approach to the primary of mass μ. In terms of heliocentric, osculating elements, it has been known for a long time that in the long-period librations the semimajor axis a oscillates about a mean value close to $a'=1$, and that, in the Sun–Jupiter case, the amplitude Δa of this long-period oscillation must be smaller than about 0.052, for any stable orbit around L_4 or L_5. In the light of the results for the motion of P/Slaughter-Burnham, to be discussed next, it appears that the requirement $0.95 \leq a \leq 1.05$ determines the limits, not only for the existence of stable solutions of long period, but also those for the possibility of a temporary or unstable Trojan status for motions incorporating a substantial short-period component.

Marsden kindly provided me with the osculating values of the elements a, e, and i, at 10 000-day intervals, from his original numerical integration of the motion of P/Slaughter-Burnham over more than 1400 years (Marsden, 1970) and also agreed to continue the backward integration for the further exploration of the orbital development of this first known comet in an extended 1:1 resonance with Jupiter. It was found that the librational stage begins around the year -450, after a very close approach to Jupiter, to about 0.02 Jupiter units. The end of the librational motion will come near the year 2075, with an approach to about 0.06 Jupiter units. Now it should be stressed that the early results far away from the basic epoch in 1959 are very uncertain, due to the limited accuracy of the elements as determined from all the 1958/59 observations and the semiapproximate recovery observations on two consecutive nights in 1969. Nevertheless, the results are of definite mathematical interest, thanks to the high computational accuracy employed, because they give us the true dynamical history of a small body for which these initial elements would be rigorously correct. Any subsequent improvement of the elements of the real comet Slaughter-Burnham may, of course, lead to a modification of the early librational features depicted in Figures 1 and 2, and to a changed total duration of the librational stage, which presently is found to cover about 2500 years. Sitarski (1968) has also remarked on the 1:1 resonance with Jupiter and the fact that the two objects are currently on opposite sides of the Sun.

Figure 1, provided by Marsden as an extended version of his original diagram (Marsden, 1970, Figure 8), shows the long-period component of the libration in longitude, in the form of the difference $\lambda - \lambda_J$ of the mean longitudes of the comet and of Jupiter, respectively. Since through the librational stage the comet's orbital eccentricity e increases from about 0.43 to roughly 0.50, accompanied by a simultaneous decrease of the inclination i from 16° to 8°, very substantial short-period oscillations have to be superposed on the $\lambda - \lambda_J$ of Figure 1, in order to obtain the rather complicated true longitude libration. Actually, the short-period libration component dominates in size during the initial centuries of the long lasting association with L_5, which begins near the year 0, but which around the year 1400 changes

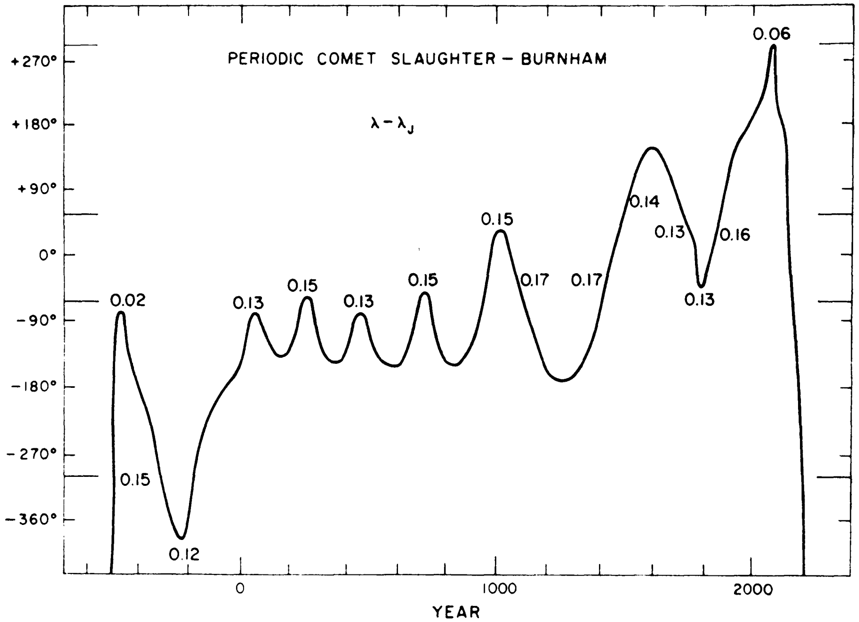

Fig. 1. The libration argument $\lambda - \lambda_J$ for P/Slaughter-Burnham. The long horizontal lines directed inward from the sides represent the Lagrangian points L_4 and L_5, at $\lambda - \lambda_J = \pm 60°$ and $\pm 300°$.

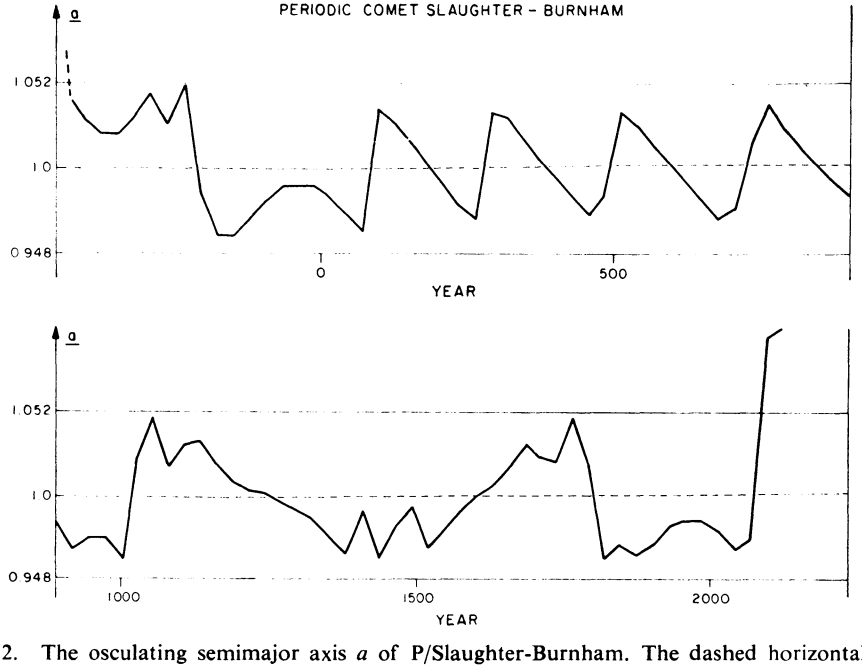

Fig. 2. The osculating semimajor axis a of P/Slaughter-Burnham. The dashed horizontal line represents Jupiter's mean solar distance $a' = 1.0$.

https://doi.org/10.1017/S0074180900006264 Published online by Cambridge University Press

over into a more short-lived association with L_4. The spiraling effect is clearly exhibited in the $\lambda - \lambda_J$ of Figure 1, with a minimum amplitude appearing not far from the year 100. The recurring approaches to Jupiter have been indicated by the approximate minimum distances involved. It is seen that the two terminating approaches are much closer than the other 13 between them.

The 10 000-day values of a have been plotted in Figure 2. However, they have been connected simply by straight lines, which accounts for the lack of smoothness. The most striking feature is the strict compliance with the requirement $0.95 \leq a \leq 1.05$ over the whole librational stage of the motion, and the abrupt termination of this stage when these limits are penetrated. It appears that the long-period component is the one of principal significance for the maintenance and duration of the librational status. Some light seems to be shed on the requirements for capture into such a libration by those 'convex' arcs of the a-curve in Figure 2 which are curved away from the horizontal line for $a = 1.0$. These arcs appear directly after capture and before escape, but also just before the comet's entry into an extended period of close association with L_5. The dashed line $a = 1.0$ is being approached but not reached by the central part of each of these arcs. The times of closest approach to $a = 1.0$ in the various arcs of this sort coincide with those times when the $\lambda - \lambda_J$ of Figure 1 passes through the opposition point $\pm 180°$, at least approximately. It seems thus that temporary libration capture becomes possible when a approaches the range $0.95 \leq a \leq 1.05$ in such a manner, that a minimum (or maximum) of a is reached inside this range, but in addition to this also at a time when the mean longitude of the comet or asteroid differs from that of Jupiter by approximately 180°. This evidently is a rather restrictive condition and may account for the rareness of such captures. Some less pronounced convex arcs are indicated in Figure 2 around the times when $\lambda - \lambda_J = 0$, or when the mean longitude of the comet passes Jupiter during one of the few 'transfers' (as from L_5 to L_4) which can be recognized in Figure 1.

The case of P/Slaughter-Burnham shows that under the right conditions even comets in rather eccentric orbits may enter the Trojan-type 1:1 resonance for a limited period of time, but that the probability of such an occurrence appears to be small. On the other hand, over very long periods of time, most of the known comets of the Jupiter group may occasionally experience the appropriate perturbations in the all-important element a to make such a Trojan capture possible. The Jacobi 'constants' C of practically all comets of the Jupiter group lie in the range between 2.5 and 3.0, so that their temporary Trojan status, involving a more or less substantial short-period component, is quite conceivable. For P/Slaughter-Burnham, the e of order 0.5 is reflected in $C = 2.72$. On the other hand, a capture into satellite status requires a C-value just slightly larger than 3, or relatively small values of e and i immediately before and after capture, so that such an event is rather unlikely for most of the Jupiter comets. Some asteroids are better candidates for such an occurrence.

Quite recently it has been estimated that about 700 Trojan planets brighter than magnitude 20.9 are associated with the Lagrangian point L_4 preceding Jupiter (van Houten *et al.*, 1970). Thus it seems that the Trojans are much more numerous than originally thought, or that relatively dense 'clouds' of these asteroids exist in the neighborhoods of the equilateral points. Obviously, then, a great number of small bodies has come into existence in these regions during the earlier stages of the solar system. The Trojans which remained there in rather stable orbits are characterized by small eccentricities, up to about $e=0.15$. If additional Trojans with somewhat larger e-values or/and larger libration amplitudes Δa also existed there long ago, but eventually escaped from libration due to various perturbing effects, they would unavoidably have tended to transfer into motions very similar to those of the present comets of the Jupiter group. The C-values of such escapers would indeed tend to fall into just that range, between 3.0 and about 2.5, which we find associated with the Jupiter comets. Furthermore, for the 38 comets recently listed by Marsden (1967) the values of the relevant elements are such that, *if* passage through $a=1$ is assumed to happen at some time without a change in the (generally much smaller) inclination i, many eccentricities would at such time attain greatly reduced values e_1, in compliance with the Tisserand criterion. A small fraction of these e_1-values even falls into the range $0.00 \leq e \leq 0.15$ of the known Trojan planets. Consequently it may be suggested that at least some of the comets of the Jupiter group may have *originated* in these fairly dense Trojan clouds and may indeed never have departed very far from the vicinity of Jupiter's orbit. Such an origin would help to explain the complete absence of retrograde orbits among the comets of the Jupiter group. Finally, the anomalous distribution of the longitudes of perihelion noted by Marsden (1967) for the comets of the Jupiter group is very similar to the one noted by Thüring (1951) for the Trojan planets. In both groups, the perihelia prefer the semicircle centered approximately near Jupiter's perihelion longitude π' and containing also the longitudes $\pi' \pm 60°$.

It should be noted that van Houten *et al.* (1970) use the notation L_5, instead of L_4, for the Lagrangian point preceding Jupiter.

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Discussion

H. Alfvén: If you integrate the orbit of this comet two or three thousand years backward, there arises the question as to how long the comet's actual lifetime is. I understand from Vsekhsvyatskij's catalogue that he estimates that cometary lifetimes are of the order of decades or centuries, and this seems to be in agreement with other results. Has it any realistic meaning to integrate for such long time intervals?

E. Rabe: The closer a comet comes to the Sun, the more volatile material it loses, but one can perhaps expect something to be left over. How long an individual comet lives is an open question. But it is a physical problem completely separate from the mathematical one.

B. G. Marsden: It may well be that our integration of the orbit of the real P/Slaughter-Burnham is unreliable as long ago as the year −450, but comparison with another integration from rather different starting conditions suggests that its general features are certainly correct back to about 1000 years ago. The main difficulties with this kind of calculation stem from close approaches to Jupiter and nongravitational effects. But close approaches of P/Slaughter-Burnham to Jupiter have not occurred for at least 1000 years, and with a perihelion distance as large as 2.5 AU the nongravitational effects are certainly very small, and so, presumably, is the comet's rate of decay.

S. K. Vsekhsvyatskij: The main point is that the secular decreases in brightness show that comets are physically unstable and hence rather young objects. We do not know those comets of Jupiter's family that might have existed in the central region of the solar system more than a hundred or few hundred years ago. Besides, it is impossible to explain the appearance of short-period comets and the peculiarities of their motions by the capture hypothesis, if we take into account the low probability of transformation of a long-period orbit into a short-period one. As to the idea that these comets could be formed within Jupiter's libration region, this is refuted by the whole of our knowledge of the conditions in interplanetary space. The solar photon and corpuscular radiation should result in the sublimation of cometary ices and the disintegration of comets at distances up to 11 or 12 AU. In the vicinity of Jupiter cometary ices exist only on the surfaces of Jupiter's satellites.

L. Kresák: P/Van Biesbroeck moves in an orbit closely resembling that of P/Slaughter-Burnham, not only by an approximate 1:1 resonance with Jupiter and a similar eccentricity, but also by a very similar value of the libration argument, putting it at present opposite Jupiter. I wonder if anybody has tried to obtain long-term perturbations for this comet; possibly the results may be analogous to those obtained for P/Slaughter-Burnham.

B. G. Marsden: We have found that P/Van Biesbroeck was not librating in the past, perturbations by Jupiter having caused significant changes in the orbit not too long ago. I do not know about the future motion.

G. A. Chebotarev: What is the reason for the absence of retrograde motions among the comets of the Jupiter family?

E. Rabe: If some of these comets were originally members of 'Trojan' clouds, then they would be in direct orbits only. All the bodies associated with the libration points are in direct orbits because they have a mean angular distance of 60° from Jupiter and move in the same direction as Jupiter.