ON BLACK AND WHITE HOLES

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Abstract. Various possible cases of spherically systems the matter of which is localized in a domain smaller than the corresponding gravitational radius is considered.

The metic of the Friedmann closed world or of a part of it with an external continuation is suggested as a model of these systems.

There can exist black holes which are described by semi-closed metrics (black holes of the second kind). The class of systems in question may be both in the state of collapse and in the state of anti-collapse (including the state of 'white holes').

There are some grounds to suppose that collapse of celestial bodies should stop in the domain $\hbar/m_v c$, where m_v is the mass of the vector meson, and that the pair production effect due to collapse of a charged sphere should conserve the Laplace determinism of the process.

The role of the charges of sources of different fields (electromagnetic, meson vector, scalar long-range, scalar meson, various versions of neutrino fields) in the deformation of the external and internal metric of black and white holes is analysed.

In this consideration a number of problems arises (the absence of horizon in the case of any small charges of scalar fields, the presence of the generalized Gauss theorem for vector meson field etc.), which provide evidence that the assertion 'Black hole has no hair' needs further investigations. In particular, the inverse process of formation of hair (e.g. vector-meson, scalar fields) in the process of anti-collopose has not been studied yet.

For the limiting case of the Nordström-Reissner metric $m = e$ (more correctly $m \to e$) two essentially different possibilities of continuing to the internal metric are considered (the Papapetrou case and the case which we called 'friedmon metric' describing charged black holes of the second kind (friedmons)).

In the case of charged holes of the second kind (friedmons) the occurance of quantum effects (pair productions) can reduce the horizon surface and violate the Hawking theorem.

The notion of black holes may turn out to be essential in elementary particle theory: among the intermediate states in elementary particle theory there are states the characteristic feature of which is the localization of arbitrary large energies (masses) in a domain smaller than the gravitational radius.

Collapse and anticollapse of material systems have long been the object of theoretical investigations. At the present time when astrophysics becomes gradually an experimental science the interest in such systems increases greatly. Much attention is also paid to peculiar changes in the process of collapse of the global properties of matter which are being extensively discussed. In particular, a possible disappearance of some fields in the external metric of similar systems in the process of collapse (when the collapsing matter is behind the Schwarzschild sphere) is widely discussed.

This situation figuratively is defined by Wheeler (1971) as follows: 'A black hole has no hair'.

In what follows we are dealing with the systems in which matter is localized in regions smaller than the corresponding gravitational radii. The state of these systems has a certain variety. It can, in particular, be a both a collapse and an anticollapse. The systems under discussion have also other differences if sources of different fields are included into them. It seems advisable to make a certain classification of possible objects of this kind in the framework of general relativity. In what follows we restrict ourselves in the main to the consideration of the systems when the appropriate

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metric in the region of localization of gravitating matter is described by a linear element of the closed Friedmann world (Hoyle *et al.*, 1965), and the external solution matched with this 'internal' solution at a given moment is the Schwarzschild solution. We exclude thereby a very interesting class of external metrics of the Kerr type because the appropriate internal metrics matched with them have not been constructed yet. In the discussion of the fate of short-range forces in the external metric of these systems it is advisable to employ also systems of small total masses. Such systems are, in principle, allowed by theory. The gravitational radius of the systems in question may be e.g. of the order $\hbar/m_v c$, where m_v is the mass of, e.g., a vector meson. The quantity $\hbar/m_v c$ is of the order 10^{-13} cm: here we are still in the range of applicability of the non-quantum theory of gravitational field. In other words, the given length $\hbar/m_v c$ is by 20 orders of magnitude larger than the corresponding gravitational length ($l_{gr} = (\hbar c \varkappa)^{1/2} c^{-2} \sim 10^{-32}$ cm), where we may suspect the invalidity of the classical Einstein gravitational theory. In addition, we may always, if needed, put formally m_v to be very small, i.e. $\hbar/m_v c$ to be arbitrary large. There is no real physical sense of considering short-range external fields of celestial bodies at distances $\hbar/m_v c \sim 10^{-13}$ cm and discussing whether it is possible to detect them experimentally. It should be stressed that systems with gravitational radius of an order of 10^{-13} cm are by no means related to microworld objects. Such a gravitational radius is assigned to the mass

$$M_0 \sim \frac{\hbar}{m_v c} \frac{c^2}{2\varkappa} \sim 10^{14}\ \mathrm{g} = 10^8\ \mathrm{ton}. \tag{1}$$

The Friedmann linear element

$$\mathrm{d}s^2 = a^2(\eta)\,\mathrm{d}\eta^2 - a^2(\eta)\,\mathrm{d}\chi^2 - a^2(\eta)\sin^2\chi(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)$$

or

$$\mathrm{d}s^2 = c^2\,\mathrm{d}t^2 - a^2(t)\,\{\mathrm{d}\chi^2 + \sin^2\chi(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2)\tag{2}$$

is known to describe one of the models of the closed world.

Here the variable χ changes in the limits

$$0 \leqslant \chi \leqslant \pi, \qquad a(\eta) = a_0(1 - \cos\eta). \tag{3}$$

The variable η is connected with the time t by a simple relation:

$$t = \frac{a}{c}(\eta - \sin\eta).$$

The same Equation (2) can describe the internal metric of the so-called black hole, more strictly, of a certain model of the black hole, provided the matter of the system is distributed in such a manner that the region χ is filled only to $\chi_m \leqslant \pi/2$. And then the external metric, e.g. (Euclidean at infinity), should be 'matched' with the internal Friedmann metric in an appropriate manner. The black hole is usually viewed as the final state of a collapsing system when the matter of the system is behing the

Schwarzschild sphere. We imply here the state which the collapsing system tries to reach at the Schwarzschild time increasing to infinity. We call this limiting state of the system black hole of the first kind.

We may define strictly this object in the framework of the abovementioned metric with the above-mentioned restriction on the range of the χ values. Namely,

(1) Black hole of the first kind:

$$0 < \chi_m \leqslant \frac{\pi}{2}. \tag{4}$$

As far as the metric is nonstatic and the question is to discuss collapse, further fate of the system is related to decreasing $a(t)\,\chi_m$ value, i.e. to reducing sizes of the system which 'is' already behind the Schwarzschild sphere.

The variety of the objects described by the internal metric (2) is not limited by the state-black hole of this kind.

In fact, if matter fills in the region χ so that $\pi/2 < \chi_m < \pi$ then there arises a space with a semi-closed metric. This space has the following properties. If spheres or $\chi > 0$ are circumscribed around the point $\chi = 0$ at a given moment then the surface of the sphere is

$$S = 4\pi a^2(t) \sin^2 \chi. \tag{5}$$

The surface of the sphere S increases with increasing χ to $\chi = \pi/2$. However, when $\chi > \pi/2$, the sizes of the sphere decrease and for $\chi = \pi$ the sphere reduces to a point which implies that the world becomes closed.

Semi-closed worlds were first studied by Klein (1961), Zel'dovich (1962), and Novikov (1962). Klein called the system of this kind Friedmann world with external continuation '(Mit Außenwelt)'.

When $\chi < \pi/2$ (black hole of the first kind) the surface of the spheres in question increases monotonically with increasing χ at a given moment t. In the external metric the quantity $a(t) \sin \chi = r$ assumes also the meaning of a monotonically increasing radius. The semi-closed metric (when $\pi/2 < \chi_m < \pi$) is characterized by the existence of a minimal surface, a minimal value of r in vacuum, where $\partial r/\partial \chi = 0$ and $\partial^2 r/\partial \chi^2 > 0$ (for details see Markov and Frolov, 1970; Markov, 1971).

In other words, the semi-closed metric is characterized by the presence of a specific throat which links the internal and external metric.

In principle, among celestial bodies there may be objects of the second class just mentioned. Thus

(2) Objects with semi-closed metric: $\pi/2 < \chi_m < \pi$ are black holes of the second kind.

Although the objects of the second kind are, in their astrophysical nature, analogous to the black holes, they differ essentially from the black holes of the first kind. The objects of the former class can arise (for instance) in the process of evolution of white holes (see Appendix II).

Finally, the third case:

(3) $\chi_m = \pi$ – the Friedmann closed world.

The matter density $\mu(t)$ integrated over the whole space of a closed world gives a 'bare' mass of the system i.e. the total mass without taking into account the gravitational defect:

$$M_0 = 2\pi^2 \mu(t)\, a^3(t). \tag{6}$$

This value of the 'bare' mass defines the sized of the radius of the closed world at the moment of its maximum extension:

$$a_0 = \frac{\varkappa M_0}{3\pi c^2}. \tag{7}$$

The latter expression is immediately obtained from the Einstein equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{2\pi}{3}\varkappa\mu - \frac{c^2}{a^2}. \tag{8}$$

If we put in Equation (8) $\dot{a} = \mathrm{d}a/\mathrm{d}t = 0$, then according to Equation (6), $\mu_0 = M_0/2\pi^2 a_0^3$.

These relations show that a *closed world can, in principle, contain an arbitrary small amount of matter and has arbitrary small sizes.* It appears that quantum mechanichs alone can impose here definite restrictions. Thus, for

$$M_0 = \left(\frac{\hbar c}{\varkappa}\right)^{1/2} \sim 10^{-5}\ \mathrm{g}$$

the dimensions of the world

$$a_0 \sim \frac{(\hbar c \varkappa)^{1/2}}{c^2} \sim 10^{-33}\ \mathrm{cm}. \tag{9}$$

There are some grounds to believe that at these distances the notion of length losses its meaning due to quantum fluctuations of the metric (Regge, 1958; Blokhintsev, 1960).

The total mass of a part of a closed world, localized in the region from $\chi = 0$ to χ_m (i.e. the bare mass minus its gravitational defect) is given by the expression

$$M_{\mathrm{tot}} = \frac{c}{\varkappa}\, a_0 \sin^3 \chi_m. \tag{10}$$

(Klein, 1961; Zel'dovich and Novikov, 1967).

Thus, the total mass of the closed system ($\chi_m = \pi$) is zero.

The semi-closed system is characterized by two lengths. One of the lengths l_{in} characterizes the internal sizes of the system

$$l_{\mathrm{in}} = a(t)\,\chi. \tag{11}$$

The maximum value of the internal sizes may be arbitrary large

$$l_{\mathrm{in}}^{\max} = a_0 \chi_m, \tag{12}$$

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where a_0 is the radius of the system defined by the bare mass, according to Equation (7).

The other sizes are associated with the surface which surrounds a part of the Friedmann world filled in with matter. This is the surface of the 'throat' $S = a^2 \sin^2 \chi_0$ with its radius

$$l_{\text{out}} = a(t) \sin \chi_0, \qquad \chi_0 > \chi_m, \tag{13}$$

which, depending on the χ_0 value, may be arbitrary small. At $\chi_0 \to 0$, l_{out} tends to zero. The quantity l_{out} is the external dimension of the system, that is the dimensions of the system which an external observer sees.

Systems like our Universe with numerous galaxies having ultramacroscopic internal dimensions (l_{in}) may have arbitrary small external dimensions (l_{out}). Even this surface may be microscopic dimensions and, according to Equation (10), may possess a microscopic total mass*. In this sense, we may speak of a peculiar relativity of the notions of macro and micro (Markov, 1971).

Further variety of the objects under discussion may be associated with the fact that they can be in two different states defined by the initial conditions. In other words, the internal dimensions of the system can eventually either decrease

$$\left(\frac{\mathrm{d}a}{\mathrm{d}t} < 0 - \text{collapse}\right)$$

or increase

$$\left(\frac{\mathrm{d}a}{\mathrm{d}t} > 0 - \text{anticollapse}\right).$$

These objects being in the state of collapse are systems absorbing matter from the surrounding space (black holes), while in the state of anticollapse there may, in principle, be radiating objects sources emitting matter (white holes).

Strictly speaking, so far we are not aware of whether black holes exist and whether the collapsing systems are realized in nature.

However, we may apparently assert with a great probability that there exists, at least, one system in the state of anticollapse, this is precisely our Universe. It is quite probable that our Universe is just a white hole. Very little is known about the end fate of the collapsing system.

It is hard to say *a priori* whether it is possible to exclude the existence of periodic states when a collapse is changed by an anticollapse. In principle, such a possibility may be given by, e.g., long-range fields of the type of electrostatic fields (Novikov, 1966). Short-range vector meson fields may play the same role provided in the process of collapse the system turns out to be in the region $\hbar/m_v c$ (Berezin and Markov, 1969), where m_v is the vector meson mass.

But studying this situation we see that the collapse changes by an anticollapse

* In the framework of the classical theory there are not restrictions on the smallness of the total mass. It may be, e.g. of the order of the mass of an elementary particle.

which however occurs not in the space where the collapse took place (Novikov, 1966; de la Cruz and Israel, 1967; Bardeen, 1968).

On the one hand, this unexpected and unusual situation forces us to search for the explanation may be in an insufficiently real description of the process (for instance, in neglecting e.g. the huge pair production effect in big meson fields of the final stage of collapse (see Appendix 1). On the other hand, the same situation gives a possibility of making nontrivial speculations concerning topological structure of our Universe (Sakharov, 1971). Should we exclude in this case the possibility that some of the objects in question might enter in this state of anticollapse in the state of a white hole only recently, as a result of its periodic development, and this might happen precisely in our space.

These considerations would suggest an interpretation of the formation of galaxies in the spirit of the conceptions of Ambartsumian (1962) which, in his opinion, follow from astronomical observations. A similar hypothesis was proposed by Novikov (1964) and Ne'eman (1965). But finally, the problem reduces to different formulations of the appropriate initial conditions.

It is obvious that spontaneous emergence of a white hole in our space is not a very easy understandable event. However, in a certain sense the same problem arises when discussing the original moment in the development of our Universe.

Objects with semi-closed matric can be classified as an independent class of objects.

A black hole of the first kind cannot turn to, e.g. a semi-closed system. The matter is that the black hole can only absorb matter from the surrounding space, can only increase its gravitational radius, while the formation of a system with semi-closed metric requires for the Schwarzschild sphere to decrease its sizes.

Another situation arises if we deal with objects in the state of anticollapse (see Appendix 2).

Up to this point of our presentation the subject of our consideration were electrically neutral systems. The introduction of even arbitrary small electric charges in the systems in question changes essentially the situation and thereby changes the classification of the objects we are interested in.

First of all, the third case turns out to be impossible – system with closed metric. The closed Friedmann metric perturbed by the presence of an arbitrary small electric charge is the case of the semi-closed metric.

In the Friedmann metric (2) we can formally find a solution for the electrostatic potential φ, for simplicity at the moment of maximum extension of the world.

Such a solution is of the form

$$\varphi = \frac{\text{const}}{a \sin \chi}. \tag{14}$$

As $\chi \to \pi$ the expression for the potential becomes infinite. For $\chi = \pi$ there appears a particular feature characteristic of a point source the presence of which at this place has not been supposed by us. This is precisely the form in which the contradiction between the closed metric and the attempt to introduce in this metric the total non-zero electric charge is revealed. In fact, at the place where $\chi \to \pi$ there arises something representing the image of the field source localized at $\chi = 0$. The contradiction with the closed metric disappears if we imply the charge of an opposite sign. Then the total electric charge of the system turns out to be equal to zero. If the strength lines go out from the charge at the point $\chi = 0$, then they must end in this case on the charge of an opposite sign at the point $\chi = \pi$.

The character of the deformation of the closed world metric by a small electric charge was considered in detail by Markov and Frolov (1970). When the electric charge is arbitrary small a noticeable deviation from the Friedmann metric arises only for χ arbitrary close to π.

In other words, if we circumscribe spheres with $\chi > 0$ around a small charge ε localized at $\chi = 0$ then these spheres are characterized by the expression (5) up to χ very close to π.

In the domain $\chi > \pi/2$ the spheres will greatly decrease with increasing χ, as in the absence of the electric change. However, in this case (at $\chi > \pi/2$) the spheres play the role of peculiar focusing lenses for strength lines of an electrostatic field.

A detailed study shows that when the density of strength lines ('hair' of the electromagnetic field) becomes such that the corresponding value for the electrostatic potential reaches a value close to

$$\varphi = \frac{c^2}{\varkappa^{1/2}}, \tag{15}$$

then with further increase of χ the spheres in question begin growing again and the metric turns to a particular case of the well-known Nordström-Reissner metric. This particular case is characterized by the Schwarzschild mass

$$M_{\text{tot}} = \frac{\varepsilon}{\varkappa^{1/2}} \tag{16}$$

and the metric itself (outside throat) is

$$ds^2 = \Phi^2 c^2 \, dt^2 - \frac{dr^2}{\Phi^2} - r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2), \tag{17}$$

where

$$\Phi = \left(1 - \frac{\varepsilon \varkappa^{1/2}}{c^2 r}\right)^2. \tag{18}$$

The radius of a minimal sphere which is allowed by the electrostatic charge is proportional to the electric charge

$$r_{\min} = \frac{\varepsilon \varkappa^{1/2}}{c^2}. \tag{19}$$

The contradiction with the closed metric is 'created' by the Gauss theorem. Figuratively speaking, the electric strength lines ('hair') become more dense for χ close to π

so that they 'punch' in the closed metric a 'wormhole' (throat) into which the electric vector flux rushes forming outside the given material system a particular case of the Nordström-Reissner metric. Outside the throat this metric is quite analogous to the metric suggested by Papapetrou (1945).

But the physical object characterized by this metric and the complete space-time description of the metric at small distances ($r < \varepsilon\varkappa^{1/2}/c^2$) are of quite different nature. Special attention should be paid to this fact.

The Papapetrou metric describes the static case when gravitational attraction of particles is equilibrated by electrostatic forces of repulsion of their electric charges. Roughly speaking, from the equality $\varkappa m^2/r = e^2/r$ it follows the relation (16) $m = e/\varkappa^{1/2}$.

A detailed analysis (Markov and Frolov, 1972) of the Papapetrou metric shows that in this case as one should except for the static case, matter cannot be behind the Schwarzschild sphere. In the Papapetrou model the sizes of the domain in which matter is localized is necessarily large than the gravitational radius of the system.

In our case we are dealing with the system which is inside the Schwarzschild sphere. In this model the metric is nonstatic. The external metric in the form (18) is a limiting case of the Nordström-Reissner metric for $M > \varepsilon/\varkappa^{1/2}$ when $M \to \varepsilon/\varkappa^{1/2}$.

The general case of the Nordström-Reissner metric is known to be described by the expression (17) as well, but now Φ is of the form

$$\Phi^2 = 1 - \frac{2\varkappa m_0}{c^2 r} - \frac{\varkappa\varepsilon^2}{c^4 r^2}. \tag{20}$$

The expression (20) has two roots

$$r_\pm = \frac{\varkappa m_0}{c^2}\left\{1 \pm \left(1 - \frac{\varepsilon^2}{\varkappa m_0^2}\right)^{1/2}\right\}. \tag{21}$$

In our limiting version of this metric, according to (16) $M_{\text{tot}} = \varepsilon/\varkappa^{1/2}$, r_+ and r_- are equal to each other.

The physical picture of the Friedmann metric distorted by the presence of the small charge differs from the corresponding picture of the Papapetrou model by that in the latter the values of the total mass of the system and its bare mass coincide. While in our case the total mass is of essentially electrostatic nature. The value of the bare mass (say, the number of nucleons in the system) may be arbitrary. Here the bare mass is completely cancelled by the gravitational defect of the system. The internal metric together with the external metric (18) which at $\varepsilon \to 0$ transforms to the metric of the Friedmann closed world is given by us the name of friedmon metric and the object itself – 'friedmon'.

We have considered in detail the friedmon metric and have introduced for the given object a special term 'friedmon' because in literature the case $\varepsilon^2/\varkappa \to M^2$ is interpreted as the Papapetrou case, or which is the same, as the Bonnor case (1960) even when one considers an entire analytic continuation of the metric for $\varepsilon^2/\varkappa = M^2$ (Carter, 1966). According to (19) the radius of the throat increases proportionally to the total electric charge.

This is the description of the throat from the point of view of classical (nonquantum) physics. But from the viewpoint of quantum physics the state of the throat cannot be stable.

Indeed, if at some initial moment a throat with the above mentioned properties arises then in its superstrong electrical field* there inevitably occurs a violent process of production of any kind of electrically charged pairs: proton-antiproton pairs, any kind of meson pairs, and finally, electron-positron pairs. Charges of opposite signs will attempt to decrease the effective charge of the throat while the charges of the other component of pairs will flow to the euclidean infinity.

The decrease of the electric charge of a system due to the production of charged particle pairs results in the decrease of the electric charge of the throat. The peculiarity of the quantum effect of electron-positron pair production consists in that the pair components may be spaced by a considerable space-like interval (Zel'dovich, 1971). *It may happen that the electron and positron are created on opposite sides from the event horizon.* A 'subbarrier' penetration of particles with opposite charges through the event horizon (throat), particle pairs produced outside the throat, is also possible.

From the viewpoint of the Schwarzschild observer a particle approaches the gravitational radius of the black hole during an infinitely long time. But in this process there is a striking peculiarity which should be borne in mind in a number of facts.

The law of change of this spacing with time is given by the formula (Zel'dovich and Novikov, 1967).

$$r = r_{gr} + (r_1 - r_{gr})\, e^{-c(t-t')/2r_{gr}},$$

here r is the location of the particle at a moment t', r_{gr} is the gravitational radius, $(r_1 - r_{gr}) \ll r_{gr}$.

Here the numbers are very instructive. In fact, let $r_{gr} \sim 10^5$ cm, and $(r_1 - r_{gr}) \sim 10^3$ cm. As is seen from the formula, even this short way, amount 10 m, is covered by the particle during an infinite time. The main thing is however that in a second the distance between the Schwarzschild sphere and the test particle becomes infinitely small:

$$\Delta r \sim 10^3\, e^{-10^5}\ \text{cm} < 10^{-10000}\ \text{cm},$$

which is infinitely small even compared with the characteristic length of quantum fluctuations of the metric $l \sim 10^{-33}$ cm.

Because of the presence of huge fields in the systems in question (friedmons) near the throat the pair production process under given initial conditions occurs mainly directly in the throat region. *But spontaneous appearance of a charge of an opposite sign 'inside' the fridmon decreases the internal charge of the friedmon.* Consequently, according to (19), the radius of the throat and the horizon surface, the Schwarzschild sphere surface, decrease as well.

In other words, in quantum domain quantum effects can violate the Hawking

* The value of the potential $\varphi = c^2/\sqrt{x}$ appears to be the maximum one which is allows by nature in R space. This expression contain no electric charge (!!).

theorem. Another case of quantum violation of the Hawking theorem was recently indicated by Bekenstein (1973). If we suppose that the electron of a pair produced near the throat carried away at infinity an energy $E=\varepsilon e/r_{gr}=e\varkappa^{-1/2}c^2$ when it pushes off from the throat charge (ε), then the remainder conserves the characteristic property of the friedmon:

$$M'=\frac{\varepsilon'}{\varkappa^{1/2}}.$$

Thus, in the process of spontaneous pair production near a given friedmon its total charge, total mass and the sizes of the throat can decrease. This process has not been studied yet in detail, but the estimates (Markov and Frolov, 1970) show that the pair formation process results in a decrease of the friedmon charge in the final state to a value $Ze<137\,e$. According to Landau (1955), further vacuum polarization appears to be capable of decreasing the charge to $\varepsilon=1$. It is essential that the same value of the electric charge of the final state arises for any charge value of the initial friedmon. In any case, the latter assertion holds for relatively small initial Z, namely for $Z<10^{20}$ for initial

$$r_h=\frac{\varepsilon\varkappa^{1/2}}{c^2}\sim 10^{-14}\ \text{cm}.$$

The appearance of a friedmon state in the process of evolution of white holes accompanied by radiation of charges was considered by Frolov (1973) (see Appendix 2).

If in this process the Hawking theorem is not violated then the mass of the initial system may be only half of the original mass. It is however quite possible that the whole process occurs under these conditions with a complete violation of the Hawking theorem.

In any case a friedmon with electric charge equal to unity is a stable object of friedmon metric. The parameters of this friedmon are as follows: mass $m_f^e\sim 10^{-6}$ g, sizes $r_f^e\sim 10^{-33}$ cm.

If there exist baryon hair of black holes then in addition to the electrostatic friedmon there might be meson friedmons:

$$m_f^g=\frac{g}{\varkappa^{1/2}}\sim 10^{-5}\ \text{g},\qquad r_f^g=\frac{g\varkappa^{1/2}}{c^2}\sim 10^{-32}\ \text{cm},$$

where $g^2/\hbar c\sim 1$, g is the baryon charge, the source of a vector meson field (Markov, 1970). It should be stressed once more that r_f^e and r_f^g are the external sizes of the system. Its internal sizes $a(t)\,\chi_0$ and its bare mass, the number of nucleons, may be expressed, e.g. by ultramacroscopic numbers. The particles under discussion and the particle characterized by the constants $\hbar$, c, $\varkappa$, i.e. $(\hbar c/\varkappa)^{1/2}\sim 10^{-5}$ g – Plank-Wheeler particle – may be regarded as possible 'elementary' particles of limiting maximum large masses, that is they can form a group of 'maximon' (Markov, 1965). Maximons may be of a relict origin, i.e. may exist in our Universe perpetually. Hawking (1971) has also assumed the existence of such particles with charge ~ 30. But the object of such small sizes ($\sim 10^{-}32$ cm) with such a charge will turn out to be unstable due to vacuum polarization.

Thus, we see that the presence of electric charges, sources of the electrostatic field changes essentially the classification of objects the internal metric of which is partially or completely characterized by the metric of Friedmann closed world. In the latter case there naturally and inevitably arises a semi-closed metric. There arises the question of what the effect of the change of the closed world metric is if it is perturbed by small charges of other fields: for example, scalar, massive vector fields or fields related to weak interactions.

Numerous papers assert that the sources of other fields (for the exception of electromagnetic field) localized in matter inside the Schwarzschild sphere (hole) do not excite appropriate fields outside given objects: 'Black hole has no hair'. But here many problems arise which are still open.

1. The Meson Vector Field

At first glance it seems that, as in the case of electrostatic, the presence of sources of the vector meson field is incompatible with the closed metric.

Indeed, using the standard procedure, it is easy to obtain, at the moment of maximum expansion, for the meson vector field potential the following expression

$$\varphi_0 \sim \frac{\beta e^{-\lambda\chi}}{a_0 \sin\chi}, \qquad \lambda = (a_0^2 m_v^2 - 1)^{1/2}, \tag{22}$$

where m_v is the vector meson mass. This expression can naturally be thought of as an analog, in the euclidean space, to the ordinary expression $\varphi_0 \sim \beta(e^{-m_v r}/r)$. On the basis of (22), it might be concluded that the potential φ_0 at $\chi \to \pi$ is divergent as in the electrostatic case that the presence of vector meson field sources with non-zero total charge is incompatible with the closed metric, and that the given system must have a continuation – an external metric, i.e. an external meson vector field.

But this assertion is wrong: it is based on the use of the boundary conditions which are ordinary for the euclidean space. These conditions of finiteness of the solution select solutions exponentially damping with increasing r.

In the case of a closed world there is no spacial infinity, and therefore a more general type of the solution is possible (Markov and Frolov, 1973):

$$\varphi_0 = \frac{\beta e^{-\lambda\chi}}{a \sin\chi} + \frac{\gamma e^{+\lambda\chi}}{a \sin\chi}. \tag{23}$$

The requirement that the solution should be finite and continuous is satisfied by the condition imposed on the coefficients β and γ:

$$\beta e^{\lambda\pi} + \gamma e^{-\lambda\pi} = 0. \tag{24}$$

Thus, outside the point source g_0 the solution is

$$\varphi = \frac{g_0 \operatorname{sh} \lambda (\pi - \chi)}{a \operatorname{sh} \lambda \pi \sin \chi}. \tag{25}$$

Now φ_0 at $\chi \to \pi$ is finite, and the field $\partial \varphi_0 / \partial \chi$ at $\chi \to \pi$ vanishes.

Thus, we are led to the conclusion that the closed world may contain the total nonzero baryon charge (vector meson field source). This establishes the essential difference of the massless vector field (i.e. electrodynamics) from the meson vector field. The obtained results do not contradict the assertion that the meson vector field is absent outside black holes. At the same time, it is not a proof in favour of this assertion.

Meanwhile, as in the case of electrodynamics, the previous consideration is a rigorous proof of the presence of an electrostatic field sources are found behind the Schwarzschild sphere the vector meson field outside the Schwarzschild sphere vanishes (Bekenstein, 1972; Teitelboim, 1972; Thorne, 1971).

It is worth noting that, in the case of the meson vector field as well, there is a certain peculiar analog of the Gauss theorem, more correctly, its peculiar generalization. More detailed consideration of the available relations leads us to the conclusion that the situation is really more complicated than seems at first.

We consider a mesodynamic analog of the Gauss electrodynamic theorem in an Euclidean metric, for simplicity (Markov, 1970).

Let a baryon charge ϱ be localized in a certain domain so that

$$\begin{aligned} &\varrho \neq 0, \qquad r < r_0, \\ &\varrho = 0, \qquad r > r_0. \end{aligned}$$

For the mesodynamic vector E_n flux through a closed surface surrounding the charge, on the basis of the equation

$$F^{\mu\nu}{}_{,\nu} - m_v^2 \varphi^\mu = -4\pi j^\mu, \qquad j^4 = \varrho, \tag{26}$$

we get the following expression

$$\int E_n \, \mathrm{d}s = m_v^2 \int \varphi^0 \, \mathrm{d}V - \int \varrho \, \mathrm{d}V. \tag{27}$$

After circumscribing a sphere of radius $r > r_0$ about the charge we obtain E_n flux in the form

$$\int E_n \, \mathrm{d}s = 4\pi m_v^2 \int_0^r \varphi^0(r) \, r^2 \, \mathrm{d}r - 4\pi g \tag{28}$$

If

$$\varphi^0 = \frac{g}{r} e^{-m_v r},$$

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$$\int E_n \, \mathrm{d}s = 4\pi g \left[m_v^2 \int_0^r e^{-m_v r} r \, \mathrm{d}r - 1 \right]. \tag{29}$$

In electrodynamics the vector E_n flux has the same value on the sphere of any radius, while in mesodynamics decreases with increasing radius of the sphere. When the sphere radius tends to infinity, the r.h.s. of Equation (29) vanishes; the flux E_n is completely damped. If in mesodynamics the term 'strength lines' is allowed then in the case of vector meson field the strength lines do not end, as in electrodynamics, in charges of opposite signes. Instead, they are damped in an original manner, by the 'field charge', if it is possible to say so, which is realized by the field potential, namely by the term $4\pi m_v^2 \int_0^r \varphi^0(r)\, r^2 \, \mathrm{d}r$ playing the role of such a charge opposite in sign. This 'charge' is distributed over the whole space.

The sphere of finite radius r is crossed by a nonzero vector flux – this is the 'hair' of the vector meson field. In this case it is useful to consider (as we have agreed earlier) collapse of small masses, whose gravitational radius is much smaller than $\hbar/m_v c \sim \sim 10^{-13}$ cm. The possibility of principle of collapse of small masses was discussed by Hawking (1970) and Zel'dovich (1962), An arbitrary small mass of a neutral matter can be brought into collapsing state by means of a huge pressure.

If the problem is to 'organize' a collapse of, e.g. one gram of neutrons then when neutrons are localized in a domain far smaller than $\hbar/m_v c$ the numerical value of the second term in Equation (28), 'field charge', becomes negligible. When the domain of localization is still larger than the gravitational radius of the system ($r_{\mathrm{gr}} \sim 10^{-28}$ cm) by many orders of magnitude, when the gravitational forces may still be neglected the generalized Gauss theorem is of the form

$$\int E_n \, \mathrm{d}s \cong -4\pi g. \tag{30}$$

In other words, there arises a purely electrodynamic analog of the Gauss theorem with all its consequences, in particular, and with respect to the baryon field of a system of neutrons, i.e. the vector meson field. If such a system was brought into a collapsing state then, in virtus of (30), this black hole would possess baryon 'hair'.

The assertion that outside a black hole the vector flux E_n vanishes implies that inside the black hole this flux vanishes too when approaching the surface of the event horizont. This means that the 'field charge' integrated over the internal space of the black hole increases in some way to the extent that it becomes able of compensating the disappearance of the potential which has occured outside the black hole. We cannot let the flux E_n vanish near the Schwarzschild sphere without an appropriate increase of the potential inside the black hole.

If we consider the situation with the flux in question in a closed world then it is easy to check that the potential (25) provides vanishing of the flux E_n on the 'boundary' for $\chi=\pi$. This possibility is explained by the fact that inside the closed world a term with increasing exponential is added to the expression for the potential. At this expense the density of the 'field charge' increases to the extent needed for the baryon charge to be compensated. *In this sense the total 'baryon charge' (r.h.s. of Equation (27)) in a closed world is zero. In this sense the closed world is also neutral in the baryon charge too.* Due to the change of the potential 'hair' of the vector meson field goes into the closed world.

The problem arises as to what is the reason for which the integral of the vector meson field potential inside a black hole increases when this object emerges in the process of collapse. The fact is that the boundary conditions at the euclidean infinity for a collapsing system are known to remain unchanged.

In other words, it is impossible to make the external field of a black hole only vanish. We should, figuratively speaking, 'drive' it inside the black hole so that to increase the integral $m_v^2 \int \varphi^0 \, \mathrm{d}V$ inside the black hole up to a value which would compensate the total baryon charge (9).

In any case it is still unclear how the generalized Gauss theorem is fulfilled in the process of collapse. It is impossible for the time being to assert that black holes have no external baryon field.

On the other hand, the evidence that black holes have no external vector meson field under definite initial assumptions seems also to be convincing. The initial assumptions for this proof seem also to be quite natural: it is assumed that the weak meson vector field does not change the Schwarzschild metric, that there exists an event horizon and that the potential is finite at the event horizon.

The problem would be completely resolved if we succeeded in finding matchable internal and external solutions. Unfortunately, we have not yet derived an external static solution of the Nordström-Reissner type for the short-range, i.e. meson vector field. Until this programme is realized the above questions cannot be answered. An exact solution of the problem can give rise to some surprises. The example of the scalar field is, in this case, the most instructive one. In the case of the scalar field it is surprising that the event horizon is absent, and in those exceptional cases when it exists the field potential on the Schwarzschild sphere is found to be divergent.

2. The Scalar Long-Range Field

For the scalar field a problem similar to the Nordström-Reissner one was solved by Fischer (1948) a quarter of a century ago. Fischer obtained the metric in the form

$$\mathrm{d}s^2=\left(\frac{Z-Z_0}{Z+Z_1}\right)^p \mathrm{d}t^2-\frac{r^2}{Z^2}\left(\frac{Z-Z_0}{Z+Z_1}\right)^p \mathrm{d}r^2-r^2(\mathrm{d}\theta^2+\sin^2\theta\,\mathrm{d}\varphi^2), \tag{31}$$

here

$$Z_{0,1}=(\varkappa^2 m^2+\varkappa G^2)^{1/2}\mp\varkappa m, \tag{32}$$

G is the scalar charge,

$$p = \varkappa m(\varkappa^2 m^2 + \varkappa G^2)^{-1/2},$$

$$Z(r) = re^{(\nu-\lambda)/2} \underset{r\to\infty}{\to} r, \qquad g_{00} = e^{\nu}, \qquad g_{11} = -e^{\lambda}, \tag{33}$$

and

$$(Z - Z_0)^{1-p}(Z + Z_1)^{1+p} = r^2. \tag{34}$$

The calculations of Fischer were independently obtained by Janis *et al.* (1968). By a simple transformation the metric of the latter authors transforms into the metric (31). According to (31) g_{11} nowhere becomes infinite. Both g_{11} and g_{00} tend to zero, at $Z \to Z_0$.

Both papers contain some errors in the analysis of the asymptotic behaviour of the metric, but these errors do not concern the form of the linear element (31) which is calculated correctly. The metric (31) privides evidence that in the case of the long-range scalar field the horizon is absent, the Schwarzschild sphere is absent. In this case an object like the black hole is impossible.

It is remarkable that the event horizon is absent for any weak scalar field. But it is interesting that the transition to the limit (for a scalar charge $G=0$) transforms finally the metric (31) to the Schwarzschild metric although not continuously.

In fact, for $G=0$,

$$\begin{aligned} &p = 1, \qquad Z_0 = 0, \\ &Z_1 = 2\varkappa m, \qquad Z + 2\varkappa m = r. \end{aligned} \tag{35}$$

Inserting the values for $G=0$ in the expression (31), we obtain the Schwarzschild metric. The metric (31) at $r \to 0$ and $G \to 0$ behaves in a nonanalytic way: Z as $r \to 0$, tends to zero too, but at the very limit ($r=0$) according to (35), Z assumes by jump the value (see Figure 1)

$$Z = -2\varkappa m. \tag{36}$$

If the external metric (31) is considered as valid for arbitrary small distances, then at $r \to 0$ there arises the case of a bare singularity with all serious consequences following from it.

Such a behaviour of the metric for small r testifies rather in favour of the invalidity of the metric (31) in vacuum for any small r.

It is appropriate to give certain simple considerations which stress particular features of the systems charged by the scalar field source. Let the bare mass of a matter distributed over a domain of radius r_0 be M_0 and the total scalar charge be given by G. Generalizing to this case the well-known relation (Arnowitt *et al.*, 1960) for the total mass we get

$$M_{\text{tot}} = M_0 - \frac{\varkappa M_{\text{tot}}^2}{2c^2 r_0} - \frac{G^2}{2c^2 r_0}, \quad \text{or} \tag{37}$$

$$M_{\text{tot}} = -\frac{2r_0c^2}{\varkappa} + \left(\frac{4r_0^2c^4}{\varkappa^2} + \frac{2r_0c^2M_0}{\varkappa} - \frac{G^2}{\varkappa}\right)^{1/2}. \tag{38}$$

According to (38), the total mass of the system vanishes for

$$r_{\min} = \frac{G^2}{2M_0c^2}. \tag{39}$$

The system in question cannot be localized in the domain $r < r_{\min}$. Here we meet one of the numerous peculiar properties of the scalar field.

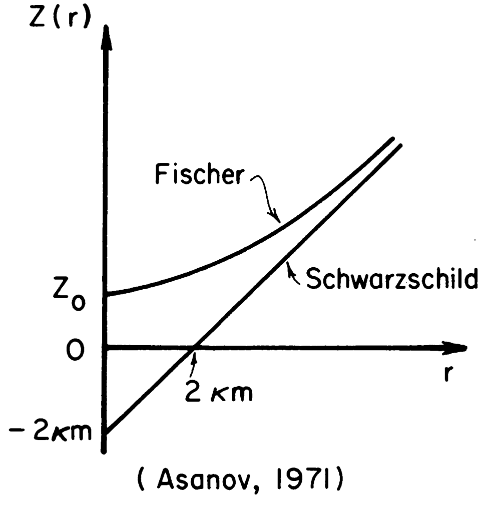

Fig. 1. Fischer's solution for a point source of massless scalar field and the corresponding Schwarzschild solution.

From the above considerations concerning the metric (31) and the specific properties of the scalar field we draw the conclusion that the problem of the scalar field in the process of collapse (let us be careful) is still waiting for its solution. All the aspects of this problem will are clarified when one finds the internal solution of a collapsing system (taking into account the effect of the scalar field on the metric) and the external solution matched with it, and, which is essential, *outside the framework of perturbation theory*.

This problem is essentially nonstatic. It becomes more complicated by that, contrary to electrodynamics, here the central symmetric motion of matter can give rise to a monopole emission of a scalar field which alters the mass of the system. Above we were dealing with the scalar field, more exactly, with the traditional equation of the scalar field

$$\nabla_\sigma \nabla^\sigma U = 4\pi j.$$

But there exists another form for the scalar field equation (Penrose, 1963; Chernikov and Tagirov, 1968)

$$\nabla_\sigma \nabla^\sigma U + \frac{R}{\sigma} U = 4\pi j, \tag{40}$$

where R is the scalar curvature.

The problem is as to whether certain surprising results of the foregoing consideration may be due to the unfortunate form of the scalar equation or not. Sometimes authors stress that the conformal invariant Equation (40) has some advantages. However, the analysis of the problem with recourse to Equation (40) makes the situation more critical.

In this case the event horizon is also absent (Bocharova *et al.*, 1971).

But in a particular case for

$$G^2 = 3\varkappa m^2 \tag{41}$$

there arises the event horizon (the Schwarzschild sphere). There arises an external metric of the type (18):

$$\begin{aligned} g_{00} &= e^\nu = e^{-\lambda} = \left(1 - \frac{a}{r}\right)^2, \\ a &= \varkappa m = \left(\frac{\varkappa G^2}{3}\right)^{1/2}. \end{aligned} \tag{42}$$

However, contrary to the electrostatic case, in this metric, the scalar potential on the event horizon surface becomes infinite:

$$U = -\frac{G}{r-a}. \tag{43}$$

In this case we have explicitly a couner-example which prŏvides direct evidence that the black hole possesses an external scalar field.

As far as at the event horizon the potential U becomes infinite then the theorem that the field should vanish outside the black hole is inapplicable to the present case (Chase, 1970): it is essentially associated with the assumption about the finiteness of the potential on the Schwarzschild sphere.

We should bear in mind that the scalar curvature R in Equation (40) for the given case (in vacuum) vanishes. The equation for the scalar potential takes on the usual form. This fact gives, however, no grounds for concluding that a conformal invariant case of the theory must not differ from the ordinary one.

The matter is that we should consider the system of equations as a whole, while the Einstein equations have in this case, essential differences. So, one of the Einstein equations, in the writing $G_\mu^\nu = -8\pi\varkappa T_\mu^\nu$ is of the form

$$\left(1 - \frac{\varkappa U}{3}\right)\left(\frac{1}{r^2} - \frac{\lambda'}{r} - \frac{e^{-\lambda}}{r^2}\right) = -\frac{\varkappa}{3} U'^2 - \varkappa \nu' (U^2)'.$$

While, in a nonconformal invariant case:

$$\frac{1}{r^2}-\frac{\lambda'}{r}-\frac{e^{-\lambda}}{r^2}=-\varkappa U'^2.$$

The difference between these equations is a consequence of different expressions for the energy-momentum tensor T^{ν}_{μ}.

Attempts to consider the problem in a consistent way, find the appropriate internal and external solutions for a nonstatic metric, were made by Price (1971). Unfortunately these attempts were made in the framework of perturbation theory, under the assumption that it is possible to disregard the effect of the scalar field on the metric. If the metric (31) holds then this consideration is invalid. When using perturbation theory we should bear in mind that the peculiarity of the metric (31) consists in that the event horizon disappears for any value of the charge of the scalar field, for the exception of the limiting value $G=0$. However, using a particular example, the author comes to the conclusion that any catastrophic values of the scalar potentials do not arise at the horizon. But one should bear in mind that the properties of the scalar field are very perculiar (Dicke, 1964), and the particular example considered by the author is not free of objections (Markov, 1972). Besides, there is an explicit counter-example (42). This example gives also evidence for the invalidity of perturbation theory since any small violation of equality (41) results in vanishing of the horizon.

The instability of this solution recalls the instability of the Papapetrou solution. In fact, any small violation of the rigorous equality of gravitational attraction and electrostatic repulsion leads to a collapse. But the main point is that there is an explicit counter-example (43).

The most essential critical remark that can be made concerning the metric (31) is the following. It is doubtful whether the consideration of a purely static problem in the collapse of a matter charged by the scalar field sources is valid. The possibility of a monopole radiation of the scalar field can just change essentially the situation.

It may be noticed that the consideration of long-range scalar forces is of purely abstract interest, since it is very likely that this kind of forces does not exist in nature. May be, of more importance are the short-range scalar fields, scalar meson fields. Recently, these fields have been discussed in elementary particle theory and are of fundamental importance when attempting to construct a unified theory of weak and electromagnetic interactions (Weinberg, 1967). But the monopole radiation of scalar mesons in the process of collapse can be arbitrarily strongly suppressed by the large mass of the quanta of this field, and thus in this case the reproach for ignoring monopole radiation is cancelled. Moreover, the Price considerations about the role of the effective potential barrier are no longer applicable to this 'short-wave' field.

The finding of the external metric of a spherical symmetric source of the meson scalar field still encounter unsolved difficulties.

Recently Asanov (1973) obtained a numerical solution for the system of the Einstein equations and the Klein equation

$$\left(\nabla_\sigma \nabla^\sigma + \frac{\mu^2 c^2}{\hbar^2}\right) U = 4\pi j \tag{44}$$

for the spherical symmetrical case we are interested in.

The numerical solution was found for the following values of the parameters

$$\varkappa G^2 = \varkappa^2 m^2, \qquad \mu = \frac{1}{\varkappa m} = \frac{1}{\varkappa^{1/2} G}. \tag{45}$$

The results of calculations are given in Figure 2. The function e^λ of its values at infinity ($e^{-\lambda} \approx 1 - (2\varkappa m/r)$) increases smoothly and reaches its maximum ($e^\lambda \approx 9.5$)

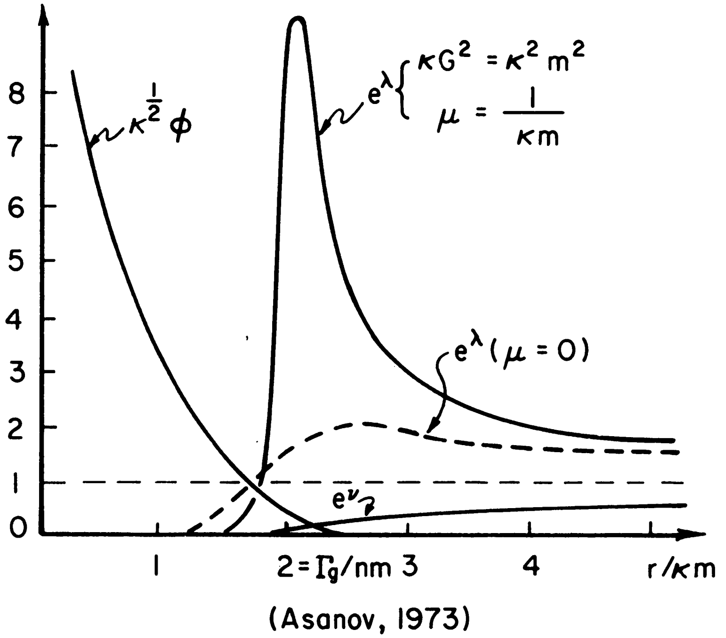

Fig. 2. $g_{00} = e^\nu$ and $g_{11} = e^\lambda$ in the case of scalar field of a point source of mass μ. e^ν and $e^\lambda$are equal to zero at $r \to 0$.

near 'the gravitational radius' $r_{gr} = 2\varkappa m$. Then e^λ falls tending smoothly to zero for $r = 0$. The function e^λ is equal to unity for $r \approx 1.8\ \varkappa m$. The function e^ν is monotonous, it is equal to zero for $r = 0$ and to unity at the spacial infinity.

If we assume that at present the existence of the black hole is confirmed by experimental data, and that on the other hand, the above theoretical results concerning the specific properties of the scalar field, incompatible with the existence of black holes, are valid, then we should conclude that the presence of black holes would testify to the absence of scalar mesons in nature. This would mean that the well-known theoretical attempts trying to unify weak and electromagnetic interactions on the basis of symmetry breaking are groundless. And, on the contrary, if the presence of scalar mesons is confirmed by experiment then the objects which at present are thought of as black holes will be interpreted in some other manner. In the theoretical aspect we should be led to the conclusion that in real situations there never arise $T_{\pm}$ domains the possibility of which we have already got accustomed to, and the metrics of Figure 3 becomes much simpler. At the same time, there arise objects close to bare singularities with all the consequences resulting from this. In general,

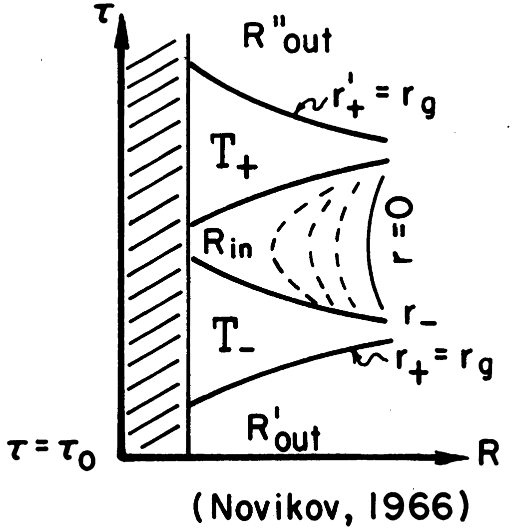

Fig. 3. The evolution of a charged sphere after collapse behind the Schwarzschild surface without taking into account the process of pair creation.

the 'price' of the proof of the validity or invalidity of the static metric (31) up to distances smaller than the gravitational radius is very high, and we should be very careful in estimating all arguments for and against. The numerical solution for, e.g. a model with scalar and electrostatic field sources, shows that up to $r=0.9\varkappa m$, where $e^{\lambda}>1$ and it is possible to match the internal and external solutions, the horizon is absent. (Asanov, 1972.)

3. The Neutrino Field

The situation with neutrino forces is far more complicated. In fact, if there is a direct interaction of the type $(e\nu)(e\nu)$ then the system consisting e.g. of hydrogen should induce in the surrounding space a neutrino-antineutrino field with potential (Tamm, 1934):

$$B \sim \frac{1}{r^5}.$$

There is a direct calculation (Hartle, 1971) for the vector-pseudovector case where it is shown that when neutrino field sources come nearer the Schwarzschild sphere the neutrino field in the external space tends to zero. But the calculation of the scalar case by this method does not result in the disappearance of neutrino hair outside the black hole (Berezin, 1973). Moreover, for a point of neutrino forces of the vector case localized at the point $\chi=0$ of a closed world, as in electrostatics, there arises a mirror image of the sorce for $\chi=\pi$ (Berezin).

In other words, one would think that the appropriate neutrino hair should not disappear. On the other hand, it would seem that Hartle (1971) gives for the Kerr metric a general proof of the absence of the external neutrino field of holes independently of the type of forces. This proof is also valid for the Schwarzschild metric.

If in further investigation we do not find errors in the calculations for the scalar case then the general consideration of Hartle contains some defects.

Unfortunately, the field under discussion is not the solution of some equation of the Maxwell type, and the conservation law for leptons is not associated with any analog of the Gauss theorem. It appears that in the case of the neutrino field, as in the foregoing cases of meson fields, the ultimate solution for the behaviour of these fields outside black holes arises in the process of finding matchable internal and external solutions for the systems in questions.

The foregoing consideration of the external metrics of black holes with different charges should be supplemented with one general, and in our opinion, essential remark.

In fact, as yet studies have not been made of the inverse process, process of formation of hair (e.g. vector meson field, scalar field) which inevitably occurs in the process of anticollapse, when matter goes out of the Schwarzschild sphere.

4. Black Holes as Intermediate States in Elementary Particle Theory

We are not aware of whether black holes exist in nature. But there some grounds to suppose that these objects and the corresponding notions may turn out to be essential in elementary particle theory.

In fact, in contemporary theory of elementary particles the calculation of proper masses leads to divergent values. This is due to the fact that the intermediate states in these calculations may have energy, and consequently, mass of arbitrary large values.

In contemporary elementary particle theory a striking violation of logic became historically legitimate: one introduces in consideration intermediate states with arbitrary large masses and, at the same time, entirely ignores their gravitational effects. It is of special importance that these masses must be localized in a very small domain so that there arises the possibility of huge gravitational mass defects capable of changing in a cardinal manner the energy spectrum of intermediate states.

In fact, if a particle in the intermediate state emits a quantum of mass $m = \hbar\nu/c^2$ then, according to the Heisenberg relation*, this mass is localized in the domain

$$l \sim \frac{\hbar}{mc}, \quad \text{or} \quad m = \frac{\hbar}{lc}. \tag{46}$$

* When emitting a quantum of an energy $E = \hbar\nu$ a particle, using the Fermi terminology, 'borrows' an energy $E = mc^2$. According to the uncertainty relation, the time of 'borrowing' may not be longer than $\hbar/mc^2$. During this period the emitted quantum cannot go away from the particle not far than at a distance $\sim \hbar/mc$.

When a mass of the order of

$$m \sim \left(\frac{\hbar c}{\varkappa}\right)^{1/2}$$

appears in the intermediate state, then the gravitational radius of this system is

$$r_{gr} = \frac{2\varkappa m}{c^2} = \frac{2(\varkappa \hbar c)^{1/2}}{c^2}. \tag{47}$$

On the other hand, with this mass the dimension of the domain in which, according to (46), the mass is localized

$$l \sim \frac{\hbar}{mc} \sim \frac{(\hbar c \varkappa)^{1/2}}{c^2} \tag{48}$$

coincides with the gravitational radius of the object in this state. With further increasing intermediate state energy the gravitational radius should have increased. But, on the other hand, the domain of localization of the intermediate state energy should, according to the Heisenberg relation, have decreased and for $m > (\hbar c/\varkappa)^{1/2}$ should have became smaller than the gravitational radius.

If such a situation arose in the range of applicability of classical physics we would say that we are dealing with a system whose mass is behind the gravitational Schwarzschild sphere. In other words, we would imply a system in the collapsing state. This might be either the state of black hole of the first kind or rather the state of systems with semi-closed matric if the bare mass of the intermediate state decreases strongly due to gravitational defect. From this viewpoint it would seem that the states of semi-closed systems or the states of black holes must belong to the complete set of states that can arise spontaneously in these cases. Moreover, energetically these states are the lowest states. Although at present we do not know to what extent our understanding of the metric remains valid in this state. However, in modern theory one got accustomed to use the euclidean metric at arbitrary small distances from the point particle. If black holes have really no hair then in the spectrum of the mediate states such states must in a cardinal manner, affect the results of calculations, since in these states there are no longer interactions with the given fields.

Appendix 1

(The role of the vector meson field in a distant stage of collapse of stars).

Novikov (1966) considered the evolution of a charged sphere after collapse behind the Schwarzschild sphere. The spacetime picture of this evolution is given by Novikov in Figure 3 in the Cruscal type coordinates. The region occupied by matter is shaded.

In the course of contraction its boundary crosses the Schwarzschild sphere. For

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a charged sphere of mass M and charge ε

$$r_{gr} = r_+ = \frac{\varkappa M}{c^2}\left[1 + \left(1 - \frac{\varepsilon^2}{\varkappa M^2}\right)^{1/2}\right],$$

where $\varkappa$ is the gravitational constant, c – the light velocity. From the external domain (R'_{out}) the system falls in a contracting T_- – domain.

For the sizes

$$r_- = \frac{\varkappa M}{c^2}\left[1 - \left(1 - \frac{\varepsilon^2}{\varkappa M^2}\right)^{1/2}\right]$$

contraction can be changed by expansion. This occurs in the internal (R_{in}) domain. After passing the expanding T_+ domain the sphere boundary crosses again the Schwarzschild sphere and goes out in the external (R''_{out}) domain (Figure 3). However, as is seen from the figure, the domain R''_{out} is not the same in which the contraction of the sphere occurs. The R''_{out} domain lies in an absolute future with respect to R'_{out}. The peculiarity of the situation consists in that the behaviour of the system after crossing the surfaces r_- is not defined by the initial conditions in the R'_{out} space. Thus, in mechanics there first arises the possibility for the Laplace determinism to be violated*.

As it happens always in these cases, congenial scientific conservatism searches naturally for possibilities of solving the problem in some other way, namely in the framework of usual regularities. It is a cruel necessity alone that can affect in a cardinal way the existing viewpoints.

In fact, it seems that there is such a natural possibility. The matter is that in the process of contraction of a charged sphere there arise outside it electric fields of a so high intensity that *a purely classical consideration of the problem neglecting the production of different kinds of charged particle pairs in these fields is quite inadmissible*.

In fact, for $\varepsilon^2 \ll \varkappa M^2$, due the process of contraction, a charged sphere on the boundary of the T_- domain acquires dimensions $r_- \sim \varepsilon^2/2Mc^2$.

The electric potential on the surface of this sphere should have taken the value:

$$V \sim \frac{\varepsilon e}{r_-} = Mc^2 \frac{e}{\varepsilon}, \quad \text{or} \quad eE = \frac{\varepsilon e}{r_-^2} = \frac{eM^2c^4}{\varepsilon^3}.$$

For $M = M_\odot \sim 10^{34}$ g, $\varepsilon < 10^{20}\, e$.

In other words, the electrostatic field energy, localized outside a sphere of radius r, turns out to be equal to the total energy of the system. However, in reality, such a potential cannot be realized due to electron-positron pairs production effect. Electron-positron pairs are produced long in advance until the charged sphere reduces to dimensions r_-.

Thus, the situation arises when because of production of new particles we are

* The reference of Penrose (1968) to the fact that in quantum theory we have got accustomed to indeterminism is a misunderstanding: the behaviour of the wave functions is completely defined by the initial data.

deprived of the convenient *comoving coordinate system* for describing the process of collapse and, moreover, the classic space-time description losses its meaning when a pair emerges, electrons and positrons may be spaced from one another by an essential space-like interval (Zel'dovich, 1971). It is possible that electrons should not reach the surface r'_- if, (for instance) in T_-, and R_{in} domains will appear a charge of opposite sign (Novikov, 1970) or the all picture may be essential the other.

It should be noticed that the inclusion of the electric charge in the system is a very artificial method and bears no direct relation to a really collapsing star. However, the really collapsing star is immediately associated with the meson vector field.

The vector meson field is a short-range one, therefore until a collapsing system turns out to be contracted up to dimensions of the order $\hbar/m_v c$ (m_v – the vector meson mass), this field may not be of importance in the process of collapse. However, with further contraction ($r<\hbar/m_v c$) the vector field becomes practically a long-range one, but at the same time it is stronger than the electromagnetic field. The meson field is capable of producing nucleon-antinucleon pairs.

The model of the electrically charged sphere has the shortcoming that the electric charges are, in this model, a mechanical admixture to neutral matter. Strictly speaking, the whole previous consideration of collapse of electrically charged matter is meaningful when each particle of this matter is electrically charged.

In the case of an electrically neutral star, each of its nucleons is a carrier of the meson vector field charge.

It should also be added that the collapsing mass of a star of the order of the solar mass has a density $\varrho \sim 10^{72}$ g cm^{-3} at dimensions $\sim \hbar/m_v c \sim 10^{-13}$ cm which is by 20 orders of magnitude smaller than the critical 'quantum' density at which, as one hopes without any definite grounds, collapse of a star can change by anticollapse. Thus, there are serious grounds to assume that the appearance of R_{in} and R''_{out} in Figure 3 and the violation of the Laplace determinism is a consequence of an abstract approach to the process.

It is remarkable that the critical density $\varrho_q \sim 10^{93}$ g cm^{-3} would be reached for collapsing masses

$$m \sim \varrho_q \left(\frac{\hbar}{m_v c}\right)^3 \sim 10^{55} \text{ g}$$

i.e. of the order of the mass of the whole Universe.

Appendix 2

A possible appearance of a semi-closed metric in the process of anticollapse (in particular, white holes) was considered by Frolov (1973).

Frolov considered a more general case of motion of a massive charged, radiating spherically symmetrical shell. The external metric of a radiating system found by Vaidya (1971, 1953) was used in his paper. The equations of motion of the shell are derived following Israel (1967, 1968). A detailed analysis is given for the case, when radiation occurs during a short period of the proper time. In this case the connection is found of the parameters of the system before and after radiation or absorption of both energy and charges of the system.

The effect of these processes on the total mass of the system for different types of the shell (anticollapse, collapse, usual motion) is analysed. It is shown that in the case of an open system with small charge, a transition to semiclosed state can occur due to energy radiation. If the radiation transfers the charge, then the final states can have the friedmons metric.

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