7. COLLECTIVE PLASMA PHENOMENA AND THEIR ROLE IN THE DYNAMICS OF THE INTERSTELLAR MEDIUM

Introductory Report

(Wednesday, September 10, 1969)

B. B. KADOMTSEV and V. N. TSYTOVICH

Fizicheskij Institut P. N. Lebedev, Akademiya Nauk S.S.S.R., Moskva, S.S.S.R.

1. Introduction

The First Symposium on cosmical gas dynamics clearly demonstrated that magnetic fields play an essential role in the dynamics of the interstellar matter. The Symposium emphasized that the behavior of interstellar gas should be described not by traditional gas dynamics but with the help of magneto-gas dynamics. As we now realize this was only the first step, since in plasmas a very broad range of phenomena can occur which are coupled not with large-scale magneto-gas dynamic motions but rather with more subtle collective plasma motions. To describe properly the dynamics of an ensemble of charged particles we should use not only average characteristics such as density, pressure and mean velocity, but also more detailed features of the particle distribution function.

At present, plasma physics is becoming a well-developed branch of modern physics. It contains an elaborate system of concepts, based both on theoretical investigations of simplified models and on many experimental studies. These investigations have led to the general picture of a plasma as a very unstable medium, in which many types of oscillations and noises can be easily excited. Such oscillations are not only the large-scale Alfvén and magnetosonic waves, which are already known from magneto-gas dynamics, but also the pure plasma oscillations. These oscillations can interact with plasma particles leading toward energy and momentum exchange. If the amplitudes are sufficiently high, nonlinear interaction between these oscillations is significant for the behavior of the plasma.

It is the purpose of the present paper to discuss the essential role that collective processes play in cosmic plasmas. The first part (Sections 1 through 5) deals in a general way with the concepts developed in recent years of collective phenomena in plasmas. Various types of instabilities, which are associated with the excitation of waves by particles (for example by cosmic rays), are considered. A picture is presented of nonlinear interactions and wave transformations that produce the spectra of oscillations in a weakly turbulent plasma. In the second part of this Report (Sections 6 through 10) acceleration of particles in a turbulent plasma is considered, and the role of this process in the generation of cosmic rays is discussed. The role of Langmuir turbulence is emphasized particularly. The turbulence together with the electromagnetic radiation that it excites and with the induced Compton scattering, gives a power-type spectrum of relativistic particles.

Habing (ed.), Interstellar Gas Dynamics, 108–132.

The first part of the report is written largely by B. B. Kadomtsev, the second part largely by V. N. Tsytovich.

2. Waves in a Plasma

The presence of electromagnetic forces in an ensemble of charged particles leads to an elastic behavior. For example, if electrons are displaced relative to ions, an electric field is produced which tends to return the electrons to their initial position. When a magnetic field is present the quasi-elastic forces occur even if the electrons and ions are displaced together. In contrast to usual hydrodynamics, where small velocities can give rise to large displacements, a weak motion in plasmas does lead to small oscillations around some stationary state. It appears that such oscillations play a significant role in plasma dynamics. First we consider three cases with no external magnetic field present. Then we turn to the more complex case of plasma oscillations in the presence of an external field.

A. LANGMUIR OSCILLATIONS

Consider first the 'Langmuir' or 'plasma' oscillations which arise when the electrons are displaced relative to the ions, causing small perturbations in the state of quasi-neutrality. This type of oscillation is the simplest example, in which the long-range Coulomb forces are essential. Let the electrons be cold and let their density n_0 be constant in space. Give the electrons a small displacement $\boldsymbol{\xi}$. For simplicity take the magnetic field zero. Then

$$\ddot{\boldsymbol{\xi}} = -\frac{e}{m_e}\mathbf{E}, \tag{1}$$

where e is the charge of an electron, m_e its mass, and $\mathbf{E}$ the electric field strength. Such displacement leads to a perturbation of the charge density

$$n'_e = n_0 \operatorname{div}\boldsymbol{\xi}. \tag{2}$$

Since the ion mass is much larger than the electron mass, we may consider the ions to be at rest and, therefore,

$$\operatorname{div}\mathbf{E} = -4\pi e n'_e = 4\pi e n_0 \operatorname{div}\boldsymbol{\xi}. \tag{3}$$

Since curl $\boldsymbol{\xi}$ and curl $\mathbf{E}$ both vanish, it follows from Equations (1) and (3) that, in an infinite plasma,

$$\ddot{\boldsymbol{\xi}} = -\omega_{pe}^2\boldsymbol{\xi} \tag{4}$$

where the 'Langmuir frequency' $\omega_{pe} = \sqrt{(4\pi e^2 n_0/m_e)}$. Equation (4) holds for any displacement $\boldsymbol{\xi}(\mathbf{r})$. As we shall see below it is very useful to expand $\boldsymbol{\xi}(\mathbf{r})$ in a Fourier series, i.e. to consider $\boldsymbol{\xi}(\mathbf{r})$ as a superposition of plane waves

$$\boldsymbol{\xi}(\mathbf{r}) = \int \boldsymbol{\xi}_k \exp(i\mathbf{k}\cdot\mathbf{r})\,\mathrm{d}\mathbf{k}. \tag{5}$$

Each harmonic ξ_k in this expression corresponds to a plane wave, which propagates in the direction of the wave vector $\mathbf{k}$ with the phase velocity ω/k. If the electron temperature T_e (the dimension of T_e is energy, it is expressed in eV (Ed.)) is not zero, the gradient of the electron pressure should be taken into account in Equation (1). But as this gradient is proportional to k, it may be neglected for $k \ll \omega_{pe}\sqrt{(m_e/T_e)}$. So far we have assumed the absence of an external magnetic field. If such a field is present, the equation of motion (Equation (1)) is more complicated. But in practical astrophysical cases the cyclotron frequency $\omega_{Be} = eB/m_e c$ is much less than the plasma frequency ω_{pe}, and the magnetic field does not influence the frequency of the Langmuir oscillations.

B. ION-SOUND WAVES

If the electron temperature T_e is at least three times as great as the ion temperature, then in a plasma in the absence of a magnetic field another branch of longitudinal oscillations occurs, the so-called 'ion-sound waves'. In such waves the ions oscillate inertially and we have

$$\ddot{\xi}_i = \frac{e}{m_i}\mathbf{E} \tag{6}$$

where m_i is the ion mass. Since the electrons are more mobile, they can come to an equilibrium. Hence, we have

$$T_e \nabla n_e = -\,e\mathbf{E}\,. \tag{7}$$

In the equation for $\mathbf{E}$, we must take into account the perturbations in both the ion and the electron densities. Using, again, $n_e' = n_e - n_0$ we get

$$\operatorname{div}\mathbf{E} = -\,4\pi e n_0 \operatorname{div}\xi_i - 4\pi e n_e'\,. \tag{8}$$

From these equations, it is easy to find the expression for the frequency ω of a plane ion-sound wave

$$\omega^2 = \frac{k^2 V_s^2}{1 + k^2 d_e^2}\,. \tag{9}$$

Here $V_s[=\sqrt{(T_e/m_i)}]$ is the sound velocity and $d_e(=\sqrt{[T_e/(4\pi e^2 n_0)]})$ is the Debije length. The wave propagates for small k with the sound velocity V_s, whereas for large k, the frequency is near the ion Langmuir frequency $\omega_{pi} = \sqrt{(4\pi e^2 n_0/m_i)}$.

C. ELECTROMAGNETIC WAVES

If there is no external magnetic field, the transverse waves (for which $\operatorname{div}\mathbf{E} = 0$) are completely separated from the longitudinal waves. They are described by Maxwell's equations, in which the current density $\mathbf{j}$ is expressed as a function of $\mathbf{E}$ by the equations of motion. The calculations give for the frequency ω

$$\omega^2 = \omega_{pe}^2 + k^2 c^2\,. \tag{10}$$

For large k this is the usual electromagnetic wave. For small k the frequency ω does not become arbitrarily small but approaches ω_{pe}, and if $\omega < \omega_{pe}$ the electromagnetic waves cannot penetrate in the plasma.

D. PLASMA OSCILLATIONS IN A MAGNETIC FIELD

If an external magnetic field is present, but if it is sufficiently weak (i.e. if $\omega_{Be} \ll \omega_{pe}$), the above expressions for Langmuir and electromagnetic waves remain valid, but a new qualitative feature appears – the possibility of penetration of low frequency waves in a plasma. In the language of magnetohydrodynamics, these are known as Alfvén waves and as (fast and slow) magnetosonic waves. Alfvén waves correspond to the displacement of plasma across **B** and **k** without compression. The frequency is $\omega = k_{\parallel} V_A$ where $V_A [= B/\sqrt{(4\pi n_0 m_i)}]$ is the Alfvén velocity and where $k_{\parallel}$ is the wave vector in the direction of the magnetic field. In magnetosonic waves, plasma compression occurs, which (for a fast wave) is in phase and (for a slow wave) is in antiphase with the enforced magnetic field. (The slow wave is, in some sense, a modified ion-sound wave.) The high-frequency behavior of the oscillations is strongly influenced by the dispersion, i.e. by the dependence of the phase velocity ω/k upon the wave number k. It is necessary to distinguish between waves of different circular polarization (in the case of propagation along the magnetic field lines) or between waves of different elliptical polarization (in the general case). Alfvén waves have a strong dispersion and become polarized in the direction of the ion Larmor rotation when the frequency of the waves ω is near the ion cyclotron frequency ω_{Bi}. An Alfvén wave can propagate only if $\omega < (k_{\parallel}/k)\omega_{Bi}$. A magnetosonic wave can propagate at much greater frequencies up to the electron

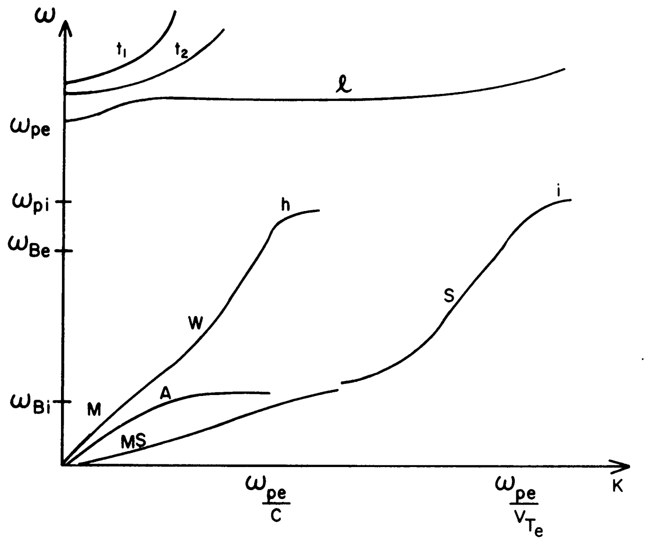

Fig. 1. Wave number k vs frequency for various oscillation modes. MS = slow magnetosonic, S = ion-sound, A = Alfvén, M = fast magnetosonic, w = whistlers, h = hybrid modes, l = Langmuir waves, and t_1 and t_2 are the two polarizations of the transverse waves.

https://doi.org/10.1017/S0074180900004848 Published online by Cambridge University Press

cyclotron frequency ω_{Be}. If $\omega \gg \omega_{Bi}$ the ions are at rest and the oscillations are known as 'whistlers'. These conclusions follow from the equation for the frequency of a wave that propagates along the magnetic field:

$$\frac{k_\parallel^2 c^2}{\omega^2} = 1 - \frac{\omega_{pe}^2}{\omega(\omega \pm \omega_{Be})} - \frac{\omega_{pi}^2}{\omega(\omega \mp \omega_{Bi})}. \tag{11}$$

Here the upper and lower signs correspond to the polarization in the direction of ion and electron rotation, respectively. From Equation (11) one can see that for very low frequencies $\omega^2 = k^2 V_A^2$, as found before. For the whistlers (upper sign) we have

$$\omega \approx \frac{k_\parallel^2 c^2 \omega_{Be}}{\omega_{pe}^2}. \tag{12}$$

The qualitative dependence of the frequency ω upon the wave number k is shown for different branches in Figure 1.

3. Plasma Instabilities and Excitation of Oscillations *

The importance of oscillations in plasmas is determined by the question how special the conditions are by which the oscillations are excited. In this respect, it is amazing that many types of oscillations can be excited so easily. By intuition we connect this with the great mobility of a plasma. The interaction with the magnetic and electric fields gives the particles a collective behavior that very easily entails energy exchange between waves and particles. Spontaneous wave excitation in plasma is a result of its instability – i.e. a large number of theoretical investigations show that various different, infinitesimally small deviations from a stationary equilibrium will grow exponentially. We shall not consider configurational instabilities produced by large scale inhomogeneities in the magnetic fields and in the plasma, because such instabilities can be well described by magnetohydrodynamics and are not called plasma instabilities in the proper sense. But outside the framework of magnetohydrodynamics there exist kinetic instabilities, in which oscillations are excited by resonant particles and which are very sensitive to the form of the particle distribution functions.

A. AMPLIFICATION AND DAMPING OF LANGMUIR OSCILLATIONS; CHERENKOV RESONANCE

The simplest example of the kinetic instabilities is the excitation of Langmuir oscillations by an electron beam. Suppose that on the tail of the electron distribution function $f(V)$(where V is the electron velocity) there exists a 'hump' so that $(\partial f/\partial V)_{V=V_0} > 0$. Now consider the Langmuir wave with the phase velocity $V = V_0$, i.e. with the wave number $k = \omega_{pe}/V_0$. It can be easily seen that such a wave will interact very strongly with the particles near $V = V_0$ (resonance). Indeed, in the frame of reference that moves with such particles, the wave is at rest, the particles feel the same phase for a long time, and the wave can change the particle energy considerably. If $(\partial f/\partial V)_{V_0} > 0$ there are more

* The authors use the terms 'oscillation' and 'wave' without distinction. (Ed.)

particles faster than the wave than there are slower than the wave. So the transfer of energy from the particles to the wave is greater than the transfer in the reverse process and, consequently, the amplitude of the wave grows with time. However, if $(\partial f/\partial V)_{V_0}<0$ the amplitude decreases with time ('Landau damping'). The amplification of waves by a particle beam corresponds to the emission of induced Cherenkov radiation. Particles moving in a medium with a velocity greater than the phase velocity $V_0=\omega/k$ emit Cherenkov radiation. The condition $(\partial f/\partial V)_{V_0}>0$ means that near $V=V_0$ the population of the energy levels is inverted, i.e. the temperature is negative, which is a necessary condition for the maser effect to occur. From this point of view the expansion of the oscillations in plane waves is not only a formal mathematical procedure but it also has a real physical sense – each wave has its own phase velocity and interacts with a separate group of resonant particles. For large wave number k the phase velocity becomes small and for $k>d_e^{-1}$, where d_e is the Debije length, the damping of the wave by the thermal electrons becomes very large. Excitation of oscillations by a beam causes energy losses for the resonant particles. As a result, a 'plateau' is created in the distribution function, i.e. $\partial f/\partial V$ vanishes in vicinity of $V=V_0$. More exactly, the 'plateau' exists only around those velocities where the possibility for the induced emission is satisfied, i.e. $\partial f/\partial V>0$ initially. Theoretically this effect has been investigated in the so-called quasi-linear approximation. The creation of a plateau and anomalously large losses of kinetic energy were found in a great number of experiments dealing with the interaction of particle beams and tenuous plasmas. In the preceding paragraph, V represented the longitudinal component of the velocity. Using the same letter for $|\mathbf{V}|$ we can write the condition for Cherenkov resonance

$$V \cos \Theta = \omega/k, \tag{13}$$

where Θ is the angle between $\mathbf{V}$ and $\mathbf{k}$. From Equation (13) it follows that if the phase velocity ω/k is fixed, a larger value of V gives a smaller interval for Θ in which interaction is possible.

B. ANOMALOUS RESISTIVITY

The instability described in the preceding paragraph can also occur when a current exists in a plasma. The instability is most pronounced when the drift velocity of the electrons $U=|\mathbf{j}|/en_0$ ($\mathbf{j}$ is the current density) is greater than the ion-sound velocity $V_s=\sqrt{(T_e/m_i)}$. In this case the Joule heating makes the electron temperature larger than the ion temperature and so the necessary condition for the existence of ion-sound waves is fulfilled. A similar condition as before $(U>V_s)$ leads to amplification of the ion-sound oscillations and the current density saturates even if the electric field applied to the plasma continues to increase. Usually this fact is called 'anomalous resistivity' or 'turbulent heating'. The role of the fast electrons in this process is not yet completely clear but the existence of anomalous resistivity has been proven experimentally. In cosmic conditions anomalous resistivity will probably be important near surfaces with opposite magnetic fields where a high current density can exist. Such regions can at the same time be the sources of the plasma waves.

C. CYCLOTRON RESONANCE

In the presence of an external magnetic field the charged particle rotates with the cyclotron frequency

$$\omega_B = eBc/\varepsilon, \tag{14}$$

where $\varepsilon = \sqrt{(m^2c^4 + p^2c^2)}$ is the energy of the particle. This gives the possibility of cyclotron resonance when, in the frame of reference moving along the field, the frequency of plasma oscillations equals the cyclotron frequency or its harmonics. (The harmonics of the cyclotron frequency are effective only if the transverse wave length is large compared to the Larmor radius.) Taking the Doppler shift into account, the resonance condition is of the form

$$\omega - k_\parallel V_\parallel = l\omega_B \tag{15}$$

where ω is the plasma frequency and where $k_\parallel$ and $V_\parallel$ are the projections on $\mathbf{B}$ of the wave vector $\mathbf{k}$ and the particle velocity $\mathbf{V}$, respectively. l is an integer (positive or negative). If $l>0$, we have the normal and if $l<0$, we have the anomalous Doppler effect. The meaning of this is more easily understood if one interprets Equation (15) in terms of energy and momentum conservation laws by means of a quantum mechanic analogue. Consider the charged particle as a harmonic oscillator with equidistant energy levels (Landau spectrum). If a quantum is emitted with energy $\hbar\omega$ and momentum along the magnetic field $\hbar k_\parallel$, we have

$$\begin{aligned} \hbar\omega + V_\parallel \Delta p_\parallel - l\hbar\omega_B &= 0 \\ \hbar k_\parallel + \Delta p_\parallel &= 0 \end{aligned} \tag{16}$$

where l represents the change in the orbital quantum number. If $l>0$ the transverse energy of the particle decreases, but if $l<0$ (the anomalous Doppler effect) the emission of waves is accompanied by an increase in the transverse energy. Using this analogy it is easy to find the condition for induced emission, i.e. for instability: we require that the population of the energy levels be inverted, i.e. the value of the distribution function $f(p_\parallel, p_\perp)$ in the initial state is larger than that of final state $f(p_\parallel + \Delta p_\parallel, p_\perp + \Delta p_\perp)$. In the nonrelativistic case $\Delta p_\perp = -l\hbar\omega_B m/p_\perp$, *i.e.* for small $\Delta p_\parallel$ and $\Delta p_\perp$

$$\int_{-\infty}^{+\infty} \left(k_\parallel \frac{\partial f}{\partial p_\parallel} + \frac{l\omega_B m}{p_\perp} \frac{\partial f}{\partial p_\perp} \right) \times \delta(\omega - k_\parallel V_\parallel - l\omega_B)\, \mathrm{d}p_\parallel\, \mathrm{d}p_\perp > 0. \tag{17}$$

Here the δ-function insures that only resonant particles are able to emit or absorb the wave. For relativistic particles this condition is of the same form, but ω_B depends on the particle energy and $\omega_B m = eB/c$. Using Equation (15) we rewrite Equation (17) as follows

$$\int_{-\infty}^{+\infty} \left[\frac{\omega}{p_{\parallel}}\frac{\partial f}{\partial p_{\parallel}} + l\omega_B\left(\frac{1}{p_{\perp}}\frac{\partial f}{\partial p_{\perp}} - \frac{1}{p_{\parallel}}\frac{\partial f}{\partial p_{\parallel}}\right)\right] \times \delta(\omega - k_{\parallel}V_{\parallel} - l\omega_B)\,\mathrm{d}p_{\parallel}\,\mathrm{d}p_{\perp} > 0\,. \tag{18}$$

The second term in the integral is important only if the distribution function is anisotropic $[(1/p_{\parallel})(\partial f/\partial p_{\parallel}) \neq (1/p_{\perp})(\partial f/\partial p_{\perp})]$. If the energy in the transverse direction is larger than that in the longitudinal direction, $(1/p_{\perp})(\partial f/\partial p_{\perp}) > (1/p_{\parallel})(\partial f/\partial p_{\parallel})$ and the instability is due to the normal Doppler effect; while in the opposite case it is due to the anomalous Doppler effect.

D. INSTABILITY OF COSMIC RAYS

The instability of plasma oscillations due to normal and anomalous Doppler effects may be responsible for the observed high degree of isotropy of cosmic rays (Ginzburg, 1965; Tsytovich, 1965, 1966a; Lerche, 1967; Wentzel, 1968). Indeed, even for a small anisotropy Equation (18) shows that instability can be found for waves with frequencies much less than the cyclotron frequency, *i.e.* for Alfvén and for magnetosonic waves. When the transverse pressure is greater than the longitudinal one, waves polarized in the direction of proton rotation (cosmic rays) are unstable due to the normal Doppler effect. If the transverse pressure is smaller, the wave with the opposite polarization is excited but, in this case, since the particles move faster than the phase velocity, the resonant particles rotate in the same direction as before. The instability can also be due to the presence of a drift velocity (*i.e.* absence of right-left symmetry along the magnetic field). Estimates show that for a very small anisotropy of about 1 percent as well as for a small longitudinal disturbance in the distribution function, the instability can develop with a growth time of the order of 10^2 years, which is negligibly small compared to the characteristic diffusion time of 10^6 years for cosmic rays in the Galaxy. Isotropization of cosmic rays may be due also to the presence of fluctuations in the electric and magnetic field that exist independently of the cosmic rays. However, the instability described here is the result of a direct response of the plasma to anisotropy in the distribution function and, therefore, the isotropization mechanism will be more effective when the density of the cosmic rays is sufficiently high.

E. POSSIBLE MECHANISMS OF WAVE EXCITATION IN THE COSMIC RAY PLASMA

Global anisotropy of the cosmic rays is only one of the possible causes for kinetic instability of the cosmic ray plasma. The instability may appear also because the cosmic rays flow out of the region of their origin. Moreover, in magnetohydrodynamic waves (*i.e.* low frequencies and large scales) regions of low magnetic field are produced which act as magnetic bottles. In such bottles, as many laboratory experiments indicate, the nonequilibrium distributions of fast particles seem to produce fast oscillations. This example shows that a low frequency oscillation of the magnetic field can cause an excitation of high frequency waves. In addition, excitation of high frequency waves, *e.g.* Langmuir waves, can take place at shock fronts, which occurs, for example, when beams are injected into regions having large magnetic field variations. Also, high frequency electromagnetic radiation may excite waves of other branches by nonlinear decay processes (see *e.g.* Figure 1 for such branches). In other words, in the presence of violent plasma motions, it is not surprising that oscillations are excited in all possible branches.

4. Nonlinear Interaction of Waves

If waves with finite amplitudes are excited, nonlinear wave interaction (*i.e.* anharmonicity) becomes essential, for which several different mechanisms exist. If the excitation of the waves is weak all mechanisms are of a resonant nature: the interaction is important only when frequency and wave number fulfill certain conditions. To the lowest order of the oscillation amplitude such processes are known either as three-wave processes of coalescence and of decay of waves or as induced wave scattering by plasma electrons and ions. The frequency and wave number conditions for the first process (decay and coalescence of waves) are of the form

$$\begin{aligned} \omega - \omega' &= \omega'' \\ \mathbf{k} - \mathbf{k}' &= \mathbf{k}'', \end{aligned} \tag{19}$$

where ω, ω', ω'' are the frequencies, and $\mathbf{k}$, $\mathbf{k}'$, $\mathbf{k}''$ are the wave vectors of the interacting waves. In the second process (scattering of an ω-wave into an ω'-wave) we have

$$\omega - \omega' = (\mathbf{k} - \mathbf{k}') \cdot \mathbf{V}, \tag{20}$$

where $\mathbf{V}$ is the velocity of the particles on which the scattering occurs. In the decay process the total energy and momentum of waves are conserved, while in the scattering process the total number of plasmons* is preserved. The scattering process causes energy transfer into the region of lower k and the decay processes result in the broadening of the spectrum. In some cases it is necessary to take into account four-wave processes, corresponding to the scattering of plasmons by one another. The waves participating in the interaction may belong to different branches of plasma oscillations. In this case the nonlinear interaction is an induced transformation of one type of oscillation into another. (For example, plasma waves are transformed into electromagnetic waves, or high-frequency waves into low-frequency waves.) The characteristic times of the nonlinear interactions are very small on an astrophysical scale. This means that the plasma oscillation spectra are determined by nonlinear interactions. For example, the time τ characteristic for transformation of the energy of Langmuir waves due to the scattering on ions is about

$$\frac{1}{\tau} \approx \omega_{pi} \frac{V_s}{V_{Te}} \frac{W}{n_0 T_e}. \tag{21}$$

* Somewhat similar to the association of photons with electromagnetic waves, one associates so-called 'plasmons' with plasma waves. The energy of a plasmon is $\hbar\omega$, the momentum $\hbar k$. Equation (19) expresses conservation of energy and momentum when two plasmons merge to give a third plasmon. Equation (20) may be written as $\Delta E = \Delta p \cdot V$ where ΔE and Δp are the difference in the energy and the momentum before and after scattering. (Ed.)

Here W is the turbulent energy per cm^3, $n_0 T_e$ is the thermal energy, ω_{pi} is the ion plasma frequency, and V_{Te} is the thermal velocity of the electrons. If W is of the order of $10^{-6}\, n_0 T_e$, then for $n_0 \approx 0.01\ cm^{-3}$ τ is of the order of 10^3 sec. A comprehensive summary on nonlinear interactions of waves in plasmas may be found in the monographs by Kadomtsev (1964), by Tsytovich (1967), and on linear properties by Stix (1962) and by Spitzer (1962).

5. Weak Turbulence

It is customary to call a plasma state 'weakly turbulent' if waves are excited in large numbers but with comparatively small amplitudes. As in usual turbulence, a transfer of turbulent energy occurs from the region where it is created to the dissipation region. This transfer takes place because of nonlinear interactions between the turbulent oscillations. On the one hand, turbulence in a plasma is more complicated than that in liquids because so many types of motions and oscillations are possible. On the other hand, it is easier to deal with in the sense that if the amplitudes are low, the oscillations can be well described by means of expansion methods with a small expansion parameter – the ratio of the energy of the oscillations to the energy of the thermal particles. The corresponding methods have been recently developed and applied successfully in investigations of different types of plasma turbulence in which only one branch of oscillations is important – magnetohydrodynamic, ion-sound, Langmuir turbulence and others. However, usually oscillations in one branch also excite to some extent the other branches, *i.e.* transformation occurs of one type of the waves into another and then it is necessary to take many types of oscillations into account.

6. Turbulence of the Interstellar Medium and the Acceleration of Cosmic Ray Particles

Many observations show that apart from ordinary thermal plasma the interstellar medium also contains a large number of high-energy particles. First of all, there are the 'cosmic rays' consisting of ultrarelativistic particles (protons and heavier nuclei) with energies from 10^9 to 10^{19} eV. Second, there are particles with energies of the order of 10^7 to 10^8 eV, called 'subcosmic rays'. In the third place, the existence of electrons of energies between 10^8 and 10^9 eV is inferred, because they are responsible for the cosmic radio emission. It is possible that electrons exist with both higher and lower energies. Although the number density of high-energy particles is not large, their total energy density is quite comparable with the energy density of the thermal plasma. This implies that the high-energy particles are able to interact significantly with the thermal plasma. The fact that the cosmic rays are isotropic indicates that such interaction actually occurs. Presumably a number of local sources exist which produce fast particles that subsequently are isotropized by the plasma. For this process to take place the plasma should probably be turbulent. Therefore, we shall consider here a simple model of the interaction between high-energy particles and turbulent plasmas. It should be kept in mind that the interstellar turbulence is expected to be inhomogeneous, by which we mean that there exist both active regions with a high level of turbulence and comparatively quiet regions. For example, intensive turbulence may exist in supernovae shells, in active nuclei of galaxies, in the vicinity of pulsars, neutron stars and other active stars. Therefore, we shall consider a wide range of the temperatures, densities, and turbulence levels in the plasma.

There is one outstanding observational fact that should find a theoretical explanation, namely that the spectra of relativistic electrons and ions are usually of the type

$$f(\varepsilon) = \text{const} \times \varepsilon^{-\gamma}. \tag{22}$$

Here $f(\varepsilon)$ is the number of fast particles per cm^3 and per energy interval $\mathrm{d}\varepsilon$, so that $\int_0^\infty f(\varepsilon)\mathrm{d}\varepsilon = n$ is the total number density of the particles. It could be assumed that this spectrum is produced by statistical superposition of separate spectra arising from different active regions. In practice, however, the value of γ varies only slightly from source to source and it is within the range $1<\gamma<3$ with an average value $\gamma=2.7$. Therefore it is more probable that the acceleration mechanism itself produces this type of spectrum. The modern theory of turbulence provides such a mechanism.

7. Nature and Rate of Particle Acceleration in a Turbulent Plasma

A. ACCELERATION RATE

The rate of particle acceleration is characterized by an average energy gained by a particle in unit time. Let $\mathrm{d}\varepsilon/\mathrm{d}t$ depend on ε as

$$\mathrm{d}\varepsilon/\mathrm{d}t = \beta\varepsilon^{\mu}. \tag{23}$$

There is a qualitative difference between the two cases $\mu<1$ and $\mu>1$. When $\mu<1$, the high-energy particles are accelerated at a slower rate and they determine the time required for reaching a stationary spectrum. When $\mu>1$, the acceleration time is determined by the minimum energy, the injection energy. If we had $\mu<1$, spectra would be expected with sharp cut-offs at some high energy. Since these have not been observed, it seems that the case $\mu>1$ is preferable. In this case, one and the same mechanism could be responsible for acceleration of cosmic rays up to the highest energies and the usual argument, that the particles of highest energies have an origin outside of our Galaxy, would not work.

B. INFLUENCE OF RESONANCE OF PARTICLE-TURBULENT WAVE INTERACTION ON THE ACCELERATION RATE

A possible acceleration mechanism with $\mu>1$ is predicted by modern theory (see Section 2). According to Equation (13) Cherenkov resonance will occur if

$$\omega = kV\cos\Theta \tag{24}$$

from which follows

$$V > V_p \equiv \omega/k. \tag{25}$$

This condition shows that when the particle energy ε increases there is also an increase in the range of phase velocities at which the interaction works. Therefore, a larger number of turbulent oscillations participate in the acceleration and the acceleration rate increases with ε. In the presence of a magnetic field this remains true. To see why, consider the case when the phase velocity V_p is much less than the particle velocity, *i.e.* $\omega \ll k_{\parallel} V_{\parallel}$. According to Equation (15)

$$- k_{\parallel} V_{\parallel} \approx l \frac{ZeBc}{\varepsilon} \tag{26}$$

where Z is the charge of the accelerated particle.

From $| k_{\parallel} V_{\parallel} | < kV$ we obtain

$$\varepsilon > \frac{ZeBlc}{kV}. \tag{27}$$

When ε increases, smaller values of k are allowed and the wave number interval of interacting waves is broadened. Equation (27) assures that the acceleration rate increases with ε even in the ultrarelativistic limit $\varepsilon \gg mc^2$; this is not true in the case of Equation (25). Equation (27) implies that the Larmor radius of the particle $r_\lambda = \varepsilon V/(ZeBc)$ be larger than the wavelength $\lambda = 1/k$. When the energies are high, the Larmor radius of the particles becomes large, and only for small k will the acceleration rates depend appreciably on the energy.

It should be kept in mind that if k is very small not every type of wave is possible. But, as is seen in Figure 1, at least two types of waves with small k exist, the Langmuir and the Alfvén (or magnetosonic) waves. However, as k decreases, the phase velocity V_p of the Langmuir waves increases and becomes greater than the velocity of light for $k < \omega_{pe}/c$, so that Cherenkov resonance and cyclotron resonance are no longer possible. Corresponding to the wavenumber ω_{pe}/c there is a critical energy ε_c such that for $\varepsilon > \varepsilon_c$ the acceleration rate no longer increases with ε. Requiring that $\varepsilon_c \gg mc^2$ and by substituting $k = \omega_{pe}/c$ and putting $Z = l = 1$, we find from Equation (27)

$$\frac{B^2}{4\pi n_0 mc^2} \gg \frac{m}{m_e} \tag{28}$$

or, what is the same,

$$\frac{V_A^2}{c^2} \gg \frac{m^2}{m_e m_i} \tag{29}$$

Even when the accelerated particles are electrons ($m = m_e$) Equation (29) requires very strong magnetic fields ($V_A \gg c/40$). For nonrelativistic particles, i.e. subcosmic rays, Equation (25) gives an increase in acceleration rate with increase of ε. For magnetohydrodynamic and for Alfvén waves the phase velocity V_p does not depend on k and is less than V_A if $V_A > V_s$. Therefore, if the particle velocity V is much larger than V_A the condition $V \gg V_p$ that is required by Equation (27) is fulfilled for any small k. This means that if only very long waves are present (k very small) resonance and acceleration can take place only for particles with large ε, i.e. with large Larmor radii.

This restricts the acceleration to high-energy particles. However, it is known that the frequencies of magnetic oscillations (with wavelengths larger than the Larmor radii of thermal electrons and ions) are proportional to the cosine of the angle between the direction of wave propagation and the magnetic field, and this offers the possibility of interaction of the waves with particles with $V < V_p$. Writing $\omega = kV_A \cos\Theta$ and using the fact that $k_\parallel V_\parallel \ll \omega_B$, we obtain from Equation (15)

$$\frac{ZeBl}{mcV_A k} < 1\,. \tag{30}$$

Since Equation (30) does not contain the particle velocity it allows the injection of low energy particles. This is very essential for the explanation of the chemical composition of the cosmic rays because it gives a preferential injection of multicharged, heavy ions.

C. THE INVERSE COMPTON EFFECT ON PLASMA WAVES

In addition to the effects considered above, resonance with electromagnetic waves emitted by the turbulent oscillations is possible for high-energy particles. One such mechanism is the Compton effect, in which the particle oscillates in the turbulent field and emits an electromagnetic wave. From the point of view of elementary processes the effect corresponds to a Feynman diagram with absorption of a turbulent wave σ, and emission of an electromagnetic wave t. The resonance condition states that in the rest frame of the particle the frequency does not change:

$$\omega^\sigma - \mathbf{k}^\sigma \cdot \mathbf{V} = \omega - \mathbf{k} \cdot \mathbf{V}\,. \tag{31}$$

Here ω^σ and $\mathbf{k}^\sigma$ are the frequency and the wave vector of the turbulent wave, respectively; ω and $\mathbf{k}$ are those for the electromagnetic wave and $\mathbf{V}$ is the velocity of the particle. Let us consider the case of electromagnetic waves of very high frequencies and particles with ultrarelativistic energies. Then $\omega = kc$ and, if Θ is the angle between $\mathbf{k}$ and $\mathbf{V}$,

$$\omega = \frac{\omega^\sigma - \mathbf{k}^\sigma \cdot \mathbf{V}}{1 - (V/c)\cos\Theta}\,. \tag{32}$$

From the condition $\cos\Theta < 1$ we have

$$\omega < \frac{\omega^\sigma - \mathbf{k}^\sigma \cdot \mathbf{V}}{1 - (V/c)} \approx 2\,\frac{\varepsilon^2}{(mc^2)^2}\,(\omega^\sigma - \mathbf{k}^\sigma \cdot V)\,. \tag{33}$$

This means that the higher the energy of the particle, the higher the frequency of the electromagnetic waves that can interact with it.

The Langmuir oscillations with $V_p \gg c$ have been mentioned above. Then $\omega^\sigma \gg k^\sigma c$ and, a fortiori, $\omega^\sigma \gg k^\sigma V$. We may therefore approximate Equation (33) by

$$\omega < 2\,\frac{\varepsilon^2}{(mc^2)^2}\,\omega_{pe}\,. \tag{34}$$

This implies that the acceleration process does not depend on the wavelength distribution of the turbulent oscillations but only on the density of the turbulent energy. This is very important because in such a situation we can expect a universal spectrum of accelerated particles. But the theory should answer the question whether the main part of the turbulent energy can be concentrated in Langmuir oscillations with $V_p > c$; the theory should also predict the frequency distribution of electromagnetic radiation. We note that for other types of turbulent oscillations Equation (33) shows that the resonance depends appreciably on the wavelength of the oscillation. For instance, if $V \gg V_p = \omega^\sigma / k^\sigma$ we have

$$\omega < 2 \frac{\varepsilon^2}{(mc^2)^2} k^\sigma c \, |\cos \Theta| \,. \tag{35}$$

The problem of electromagnetic wave emission by turbulent oscillations has been discussed by Kaplan and Tsytovich (1969).

Plasma mechanisms of emission of electromagnetic waves is a subject of great interest for the interpretation of cosmic radio emission. But at the basis of such an interpretation is the spectrum of the accelerated particles, and first the mechanisms that produce such a spectrum should be understood. In radiation processes oscillations of essentially different kinds can operate, but for acceleration processes the Langmuir oscillations are of greatest interest. Indeed, transparent waves are important for cosmic radio emission and nontransparent waves for particle acceleration, due to the fact that the radiation that can be reabsorbed on the fast particles can interact with them most intensively.

D. ACCELERATION OF RELATIVISTIC PARTICLES BY THE WHOLE TURBULENT SPECTRUM

The acceleration depends upon the particle energy only if the wave numbers of the turbulent oscillations cover a wide interval, so that for particles of higher energy more oscillations can take part in the acceleration. When the whole wave number spectrum is involved (*i.e.* all the turbulent oscillations interact with the particles), the acceleration, which has a stochastic nature, will be determined by an energy diffusion coefficient that does not depend on the particle energy ε, so that

$$\varepsilon^2 = 4\,Dt\,, \tag{36}$$

or

$$\mathrm{d}\varepsilon/\mathrm{d}t = 2\,D/\varepsilon\,, \tag{37}$$

and according to Equation (23) $\mu = -1$. This statement is true only at ultrarelativistic energies. For nonrelativistic particles, even if waves of all available wave numbers are interacting, the diffusion coefficient depends on V and therefore on the energy ε, because with increasing velocity there is a decrease in the fraction of turbulent oscillations that produces acceleration. As can be easily seen from the Cherenkov condition [Equation (13)], the extra factor V^{-1} gives

$$\mathrm{d}\varepsilon/\mathrm{d}t = \beta\varepsilon^{-3/2}\,. \tag{38}$$

This situation occurs for the ion-sound waves whose phase velocities V_p do not exceed the sound velocity V_s. Therefore, if $V_p < V_s$, $\mu = -\frac{3}{2}$. Since the acceleration rate

https://doi.org/10.1017/S0074180900004848 Published online by Cambridge University Press

increases rapidly with the decreasing particle energy, it is very high at low energies, *i.e.* Equation (38) gives the possibility for an efficient injection mechanism.

These arguments show that the acceleration rate will increase with particle energy only for energies small enough, so that the particles interact with part of the turbulence spectrum. From Figure 1 it can be seen that for the whistlers the values of k are limited to a narrow interval $\omega_{pi}/c < k < \omega_{pe}/c$ and that their phase velocities are small. The ion-sound and magnetic-sound waves yield an acceleration rate given by Equation (38). Therefore, it is of primary interest to know the distribution of wave numbers for Langmuir and for magnetohydrodynamic turbulence.

8. Magnetohydrodynamic Collisionless Turbulence

A. TURBULENT SPECTRA

The recent development of the concept of plasma turbulence opens a new quarter in our knowledge of magnetohydrodynamic motions. Basically there exist two types of magnetohydrodynamic oscillations: the first (Alfvén and magnetohydrodynamic waves) are waves whose frequencies are much lower than the ion collision frequency ν_i, and whose wavelengths are much larger than the (particle) free path l_i (this is the traditional region of magnetohydrodynamics); the second are waves whose frequencies are much greater than ν_i and for which $kl_i \gg 1$ (this is the collisionless region). For collisionless waves the Landau damping and induced scattering on electrons and ions result in a transformation of turbulent energy to long wavelengths. For isotropic turbulence the distribution of the turbulent energy as a function of k may be characterized by W_k – the energy per cm^3 of the plasma in the wave number interval dk, so that

$$W = \int_0^\infty W_k \, \mathrm{d}k . \tag{39}$$

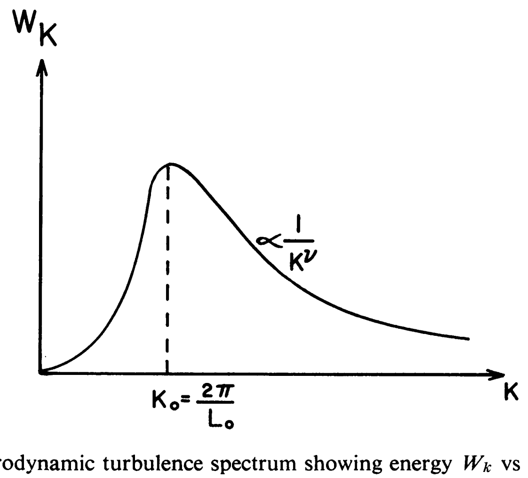

Fig. 2. Magnetohydrodynamic turbulence spectrum showing energy W_k vs k. The decrease of W_k at large wave number behaves as $1/k^\nu$.

For magnetohydrodynamic turbulence W_k differs only by a normalization factor from the distribution of turbulent energy of frequencies W_ω because of the linear dependence of ω on k.

If the magnetohydrodynamic waves have an anisotropic distribution, the turbulent spectrum characterizes their distribution integrated over all angles, and the acceleration is practically independent of angle.

The energy flow in the spectra of collisionless magnetohydrodynamic and Alfvén waves is in the direction of small values of k $(k \approx l_i^{-1} = v_i/V_{Ti})$ where the maximum of the turbulent energy will be found. Therefore in this case the basic scale of the turbulence has the order of magnitude $L_0 = 2\pi/k_0 = l_i$ at $V_A \gg V_{Ti}$. This spectrum is shown schematically in Figure 2. In the region of $k \gg k_0$, $W_k \propto k^{-\nu}$. If the characteristic dimension of the active region is less than the mean free path, as in the case of the solar wind, k_0 is defined by the dimension of the active region. For $V_{Ti} \ll V_A \ll V_{Te}$, Landau damping of magnetohydrodynamic waves is the primary cause of energy loss of turbulent oscillations because of the energy exchange between Alfvén and magnetohydrodynamic oscillations; $1 < \nu < 2$. This spectrum corresponds to that in the interplanetary magnetic field.

B. ACCELERATION OF PARTICLES

The rates of particle acceleration due to particle interaction with magnetohydrodynamic oscillations have been studied by Tsytovich (1963) and by Tverskoy (1967) (see Figure 3). For $\nu = 2$, we obtain $\mu = 1$, and the acceleration is similar to Fermi acceleration

$$\beta = \frac{V}{L_0} V_A^2 \frac{8\pi W}{B^2}. \tag{40}$$

Here $L_0 = 2\pi/k_0$ is the main scale length of turbulent spectra shown in Figure 2; this length and the Alfvén velocity, V_A, correspond respectively to the distance between

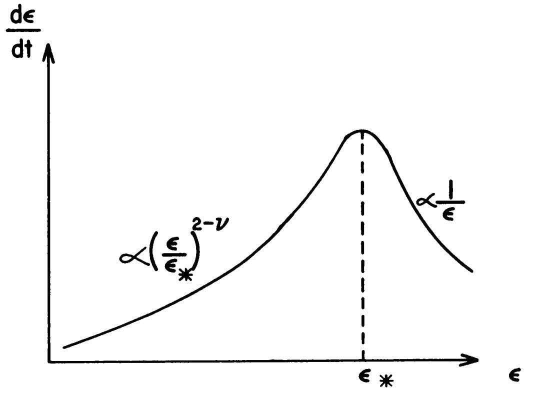

Fig. 3. Rate of energy gains, $d\varepsilon/dt$, vs energy of particles accelerated in magnetohydrodynamic turbulence. ε_* is the energy of a particle whose Larmor radius is the same as the main scale of the turbulence.

clouds and to the cloud velocity in the Fermi acceleration mechanism. In Equation (40) a new factor appears: the square of the ratio of the amplitude of Alfvén waves at the maximum of the energy-spectrum to the constant external magnetic field. According to polarization measurements this factor is of the order of 10^{-2} (Pikel'ner, 1968). We can conclude, therefore, that the acceleration due to magnetohydrodynamic oscillations is an equivalent of the Fermi acceleration in the framework of the modern concepts of plasma turbulence. In Figure 3, ε_* corresponds to the energy of a particle at which its Larmor radius becomes equal to the main scale length of the turbulence. This corresponds to the maximum wavelength for which the criterion of Equation (27) is fulfilled. The physical reason for the decrease of the acceleration rate for $\varepsilon > \varepsilon_*$ is that the particles interact with the whole turbulent spectrum and the diffusion coefficient becomes constant. It was frequently pointed out that the efficiency of the Fermi acceleration is very small (see e.g. Ginzburg and Syrovat-skii, 1963). Equation (40) is even more pessimistic because of the additional factor $8\pi W/B^2 \approx 10^{-2}$ discussed above. L_0 corresponds to the original value that was accepted for the Fermi acceleration and cannot be decreased. Moreover, $\nu = 2$ corresponds only to $\mu = 1$, but not to $\mu > 1$, and is reached only when $V_A \gg V_{Te}$. For $V_A \ll V_{Te}$ (and this is more probable under astrophysical conditions) $1 < \nu < 2$ and, therefore, $\mu < 1$. On the other hand, the injection of heavy multicharged ions can be done very effectively by Alfvén oscillations. The diffusion coefficient for such particles is proportional to $m^\nu Z^{2-\nu}$.

9. Langmuir Turbulence

It is convenient to characterize the spectrum of isotropic turbulence by W_k as was done in the preceding section. Just as in liquids, where the turbulent spectrum is defined by only one parameter, *i.e.* the turbulent energy flow, one single parameter determines the whole turbulent spectrum in a plasma. Let Q be this parameter denoting the oscillation energy generated per cm^3 per sec. Spectra of Langmuir turbulence have been calculated by Pikel'ner and Tsytovich (1968) for $T_e = T_i$ and by Tsytovich (1969) for $T_e > T_i$. The spectra are formed as a result of excitation by different kinds of instabilities. The most effective instabilities are those of low phase velocities close to the mean thermal velocity of electrons. Then the nonlinear interactions transform the phase of the waves in the following fashion. In the first stage, where $kd_e > (m_e/m_i)^{1/5}$, the transformation is due to scattering on electrons; in the second stage, $kd_e < (m_e/m_i)^{1/5}$, to scattering on ions; and in the third stage, for still greater phase velocities, the transformation takes place by collisions of plasmons. It is important to realize that the main part of the turbulent energy is concentrated in the region where the oscillations are not excited. The turbulent oscillations are very quickly transformed through the region of low phase velocities (large k). This transformation is due to a cascade process as in liquids, and the mean value for one step in this cascade is:

$$k_* = \sqrt{\left(\frac{m_e}{m_i}\right)} \frac{1}{d_e}. \tag{41}$$

If the source of turbulence (*e.g.*, the beam of particles) has phase velocities near V_{Te}, the spectrum for $k > k_{**} = 1/d_e(m_e/m_i)^{1/5}$ is $W_k \propto 1/k^{5/2}$ (see Figure 4). In the region $k_* < k < k_{**}$, the spectrum is flat and $W_k = \text{const}$ (Liperovskii and Tsytovich, 1969). If the phase velocities of the source are higher than $V_{Te}(m_i/m_e)^{1/5}$, the spectrum begins at the k number where the generation occurs. For $k \ll k_*$, collisions between the plasmons play an important role and form a 'Maxwellian' type distribu-

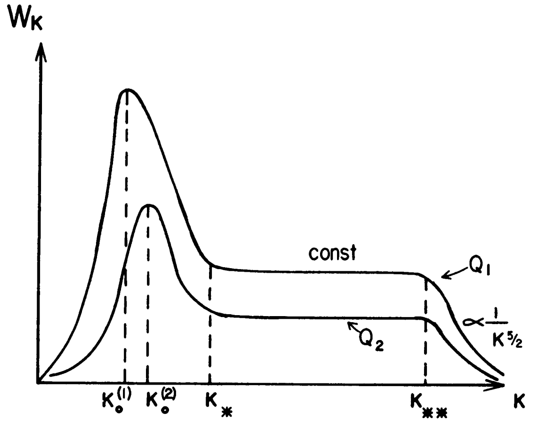

Fig. 4. The turbulent energy W_k for Langmuir turbulence shown as a function of k for two different rates of excitation Q_1 and Q_2.

tion. The maximum, $k = k_0$, of the spectrum corresponds to the energy-containing region, where the oscillations are damped by the usual collisions. In the asymptotic region, where $k \gg k_0$, Pikel'ner and Tsytovich found a spectrum $1/k^\nu$ where $2.84 < \nu < 4$. The existence of such spectra with a maximum at $k = k_0$ requires that $k_0 \ll k_*$. k_0 depends on Q (Liperovskii and Tsytovich, 1969)

$$k_0 = k_* \left(\frac{8 n_0 T_e \nu_e^2}{\omega_{pe} Q} \right)^{1/(2\nu - 2)} \tag{42}$$

where ν_e is the collision frequency of electrons. In Figure 4, the turbulent Langmuir spectra for $T_e = T_i$ are shown for two different values of the power generation, Q. For small Q the maximum in the spectrum disappears. This means that the power of turbulence generation is so low that the turbulent energy can be absorbed while transforming from the region of generation up to k_*. Since the slowest transformation takes place at large wave numbers, the growth rate of the instability must be close to threshold. Even for low Q's the maximum in the spectrum exists, and as a rule, k_0 is such that the phase velocity at k_0 is of the order of $V_p^0 \approx 10\omega_{pe}/k_* \approx 10^3\ V_{Te}$. In the majority of circumstances of astrophysical interest, this value is of the order of light velocity or higher which is important for the process of resonance emission.

10. Acceleration and Isotropization of Fast Particles by Langmuir Turbulence

A. ACCELERATION BY OSCILLATIONS WITH $V_p < c$

The subsequent events of induced emission and induced absorption of turbulent oscillations by fast particles give rise to particle diffusion both in angle and in energy. The diffusion in angle results in isotropization and that in energy leads to acceleration. The isotropization and the acceleration are produced by the same processes, but the isotropization is faster by a factor $(V/V_p)^2$. Therefore, all oscillations with low phase velocities such as ion-sound and magnetohydrodynamic waves lead more rapidly to isotropy than to acceleration. For the acceleration and isotropization by Langmuir turbulence, an essential parameter is

$$\eta = k_c/k_* . \tag{43}$$

Here the wavenumber $k_c(=\omega_{pe}/c)$ corresponds to a phase velocity equal to c, whereas k_* is the minimum value of k on the plateau of Figure 4 (p. 125). If $\eta \ll 1$ (that is $T_e < 10$ to 20 eV if $T_e = T_i$) an interval exists for $V_p < c$ where the spectrum is proportional to $\propto k^{-\nu}$; this interval is important for the interaction with subcosmic rays. The rates of isotropization and acceleration are then equal. If the plasma is hot ($\eta \gg 1$), the isotropization is faster than the acceleration by a factor of approximately $\ln [(c/V_{Te})(m_e/m_i)^{1/5}]$. Figure 5 shows schematically the acceleration rates for electrons and ions under conditions $\eta \gg 1$ and $\eta \ll 1$. If $\eta \ll 1$, the acceleration is effective and $\mu = \frac{1}{2}(\nu - 1)$, i.e. $\mu = 0.94$ for $\nu = 2.84$ and $\mu = \frac{3}{2} > 1$ for $\nu = 4$. The coefficient β is proportional to $\sqrt{Q}$. The main part of the Langmuir turbulence energy near the maximum is not effective in this mechanism. Nevertheless, very effective isotropi-

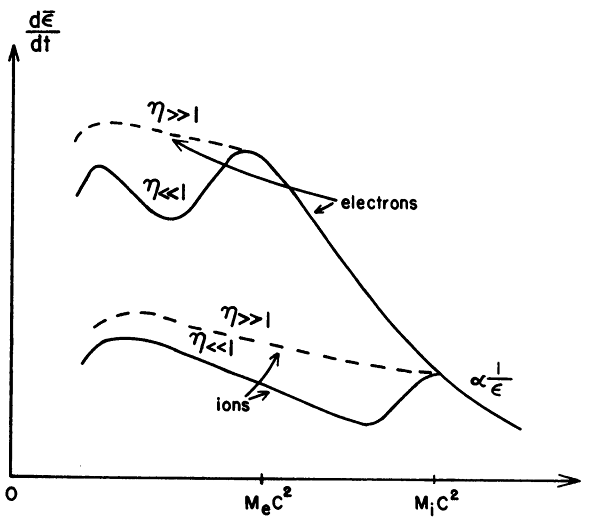

Fig. 5. Rate of energy gain for electrons and ions in Langmuir turbulence. The parameter η determines the ratio of acceleration to isotropization.

zation and acceleration of subcosmic rays can occur. According to the calculations given by Pikel'ner and Tsytovich (1969), the isotropization of subcosmic rays can be brought about by Langmuir turbulence in a heterogeneous model of the Galaxy, in which there is a mixture of active and passive regions.

B. PARTICLE ACCELERATION BY LANGMUIR OSCILLATIONS WITH $V_p > c$ DUE TO INVERSE COMPTON EFFECT

Now let us consider the acceleration arising from the interaction of relativistic particles with waves having wavenumbers at the maximum of the Langmuir turbulence spectrum. This interaction can lead to an effective acceleration only by the reabsorption of electromagnetic waves. In addition, the spectrum of the electromagnetic

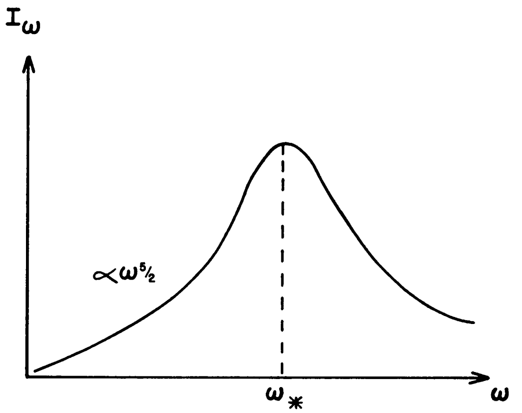

Fig. 6. Power spectrum of electromagnetic waves, I_ω, vs frequency ω in equilibrium with particles and Langmuir turbulence.

waves should have a form as sketched in Figure 6. In that figure, ω_* is the maximum frequency for which the active region is opaque for electromagnetic radiation emitted by fast particles.

Common for ultrarelativistic particles and well known for synchrotron radiation is the fact that the intensity $\propto \omega^{5/2}$. This behavior ($\propto \omega^{5/2}$) can be easily found from Equation (35) if we consider that $T_{\mathrm{eff}} \approx \varepsilon \approx mc^2 \sqrt{(\omega/2\omega_{pe})}$. Reabsorption of synchrotron radiation also gives an effective acceleration (Tsytovich, 1966b). We consider the sum of the synchrotron and plasma mechanisms of acceleration since the two mechanisms give the same energy dependences and differ from one another only by numerical factors. This can easily be seen if we compare Equation (35) with a corresponding condition for emission of synchrotron radiation

$$\omega < \left(\frac{\varepsilon}{mc^2}\right)^2 \frac{eB}{mc}. \tag{44}$$

From Equation (44) the maximum energy ε_*, for which effective acceleration can be expected, is found to be

$$\varepsilon_* = \sqrt{\left(\frac{2\omega_*}{\omega_{pe}}\right)}\, mc^2 . \tag{45}$$

For large active regions or for regions with a large number of relativistic particles, the value of ε_* can be very high. From Equation (35) it is clear that if $\varepsilon < \varepsilon_*$ the whole spectrum of radiation cannot take part and that the higher the energy of the particle the more waves can accelerate the particle. This growth of acceleration ceases at $\varepsilon \approx \varepsilon_*$, because practically all the waves take part in acceleration, and for $\varepsilon \gg \varepsilon_*$ the acceleration rate decreases as $1/\varepsilon$. Figure 7 shows the acceleration rates that correspond to these qualitative considerations (Tsytovich and Chikhachev, 1969). The relation

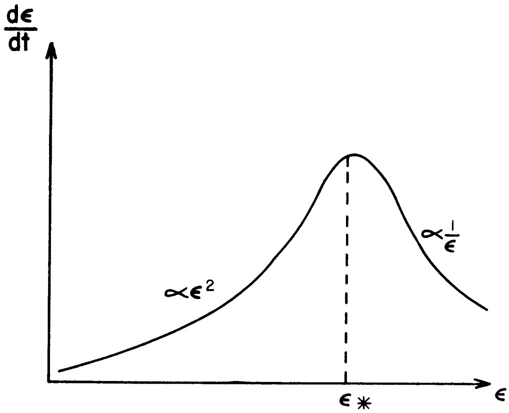

Fig. 7. Rate of energy gain of particles due to 'Compton' reabsorption of electromagnetic waves.

$d\varepsilon/dt \propto \varepsilon^2$, *i.e.* $\mu = 2$, corresponds to the above requirements of effective acceleration. It does not depend on the turbulent oscillation distribution and is therefore universal. The coefficient β is given, order-of-magnitude-wise, by

$$\beta = \frac{\omega_{pe}^4}{n_0^2 m}\left[W + a\sqrt{\left(\frac{eB}{cm\omega_{pe}}\right)}\frac{B^2}{8\pi}\right] \tag{46}$$

(Kaplan and Tsytovich, 1968) where a is a numerical factor of order unity. We can consider the Cherenkov acceleration with $V_p < c$ as an injection mechanism. Hence, the injection energy is determined by intersection of curves in Figures 5 and 7. Since both acceleration mechanisms are coupled with the same turbulent spectrum, the acceleration time depends only on the parameter Q:

$$\frac{1}{\tau} = bZ^{10/3}\left(\frac{m_e}{m_i}\right)^{4/3}\left(\frac{Q}{\nu_e n_0 T_e}\right)^{5/6} . \tag{47}$$

The parameter $Q/(\nu_e n_0 T_e)$ is the ratio of the turbulent energy to the energy of thermal motion and has a maximum value of about 1. Table I shows the coefficient b as a function of plasma density and temperature. This table shows that acceleration is extremely rapid in dense and hot turbulent plasmas. Applications to such objects as pulsars, quasars and supernova remnants can be made using the table.

TABLE I

Values of the coefficient $1/b$ for particle acceleration [see Equation (47)] (dimension of $1/b$: sec)

n_0 (cm^{-3})	T (eV) 10	10^2	10^3	10^4
10^{20}	10^{-3}	1.5×10^{-5}	3×10^{-6}	2×10^{-7}
10^{17}	0.6	10^{-2}	2×10^{-3}	10^{-4}
10^{14}	3×10^{2}	50	1	6×10^{-2}
10^{8}	10^{8}	1.5×10^{6}	3×10^{5}	2×10^{2}
10^{2}	3×10^{13}	4×10^{11}	10^{10}	6×10^{9}

11. Spectra of Accelerated Particles

A. THE ENERGY LOSSES OF FAST PARTICLES IN A TURBULENT PLASMA

To find the particle spectrum it is necessary to know not only the rate of acceleration but also the energy losses, because the balance between the acceleration and energy losses determines the stationary distribution of accelerated particles. In addition to the usual forms, a new type of energy loss exists in a turbulent plasma – the Compton effect on the plasma turbulent oscillations. It is analogous to the well-known cosmic ray energy losses from collisions with the black body radiation. In general, the energy loss on the Langmuir turbulence is largest, in which case the spectrum of radiation for a power law particle energy distribution [Equation (22)] is also a power law

$$Q_\omega = \text{const} \times \omega^{-\nu}, \qquad \nu = \tfrac{1}{2}(\gamma - 1). \tag{48}$$

In this sense, the plasma mechanism of radiation is similar to the synchrotron radiation. All the energy losses due to both the Compton effect on the turbulence and the synchrotron radiation are

$$\frac{d\varepsilon}{dt} = -\frac{16\pi\, e^4 Z^4}{3m^2c^3}\left(\frac{\varepsilon}{mc^2}\right)^2\left(\frac{B^2}{8\pi} + \frac{W}{6}\right). \tag{49}$$

The other types of energy losses, for example, the ionization or nuclear collision ones, are the same as in a quiescent plasma.

B. THE SPECTRA OF SUBCOSMIC RAYS

The stationary spectrum of accelerated particles arises as a balance of acceleration and energy losses. For low energy particles the most important losses are those of ionization and nuclear collisions. Similar to the Fermi mechanism, the acceleration by the magnetohydrodynamic and Alfvén oscillations results in a power law spectrum $\varepsilon^{-\gamma}$ only for a turbulent spectrum $\nu=2$. For $1<\nu<2$ the spectra do not follow a power law because the energy losses are due to processes that are very different from the acceleration, *i.e.* ionization and nuclear collisions. Therefore, even for $\nu=2$, there is a difference between acceleration and energy losses and the parameter γ can vary over wide intervals from 1 to ∞. Since this is contrary to the observations, it has been for a long time the principal argument against the Fermi acceleration mechanism. Therefore, this type of acceleration is probably more important as an injection mechanism, especially for heavy multicharged ions (Melrose, 1967).

Similarly, the acceleration by Langmuir oscillations with $V_p<c$ for $\varepsilon\gg mc^2$ does not result in a power law spectrum. The explanation again is due to the difference between the acceleration and energy loss mechanisms. For $\varepsilon\gg mc^2$ the distribution is Maxwellian with the effective temperature depending on the parameter Q

$$T_{\text{eff}} = \frac{mc^2}{2\gamma_0}; \quad \gamma_0 = \sqrt{\left(\frac{Q_c}{Q}\right)}; \quad Q_c = \frac{2m^2 v_e^2 n_0 T_e T_i}{27\pi m_i m_e \omega_{pe}(T_e + T_i)}. \tag{50}$$

$T_{\text{eff}}\gg mc^2$ at $\gamma_0\ll 1$, i.e. $Q\gg Q_c$. The quantity Q_c is very small. As an example, for a plasma beam instability one has $T_{\text{eff}}>mc^2$ even near the threshold of the instability.

Therefore, even the main part of the turbulent energy has $V_p>c$ the energy of the oscillations with $V_p<c$ is sufficient to give an effective injection for acceleration by radiation. For nonrelativistic subcosmic rays the distribution has the form

$$f(\varepsilon) = \frac{\text{const} \times \sqrt{\varepsilon}}{(\varepsilon + \gamma_0 T_e)^{\gamma_0}} \tag{51}$$

i.e. is a power law only if $\gamma_0>1$, $\varepsilon\gg\gamma_0 T_e$.

C. THE SPECTRA OF COSMIC RAYS

In the case of acceleration by radiation due to the Compton effect on the turbulence, the acceleration and energy losses are coupled by the same process (induced and spontaneous emission). The spectrum always is a power law for energies $\varepsilon\ll\varepsilon_*$. This is due to the fact that both the energy gain and energy losses are proportional to ε^2. Since β in Equation (23) depends on γ and γ is found independently from the equation for the particle energy distribution, we have an equation (Tsytovich and Chikachev, 1969) which shows that γ depends on two parameters $\kappa=W/(n_0 mc^2)$ and $\xi^{-1}=\omega_{pe}mc/(eB)$. The result of numerical solution of this equation is shown in Figure 8. It is interesting that γ is within a very narrow interval $1<\gamma<3$, that corresponds to the observed interval of spectra of radio sources. There exists also an extra point $\gamma=2.7$ for $\xi=10^{-2}$ to 10^{-3} that corresponds to the observed spectrum of cosmic rays. If Q and B are varied over wide limits, γ remains very close to 2.7. Therefore, the possibility exists here of determining the parameters of the turbulence from the spectral index of radio sources.

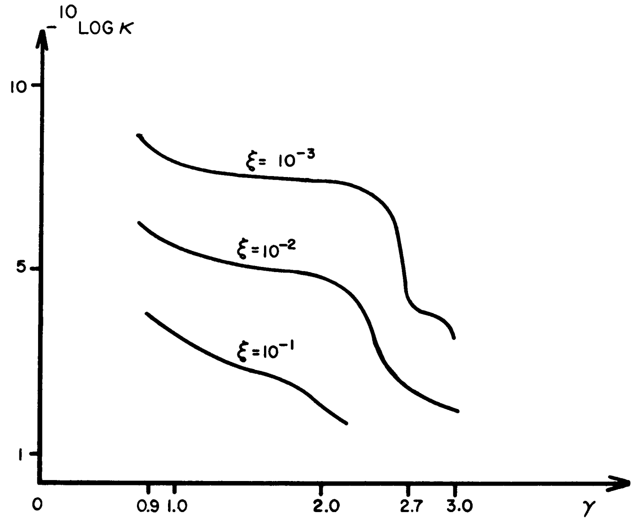

Fig. 8. The ratio of wave energy density to rest mass ($\kappa = W/n_0mc^2$) vs energy exponent γ for various values of the ratio of plasma to cyclotron frequency, $\xi^{-1} = mc\omega_{pe}/eB$.

The above analysis shows that at present radiative acceleration in a turbulent plasma may be the best mechanism for the explanation of the spectrum and the acceleration of cosmic rays.

12. Conclusion

At present, a broad system of ideas and conceptions concerning collective processes in rarefied plasmas has been developed. The application of this system to astrophysics has only begun, but even now it has led to an understanding of many processes which take place in cosmic conditions. It seems that the further development of these ideas and their wide application to astrophysics will make it possible to understand more clearly the essence of cosmic phenomena.

References

Ginzburg, V. L.: 1965, *Astron. Zh.* **42**, 1129 (1966, *Soviet Astron.* **9**, 877).
Ginzburg, V. L. and Syrovat-skii, S. I.: 1963, *The Origin of Cosmic Rays*, Izd. Akad. Nauk SSSR, Moscow (translation 1964, Pergamon Press, Oxford).
Kadomtsev, B. B.: 1964, *Plasma Turbulence*, Izd. Akad. Nauk SSSR, Moscow (translation 1965, Academic Press, New York).
Kaplan, S. A. and Tsytovich, V. N.: 1968, *Astron. Zh.* **45**, 777 (1969, *Soviet Astron.* **12**, 618).
Kaplan, S. A. and Tsytovich, V. N.: 1969, *Usp. Fiz. Nauk* **97**, 77 (1969, *Soviet Phys. Usp.*, under translation).
Lerche, I.: 1967, *Astrophys. J.* **147**, 689.
Liperovskii, V. A. and Tsytovich, V. N.: 1969, *Zh. Eksp. Teor. Fiz.* **57**, 301 (1969, *Soviet Phys. JETP*, under translation).
Melrose, D. B.: 1967, Preprint Belfer Graduate School, New York.
Pikel'ner, S. B.: 1968, *Ann. Rev. Astron. Astrophys.* **6**, 165.

Pikel'ner, S. B. and Tsytovich, V. N.: 1968, *Zh. Eksp. Teor. Fiz.* **55**, 977 (1969, *Soviet Phys. JETP* **28**, 507).
Pikel'ner, S. B. and Tsytovich, V. N.: 1969, *Astron. Zh.* **46**, 8 (1969, *Soviet Astron.* **13**, 5).
Spitzer, L.: 1962, *Physics of Fully Ionized Gases*, 2nd ed., Interscience, New York.
Stix, T.: 1962, *Theory of Plasma Waves*, McGraw-Hill, New York.
Tsytovich, V. N.: 1963, *Zh. Eksp. Teor. Fiz.* **44**, 946 (1963, *Soviet Phys. JETP* **17**, 643).
Tsytovich, V. N.: 1965, *Astron. Zh.* **42**, 33 (1965, *Soviet Astron.* **9**, 24).
Tsytovich, V. N.: 1966a, *Astron. Zh.* **43**, 528 (1966a, *Soviet Astron.* **10**, 419).
Tsytovich, V. N.: 1966b, *Usp. Fiz. Nauk* **89**, 89 (1966b, *Soviet Phys. Usp.* **9**, 370).
Tsytovich, V. N.: 1967, *Nonlinear Effects in Plasmas*, Izd. Akad. Nauk SSSR, Moscow.
Tsytovich, V. N.: 1969, Lebedev Physical Institute, preprint.
Tsytovich, V. N. and Chikhachev, A. S.: 1969, *Astron. Zh.* **46**, 486 (1970, *Soviet Astron.*, under translation).
Tverskoy, B. A.: 1967, *Zh. Eksp. Teor. Fiz.* **53**, 1417 (1968, *Soviet Phys. JETP* **26**, 821).
Wentzel, D. G.: 1968, *Astrophys. J.* **152**, 987.