CROSS-TIDAL EFFECTS AND ORBIT-ORBIT RESONANCES

THIERRY PAUWELS

Koninklijke Sterrenwacht van België, Ringlaan 3, B-1180 Brussel, Belgium

Abstract. In this paper we discuss the influence of the deformation of a planet caused by a first satellite on the orbit of a second satellite of the same planet, where both satellites are supposed to be involved in an orbit-orbit resonance. Numerical results are given for seven orbit-orbit resonances in the Solar System.

Key words: tides – orbit-orbit resonance

1. Introduction

Cross tides are the effect of the tidal deformation of a planet caused by one satellite on the orbit of another satellite of the same planet.

Since Darwin published his papers on the tidal evolution of the orbit of a satellite, it is well-known that the tidal deformation of a planet caused by one of its satellites, in combination with internal friction in the planet, will cause the orbit of this same satellite to expand or to shrink, depending on whether the spin of the planet is faster or slower than the orbital motion of the satellite. Although it is obvious that this tidal deformation of the planet must have an influence on the motion of all objects inside the gravitational field of the planet, and hence that the orbit of any other satellite of that same planet must be influenced by it, until now very little attention has been given to this cross-tidal effect.

Many of the papers about tidal evolution of orbits have the evolution of the moon's orbit in mind, so they need not worry about other satellites. Kaula (1964), however, is somewhat more general, and his comment shows how little interest has been given to cross-tides. Although formulae at the start are derived for the case where the disturbing and the disturbed satellite are different, on page 675 he writes 'In general this will be true only for ..., i.e. the tide-raising and disturbed satellite are identical.', after which he considers only this latter case.

Tidal effects on the orbit of a satellite are indeed extremely small, and only when there are non-periodic (i.e. secular) effects, the influence is worth studying. Inspection of the equations given by Kaula (1964) shows that in the case of cross tides these effects are formally purely periodic. However, it is well-known that in orbit-orbit resonances formally periodic terms can give rise to perturbations which have a non-periodic component in time. The same will happen with the cross tides between two satellites involved in an orbit-orbit resonance. Only in two papers we have found this effect mentioned.

Jeffreys (1961) merely mentions the fact that these cross-tidal effects must exist when the two satellites are involved in an orbit-orbit resonance, but he does not give any computation. On the other hand, Sinclair (1984) has computed the effect in the special case of a 1:1 resonance, i.e. for satellites librating around a Lagrangian point. In this paper we shall exclude the case of a 1:1 resonance and concentrate on the other types of orbit-orbit resonances.

S. Ferraz-Mello (ed.), Chaos, Resonance and Collective Dynamical Phenomena in the Solar System, 215–218.

2. Outline of the mathematical procedure

It is not intended to give here the full mathematical description. This will be published elsewhere. However, we can provide here a general idea of the way in which the problem can be treated.

The first thing to do is to expand the tidal potential in the orbital elements of both the disturbing and the disturbed satellite, in much the same way as a disturbing function is expanded (Kaula, 1964). Using Lagrange's planetary equations or a canonical formalism, we can find the time derivative of the semi-major axis of the disturbed satellite, and, if we wish, also that of the eccentricity and the inclination. Unlike Kaula, we do not put 'disturbed satellite = disturbing satellite', but we do eliminate all periodic terms, bearing in mind that a term containing the librating angle looks periodic, but is not. The term containing the librating angle will be the only one to remain.

We then truncate the expressions to the lowest order in the small quantities, i.e. the eccentricities, the inclinations and the ratios of the equatorial radius of the planet to the semi-major axes of the orbits of the satellites (R/a). The last step is to average the equations in time, over the whole cycle of the libration. In practice this means that we shall have to compute the mean of the sine and the cosine of the resonance angle. If we are using the pendulum model, this is readily done in terms of elliptic functions.

3. Cross tides and resonance

It is well-known that the ordinary tidal effect transfers angular momentum and energy from the rotation of the planet to the orbit of the satellite, causing it to expand. The rate of expansion is proportional to the mass of the satellite involved, and inversely proportional to the semi-major axis to the power 11/2. When two satellites are in resonance, the tidal effect will cause the orbits to expand independently, thus trying to break the commensurability. However, the resonance has a mechanism to maintain this commensurability of mean motions. By librating around a value slightly shifted from zero, the resonance can transfer just the right amount of angular momentum and energy from the orbit of one satellite to the orbit of the other one, so that the combined effect of tides and resonance causes the orbits to expand in such a way that the commensurability of mean motions is preserved. This mechanism was first described by Goldreich (1965), and is represented in figure 1. The tidal effects are always in phase. The effect of the resonance is in anti-phase and proportional to the mass of the other satellite.

The cross tides have a similar effect on the orbits of the satellites, but unlike the ordinary tidal effects the cross-tidal effects are in anti-phase and proportional to the mass of the other satellite (there is also a term which is in phase and proportional to the mass of the other satellite, but since this term has exactly the same form as the resonance term, it has no other effect than taking over part of the resonance effect). The sum of the normal and cross-tidal effects will be different from the normal tides alone, and hence the resonance will have to give a different contribution in order to maintain the commensurability. The consequence is that the total effect on the

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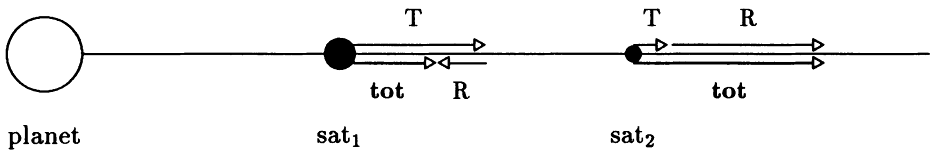

Fig. 1. Schematic representation of the interaction between tides and resonance. T = tides, R = resonance, **tot** = total.

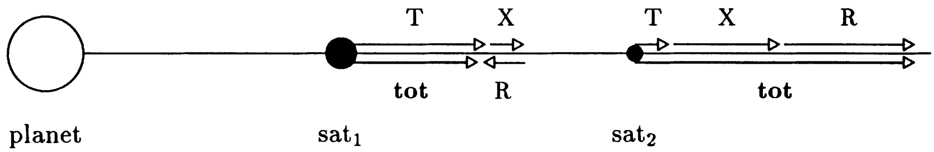

Fig. 2. Schematic representation of the interaction between tides, cross tides and resonance. T = tides, X = cross tides, R = resonance, **tot** = total.

semi-major axes of the satellites, as well as the sides effects (i.e. the evolution of the eccentricities, the inclinations and the amplitude of the libration of the resonance) will be affected by adding the cross-tidal effects to the system. This is represented in figure 2.

4. Numerical results

The effect is not expected to be very large since the expression for the cross tides contains (a) power(s) of the eccentricities and inclinations more than the normal tides. Moreover, the cross tides can contain a higher power of (R/a) than the normal tides (dependent on the parameters of the resonance). However, in the Mimas-Tethys resonance it happens that the normal tides are almost in the good ratio for preserving the resonance, so that even small cross-tidal effects could disrupt this equilibrium and have a relatively important effect on the evolution of the system.

Table I gives numerical results for seven different orbit-orbit resonances in the Solar System. Although some drastic approximations have been made, they give a good idea of the magnitude of the effect. The table lists the effect of the combination of normal tides, cross-tides and resonance on the semi-major axes, eccentricities, inclinations and amplitudes of libration, measured in units of the effect of the combination of normal tides and resonance on the same quantities. As expected the results are small. More surprising is that the numerical results are even smaller for the Mimas-Tethys resonance than for other resonances. This could be explained by the fact that the Mimas-Tethys resonance is the only one in which the coefficient

TABLE I

Ratio of the cross-tidal effect to the ordinary tidal effect in combination with orbit-orbit resonance, for seven resonances between planetary satellites.

resonance	$\dot{a}_1$ and $\dot{a}_2$	$\dot{e}_1$ and $\dot{I}_1$	$\dot{e}_2$ and $\dot{I}_2$	amplitude
I	-0.000270	0.000816	—	0.000816
II	0.0000164	—	0.0000170	0.0000170
III	-0.00266	0.00211	—	0.00211
IV	0.0000063	—	0.0000096	0.0000096
V	-0.0000041	0.0000251	-0.000270	0.00000078
VI	-0.00108	0.00297	—	0.00297
VII	0.0000018	—	0.0000018	0.0000018

Resonances:

I: $\lambda_{\mathrm{Io}} - 2\,\lambda_{\mathrm{Europa}} + \varpi_{\mathrm{Io}}$

II: $\lambda_{\mathrm{Io}} - 2\,\lambda_{\mathrm{Europa}} + \varpi_{\mathrm{Europa}}$

III: $\lambda_{\mathrm{Europa}} - 2\,\lambda_{\mathrm{Ganymede}} + \varpi_{\mathrm{Europa}}$

IV: $\lambda_{\mathrm{Europa}} - 2\,\lambda_{\mathrm{Ganymede}} + \varpi_{\mathrm{Ganymede}}$

V: $2\,\lambda_{\mathrm{Mimas}} - 4\,\lambda_{\mathrm{Tethys}} + \Omega_{\mathrm{Mimas}} + \Omega_{\mathrm{Tethys}}$

VI: $\lambda_{\mathrm{Enceladus}} - 2\,\lambda_{\mathrm{Dione}} + \varpi_{\mathrm{Enceladus}}$

VII: $3\,\lambda_{\mathrm{Titan}} - 4\,\lambda_{\mathrm{Hyperion}} + \varpi_{\mathrm{Hyperion}}$

contains two powers of eccentricities and/or inclinations, combined with the fact that among all resonances listed, the exponent of (R/a) has the largest value.

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