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A UNIFIED VIEW OF (COMPLETE) REGULARITY AND CERTAIN VARIANTS OF (COMPLETE) REGULARITY

J. K. KOHLI

1. Introduction. Regularity and complete regularity are important topological properties and several generalizations of them occur in the literature on separation axioms. The properties of certain of these variants of (complete) regularity are similar to those of (complete) regularity and their theories run, either in part or in the whole, parallel to the theory of (complete) regularity. All the more, analogies inherent in their definitions as well as the nature of results obtained in the process of their study suggest the need of formulating a coherent unified theory encompassing the theory of (complete) regularity and its generalizations. An attempt leading towards the fulfillment of this need constitutes the theme of the present paper.

Section 2 is devoted to preliminaries and basic definitions. In Section 3 we devise a framework which leads to the formulation of a unified theory of (complete) regularity, almost (complete) regularity, ([**26,**], [**27**], [**28**]), (complete) s-regularity [**13**], (functionally) Hausdorff spaces, R_1-spaces [**3**], and others. Certain aspects of the theory are then elaborated in subsequent sections. In particular, in the class of T_1-spaces the Hausdorff axiom is also viewed as a weak form of regularity.

Preservation of (complete) regularity and its variants under mappings is investigated in Section 5 and this yields improvements and refinements of known preservation results under mappings pertaining to regularity, complete regularity, R_1-spaces, Hausdorff spaces and their variants. In particular, in the process we obtain unified proofs and improvements of certain results of Chaber [**2**], Dorsett [**4**], Kohli ([**11**], [**12**]), Singal and Arya [**26**], and Singal and Singal [**19**].

Section 6 is devoted to examples which supplement the theory and reflect on the interrelations among standard separation axioms and weak forms of (complete) regularity discussed in the paper.

2. Basic definitions and preliminaries. Let X be a topological space. A subset A of X is said to be *regular closed* if it is the closure of its interior. A π-*set* [**35**] is the intersection of any finite number of regular closed sets and a δ-*closed* set [**30**] is the intersection of any collection of regular closed

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sets. The complement of a regular closed set is called *regular open*. We shall call a subset F of X a *point closure* if it is the closure of a singleton. A space is an R_1-*space* [**3**] if point closures of distinct points are contained in disjoint open sets whenever they do not coincide.

A subset A of a space X with topology τ is called *quasi H-closed* relative to X [**24**] if each τ-open family which covers A has a finite subfamily whose union is τ-dense in A.

Let E be a topological space. A subset A of X is said to be *E-closed* in X [**22**] if for some positive integer n there exists a closed subset B of E^n and a continuous function $f:X \to E^n$ such that $A = f^{-1}(B)$. An E-closed set is called *D-closed* [**8**] if E is a developable T_1-space.

A family $\mathscr{F}$ of closed sets in X is called a *strongly closed* G_δ-*family* if each $F \in \mathscr{F}$ is a countable intersection

$$F = \cap \{X - F_i: \quad F_i \in \mathscr{F}\};$$

the members of any such family $\mathscr{F}$ are called *strongly closed* G_δ-*sets*.

3. A unified framework. Let P denote a topological property possessed by certain subsets of a topological space. Properties with which we shall be dealing in this paper are quite diverse and include, among others, being a (regular) closed set, being an E-closed set, being a point closure, being a connected set; a complete list is given in the accompanying table.

3.1 *Definitions.* Let X be a topological space and let $A \subset X$. We say that

(i) A is a *P-set* if A possesses property P; and

(ii) A has *P-complement* if $X - A$ possesses property P.

3.2 *Definitions.* A topological space X is called

(i) *P-regular* if each closed P-set and a point outside it are contained in disjoint open sets;

(ii) *completely P-regular* if for each closed P-set F and a point p outside F, there is a continuous real-valued function f defined on X such that $f(F) = 1$ and $f(p) = 0$;

(iii) *weakly P-regular* if every separated pair consisting of a closed P-set and a point outside is separated by disjoint open sets.

The following implications are immediate from the definitions:

$$\begin{array}{ccccc}
\text{complete regularity} & \Rightarrow & \text{complete } P\text{-regularity} & & \\
\Downarrow & & \Downarrow & & \\
\text{regularity} & \Rightarrow & P\text{-regularity} & \Rightarrow & \text{weak } P\text{-regularity}
\end{array}$$

Examples quoted in [**13**], [**26**] and [**27**] suffice to show that none of the above implications in general is reversible.

3.1 PROPOSITION. *If P is a property which is preserved under the operation of taking closures and is enjoyed by all singletons, then a T_0 (completely) P-regular space is (functionally) Hausdorff. In particular, a T_0 (completely) s-regular space is (functionally) Hausdorff.*

A topological property P is an *absolute property* if "Z has P" does not depend on the space in which Z is embedded.

3.2 PROPOSITION. *If P is an absolute property such that the closure of a P-set is again a P-set, then (complete) P-regularity is hereditary.*

Proposition 3.2 is false with "absolute property" replaced by "property," even if the conclusion is weakened from hereditary to closed hereditary (see Example 6.9). (This was pointed out by referee who also supplied Example 6.9.)

3.3 PROPOSITION. *A space X is an R_1-space if and only if X is P-regular with P = point closure.*

Proof. Let X be an R_1-space. Clearly every point closure in X and a point outside it are contained in disjoint open sets. Conversely, suppose X has the property that every point closure and a point outside it are separated by disjoint open sets and let x, y be any two distinct points in X such that $\{\overline{x}\} \neq \{\overline{y}\}$. Then there is a point z which is either in $\{\overline{x}\}$ or in $\{\overline{y}\}$ but not in both. For definiteness suppose that $z \in \{\overline{x}\}$ and so there are disjoint open sets U and V containing z and $\{\overline{y}\}$, respectively. Thus $z \in U \subset X - V$ and since $z \in \{\overline{x}\}$, $x \in U$ and hence $\{\overline{x}\} \subset X - V$. Thus

$$\{\overline{x}\} \cap \{\overline{y}\} = \emptyset$$

and so X is an R_0-space [**3**]. By [**3**, Theorem 2], $\{\overline{x}\} \subset U$. This completes the proof that X is an R_1-space.

3.4 PROPOSITION. *A T_0 (completely) R_1-space is (functionally) Hausdorff.*

3.3 *Definition*. A topological space X is called a *semilocally P-space* if for each $x \in X$ and each open set U containing x there is an open set V such that $x \in V \subset U$ and $X - V$ is the union of finitely many closed P-sets.

The notion of semilocally P-space, introduced and developed in ([**15**], [**16**]) represents unification of the concepts of semiregular space, semilocally connected space, completely regular space, (completely) D-regular space [**9**], (countably) compact space, Lindelöf space and others. The concepts of E-completely regular space due to Engelking and Mrówka ([**6**], [**22**]) and E-completely regular space due to Herrlich (which Herrlich named E-regular spaces) also come under the purview of semilocally

P-spaces to a certain extent.

The table nearby illustrates the type of P-regularity, complete P-regularity and semilocally P-space determined by property P. References are quoted as an aid to literature and an attempt has been made to quote the reference in which a concept or a notion appears for the first time. However, no claim is made to completeness or originality of the source.

If P_1 and P_2 are two topological properties such that P_1 implies P_2, then every (completely) P_2-regular space is (completely) P_1-regular. However, the reverse implication does not hold in general (see Examples 6.1-6.8).

In particular, every Hausdorff space is c-regular and a c-regular space is finitely regular and in the case of T_1 spaces the concepts of c-regularity, finite regularity and the Hausdorff property coincide.

4. Results.

4.1 THEOREM. *A semilocally P-space is (completely) regular if and only if it is (completely) P-regular.*

Proof. Necessity is obvious. To prove sufficiency, let X be a semilocally P-space and suppose that X is (completely) P-regular. Let F be a closed subset of X and let $x \notin F$. Since X is a semilocally P-space, there is an open set V such that $x \in V \subset X - F$, and $X - V$ consists of finite number of closed P-sets, $F_1, \ldots, F_n$. By P-regularity of X, there are disjoint open sets N_k and V_k for each $k = 1, \ldots, n$ such that $x \in N_k$ and $F_k \subset V_k$. Let

$$N = \bigcap_{k=1}^{n} N_k \quad \text{and} \quad U = \bigcup_{k=1}^{n} V_k.$$

Then N and U are disjoint open sets containing x and F, respectively.

In case X is completely P-regular, for each $k = 1, \ldots, n$, there is a continuous real-valued function f_k defined on X such that $f_k(x) = 0$, $f_k(F_k) = 1$. Define $f:X \to \mathbf{R}$ by

$$f(x) = \sup\{f_1(x), \ldots, f_n(x)\}.$$

Then f is continuous, $f(x) = 0, f(F) = 1$.

4.1 *Remark*. Reading from the table, Theorem 4.1 includes several results in the literature; for example, with P = regular closed we get that a semiregular space is (completely) regular if and only if it is almost (completely) regular, a result that contains Theorem 3.1 of [**26**]. Similarly, the substitution P = connectedness in Theorem 4.1 yields [**13**, Theorem 3.2].

4.2 THEOREM. *Let P denote a property possessed by all singletons in a topological space. Suppose $f:X \to Y$ is a continuous, open and closed* *surjection defined on a P-regular space such that either X or Y is* T_1. *If* $f^{-1}(y)$ *is a P-set for each* $y \in Y$, *then Y is Hausdorff.*

Proof. In view of closedness of f, in either case we may assume that Y is T_1. First we show that the set

$$A = \{ (x_1, x_2) : f(x_1) = f(x_2) \}$$

is closed in $X \times X$. If $(x_1, x_2) \notin A$, then

$$x_1 \notin f^{-1}[f(x_2)].$$

Since Y is T_1, in view of hypothesis on f, $f^{-1}[f(x_2)]$ is a closed P-set. By P-regularity of X, there are disjoint open sets U and V containing x_1, and $f^{-1}[f(x_2)]$, respectively. Since f is closed, by [**5**, Theorem 11.6, p. 86], there is an open set W containing $f(x_2)$ such that

$$f^{-1}[f(x_2)] \subset f^{-1}(W) \subset V.$$

Then since f is continuous, $U \times f^{-1}(W)$ is an open set containing (x_1, x_2) which does not intersect A. Thus A is closed in $X \times X$.

Now, suppose that $f(x_1)$ and $f(x_2)$ are distinct points of Y. Then $(x_1, x_2) \notin A$. So, there are open sets U_1 and U_2 containing x_1 and x_2, respectively, such that

$$(U_1 \times U_2) \cap A = \emptyset.$$

Since f is open, $f(U_1)$ and $f(U_2)$ are disjoint open sets containing x_1 and x_2, respectively.

4.2 *Remark*. Using the table, Theorem 4.2 contains several results in the literature; for example, with P = closed set we get that a T_1 continuous open and closed image of a regular space is Hausdorff, a result that includes [**33**, Theorem 14.6, p. 94]. Similarly, for P = connectedness it yields Theorem 3.3 of [**13**].

None of the hypotheses of continuity, openness or closedness in Theorem 4.2 can be omitted.

4.3 THEOREM. *Let X be a Hausdorff P-regular space and let A be a closed P-subset of X. Then the quotient space obtained from X by identifying A to a point is Hausdorff.*

4.4 THEOREM. *Let X be a completely P-regular space. If K and F are disjoint subsets of X such that K is compact and F is a closed P-set, then there is a continuous function* $f : X \to [0, 1]$, *such that* $f(K) = 0$ *and* $f(F) = 1$.

4.5 THEOREM. *Let X be a completely P-regular space. If K is a compact* G_δ *which is expressible as a countable intersection of open sets having P-complements, then K is a zero set in X.*

We omit proofs of Theorems 4.3, 4.4, and 4.5. The special case of these theorems for P equal to connectedness is dealt with in [**13**] and with P = closed set they reduce to classical known results pertaining to (complete) regularity.

5. Preservation under mappings. In this section we study the behavior of P-regularity and complete P-regularity under mappings. The known results, pertaining to the preservation of (complete) regularity, almost (complete) regularity, (complete) s-regularity, R_1-spaces, and Hausdorff spaces under mappings, follow as easy corollaries to the results so obtained.

5.1 *Definitions.* A function $f:X \to Y$ from a topological space X into a topological space Y is called

(i) *P-continuous* [**15**] if for each $x \in X$ and each open set V containing $f(x)$ and having P-complement there is an open set U containing x such that $f(U) \subset V$; and

(ii) *P-proper map* if for each closed P-set $K \subset Y$, $f^{-1}(K)$ is a closed P-set in X.

Continuity and several weak forms of continuity, which occur in the literature are special cases of P-continuity. For example, the concept of almost continuous function [**25**] (respectively, c-continuous function [**7**], respectively s-continuous function [**11**]) coincide with P-continuous function with P = regular closed (respectively P = compactness, respectively P = connectedness). Again, for P = connectedness P-proper maps have been studied by Jones [**10**] and Long [**19**], and for P = δ-closed by Noiri [**23**] and Mathur [**21**]. It seems an interesting and a profitable exercise to study the category of topological spaces and P-proper maps.

5.1 THEOREM. *Let $f:X \to X$ be a P-proper closed surjection defined on a P-regular space. If either f is open or if $f^{-1}(y)$ is compact for each $y \in Y$, then Y is a P-regular space.*

5.2 THEOREM. *Let $f:X \to Y$ be a P-continuous closed surjection defined on a regular space. If either f is open or if $f^{-1}(y)$ is compact for each $y \in Y$, then Y is a P-regular space.*

Proof of Theorems 5.1 *and* 5.2. Let $F \subset Y$ be a closed P-set and suppose that $y \notin F$. Then

$$f^{-1}(F) \cap f^{-1}(y) = \emptyset.$$

Since f is P-continuous, by [**15**, Theorem 3.1] $f^{-1}(F)$ is closed. In case f is a P-proper function, then $f^{-1}(F)$ is also a P-set.

Case 1. f is open. Let $x \in f^{-1}(y)$. Then in view of regularity or P-regularity of X, as the case may be, there are disjoint open sets U and V

such that $x \in U$ and $f^{-1}(F) \subset V$. Then $f(U)$ and $Y - f(X - V)$ are disjoint open sets containing y and f, respectively.

Case 2. $f^{-1}(y)$ is compact for each $y \in Y$. Then in view of regularity or P-regularity, as the case may be, there are disjoint open sets U and V containing $f^{-1}(y)$ and $f^{-1}(F)$, respectively. Since f is closed, the sets

$$V_1 = Y - f(X - U) \quad \text{and} \quad V_2 = Y - f(X - Y)$$

are open. It is easily verified that V_1 and V_2 are disjoint and contain y and F, respectively.

5.1 *Remark.* According to the table Theorems 5.1 and 5.2 contain several results in the literature; for example, with P = closed set we get that regularity is invariant under perfect mappings [**5**, p. 235] and that regularity is preserved under continuous open closed surjections [**2**]. Again, in view of a result of Whyburn [**32**], with P = compactness, it is easily deduced that a perfect image of a Hausdorff space is Hausdorff [**5**, p. 234]. Similarly, the substitution P = point closure in Theorem 5.1 (or 5.2) yields an assertion which includes a result of [**4**] and the substitution P = regular closed yields Theorem 2.14 of [**25**]. Further, for P = connectedness we get Theorem 3.7 of [**13**].

5.3 THEOREM. *Let $f:X \to Y$ be an open closed P-continuous surjection defined on a completely P-regular space X. If either X is completely regular, or if f is a P-proper function, then Y is completely P-regular.*

Proof. Let $K \subset Y$ be a closed P-subset and suppose that $y \notin K$. Since f is P-continuous by [**15**, Theorem 3.1] $f^{-1}(K)$ is closed. In case f is a P-proper function, then $f^{-1}(K)$ is also a P-set. Take a point $x_0 \in f^{-1}(y)$. Then in view of complete regularity or complete P-regularity, as the case may be, there is a continuous real-valued function $\phi:X \to [0, 1]$ such that

$$\phi(x_0) = 1 \quad \text{and} \quad \phi(f^{-1}(K)) = 0.$$

Define $\hat{\phi}:Y \to [0, 1]$ by putting

$$\hat{\phi}(y) = \sup\{\phi(x):x \in f^{-1}(y)\}.$$

Then $\hat{\phi}(y) = 1$, $\hat{\phi}(F) = 0$ and by [**5**, p. 96, Exercise 16] $\hat{\phi}$ is continuous. Thus Y is completely P-regular.

5.2 *Remark.* Reading from the table Theorem 5.3 contains several known results in the literature; for example, with P = closed set we get that complete regularity is invariant under open-closed continuous surjections, a result due to Chaber [**2**]. Similarly, the substitution P = connectedness yields Theorem 3.10 of [**13**]. Further, with P = finite set we get that an open-closed image of a T_1-completely regular space with closed point inverses is functionally Hausdorff, a result which includes Theorem 2.14 of [**12**].

TABLE

	Property P	P-regular space	Completely P-regular space	Semilocally P-space
1	Closed set	regular	completely regular	topological
2(a) (b) (c)	Regular closed δ-closed π-closed	almost regular [**26**]	almost completely regular [**27**] [**28**]	semiregular [**29**]
3(a)	Cardinality $\leqq m$	m-regular [**18**]	completely m-regular	The space has a base of nbds having complements of cardinalty $\leqq m$
(b)	Countable set	$\aleph_0$-regular	completely $\aleph_0$-regular	cocountable topology
4	Finite set	finitely regular	completely finitely regular	cofinite topology
5(a) (b)	Connectedness Having finite components	s-regular [**13**]	completely s-regular [**13**]	semilocally connected [**31**]
6	Point closure	R_1-space [**3**]	completely R_1-space	The space has a base of nbds having complements which are closures of finite sets
7	Zero set	topological	topological	completely regular
8	Compactness	c-regular	completely c-regular	compact
9	Countable compactness	c^*-regular	completely c^*-regular	countably compact
10	Quasi H-closed	H-regular	completely H-regular	c^*-compact [**16**]
11	E-closed	–	–	E-completely regular [**22**]
12	Cl open set	topological	topological	zero dimensional
13	(Strongly) closed G_δ	(weak) G_δ-regular [**17**]	(weak) completely G_δ-regular [**17**]	(completely) D-regular [**9**]
14	D-closed	D^*-regular [**17**]	completely D^*-regular [**71**]	completely D-regular [**9**]
15	Lindelöf property	l-regular	completely l-regular	Lindelöf space

6. Examples.

6.1 *A functionally Hausdorff space which is not G_δ-regular.* Let F denote the Moore plane, i.e., the upper half plane $\{ (x, y):y \geqq 0) \}$ in $\mathbf{R}^2$ endowed with the tangent disc topology (see [**33**, p. 36]). Let D and E be the closed sets of points on the x-axis in F whose first coordinates are rational and irrational, respectively. Let X denote the quotient space of F obtained by identifying the closed set D to a point and let $q:F \to X$ be the quotient map. The space X is a functionally Hausdorff space in which the closed G_δ-set $q(E)$ and the point $q(D)$ cannot be separated by disjoint open sets.

The next example is a quotient of the Tychonoff plank [**33**, p. 122].

6.2 *A functionally Hausdorff completely G_δ-regular space which is not regular.* Let Ω denote the space of ordinals up to and including the first uncountable ordinal ω_1 and let $\Omega_0 = \Omega - \{\omega_1\}$, where Ω and Ω_0 possess their natural order topologies. Let $\Omega(\omega) = \mathbf{N} \cup \{\omega\}$ denote the one point compactification of the discrete space of natural numbers and let $T = \Omega \times \Omega(\omega)$. The subspace

$$T = T^* - \{ (\omega_1, \omega) \}$$

is called Tychonoff plank.

Let

$$A = \{ (\omega_1, n):n \in \mathbf{N}\} \quad \text{and} \quad B = \{ (\alpha, \omega): \quad \alpha \in \Omega_0\}.$$

Let X be the quotient space of T obtained by collapsing B to a point and let $q:T \to X$ denote the quotient map. The space X is a functionally Hausdorff completely G_δ-regular space in which the closed set $q(A)$ and the point $q(B)$ cannot be separated by disjoint open sets.

6.3 *A functionally Hausdorff space which is not c^*-regular.* Let T, A and B have the same meaning as in Example 6.2. Let Y be the quotient space of T obtained by identifying the closed set A to a point and let $p:T \to Y$ be the quotient map. Then Y is a functionally Hausdorff space in which the closed countably compact (in fact, sequentially compact) set $p(B)$ and the point $p(A)$ cannot be separated by disjoint open sets.

6.4 *A Hausdorff completely C^*-regular space which is not* 1*-regular.* Let X be the real line, with neighborhoods as usual except that basic neighborhoods of 0 have the form $(-\epsilon, \epsilon) - A$, where $A = \{1/n:n \in \mathbf{N}\}$. Then A is a Lindelöf closed set which cannot be separated from 0 by disjoint open sets.

6.5 The space X of Example 6.2 is a *functionally Hausdorff completely* 1*-regular space which is not regular.*

6.6 *A finitely regular space which is not c-regular.* Let $X = [-2, 2]$ and let X be endowed with the topology generated by all sets of the form

$$\begin{aligned} A_{-2_a} &= [-2, a), 0 < a < 1 \\ A_{-1_a} &= [-1, a), 0 < a < 1 \\ A_{bc} &= (b, c), -1 < b < 0 < c < 1 \\ N_{cd} &= (c, d), 0 < c < d < 2 \\ A_{a^2} &= (a, 2], -1 < a < 1 \end{aligned}$$

The space X is a completely finitely regular, non-Hausdorff, compact space which is not c-regular.

6.7 The example of a minimal Hausdorff topology [**29**, p. 119] is a *c-regular space which is not H-regular.*

6.8 *An H-regular space which is not regular.* Let $\mathbf{N}^*$ be the subspace $\{0\} \cup \{1/n:n \in \mathbf{N}\}$ of $\mathbf{R}$. Let X be $\mathbf{N} \times \mathbf{N}^*$ together with an ideal point a whose neighborhoods have the form

$$U_{n_0}(a) = \{a\} \cup \{(n, 1/m) \in \mathbf{N} \times \mathbf{N}^*:n \geqq n_0\}.$$

The resulting space is H-regular but not regular.

The example described in [**34**] of a Moore space on which every continuous real-valued function is constant suffices for a regular nonweak completely G_δ-regular space, not completely D^*-regular space.

6.9 Let P be the topological property: Z has P if and only if Z is closed and Z has a neighborhood containing no isolated points. Then P is a property such that the closure of a P-set is again a P-set. Let

$$X = \{a, b, c, d, e\}$$

with the topology

$$\{\emptyset, X, \{e\}, \{a, c, e\}, \{a, b, c, e\}, \{a, c, d, e\}\}.$$

Then X is P-regular since there is no P-set in X. Indeed, every nonempty open set contains e which is isolated. Let Y be the closed subspace $\{a, b, c, d\}$. Then Y is not P-regular. The singleton $\{d\}$ is closed in Y and it is a P-set in Y. But $\{d\}$ and b cannot be separated by disjoint open sets.

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Lamar University,
Beaumont, Texas;
Hindu College, University of Delhi,
Delhi, India