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GUEST EDITORIAL

Special Section: Topological representation and reasoning in design and manufacturing

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1. INTRODUCTION

The word "topology" is derived from the Greek word "τοπος," which means "position" or "location." A simplified and thus partial definition has often been used (Croom, 1989, page 2): "topology deals with geometric properties which are dependent only upon the relative positions of the components of figures and not upon such concepts as length, size, and magnitude." Topology deals with those properties of an object that remain invariant under continuous transformations (specifically bending, stretching, and squeezing, but not breaking or tearing). Topological notions and methods have illuminated and clarified basic structural concepts in diverse branches of modern mathematics. However, the influence of topology extends to almost every other discipline that formerly was not considered amenable to mathematical handling. For example, topology supports design and representation of mechanical devices, communication and transportation networks, topographic maps, and planning and controlling of complex activities. In addition, aspects of topology are closely related to symbolic logic, which forms the foundation of artificial intelligence. In the same way that the Euclidean plane satisfies certain axioms or postulates, it can be shown that certain abstract spaces—where the relations of points to sets and continuity of functions are important—have definite properties that can be analyzed without examining these spaces individually. By approaching engineering design from this abstract point of view, it is possible to use topological methods to study collections of geometric objects or collections of entities that are of concern in design analysis or synthesis. These collections of objects and or entities can be treated as spaces, and the elements in them as points.

The importance of topological representation and reasoning in analysis, design, and manufacturing is heightened by the contemporary view that stresses the need for conceptual design. At the conceptual design stage, a rough, overall representation of the design structure is produced. This efficient approach to engineering design that uses a rough design blueprint of the overall structure includes rapid visualization and exploration of the feasibility of the design through computational analysis (e.g., Hoffmann, 1989). A computationally intensive detailed design then follows. Thus, the topological modeling approach should make the creation of the final product more efficient by reducing some of the iterations incurred by certain expensive detailed design operations.

The exciting new area of research expressed in these special issues of ***AI****EDAM* involves the integration of topological properties in a wide variety of analysis, design, and manufacturing-related areas. The range of application of topology in computational engineering analysis, design, and manufacturing is summarized below (for more information, see Finger & Dixon, 1989; Reich, 1995; Rosen & Peters, 1996; Braha & Maimon, 1998).

2. MODELS FOR REPRESENTATION OF DESIGN KNOWLEDGE, AND CONCEPTUAL AND PRELIMINARY DESIGN PROCESSES WITH TOPOLOGICAL SPACES KNOWLEDGE STRUCTURE

Models of the design space and conceptual design process based on topological spaces—General Design Theory (GDT)—mathematically describe the design process in terms of point–set topology (Yoshikawa, 1981; Tomiyama, 1994). Set–point topology (topology) is a structured set of subsets of a given set (the subsets are called open subsets). A topological space is an ordered pair comprising the given set and a topology on that set. Many formal properties of functions depend upon the topologies imposed upon their domains and ranges. Set–point topology has been used by Yoshikawa (1981) to study the structure of abstract design concepts that include functional and structural attributes, and their relationships. The principle assumption states that

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abstract concepts are topologies of the existent or nonexistent entity set (design solutions). By defining a topology on entities, the relationship between functional specifications and potential design solutions has been defined. That is, for each functional specification there is a set of entities that can fulfill that specification (solutions to the problem). The set of entities that can fulfill a specification is an open set in the function topology. Entities that can fulfill two or more specifications are found by intersecting the sets of entities that meet each specification individually. The same type of relationship occurs with structural attributes. When defining function and attribute topological spaces, design activity can be viewed as a mapping from functional space to attribute space.

GDT has been extended to deal with real knowledge that is finite, limited in processing speed, and is iterative and evolutionary in nature (Tomiyama & Yoshikawa, 1987; Takeda et al., 1990; Tomiyama, 1994). The extension has been done by introducing a new topology of *metamodels*. The open sets (abstract concepts) in the metamodel topology represent behaviors based on physical laws. Thus, rather than treating design as a mapping from functions to attributes, the design activity is defined as a stepwise, evolutionary transformation using the concepts of behaviors as intermediate states.

GDT does not hold for real design processes for the following reasons. First, the refinement process is made easier by the use of the entity set as mediator between the specification and the design description. In the absence of the entity set the process could be more complex. GDT applies to domains with set–point topological structure, but real domains do not *necessarily* satisfy this requirement. Moreover, the restriction to domains with set–point topological structure limits the design selection to the entity set (or a *catalogue*). The second reason that GDT does not apply to real design is because all entities have the same status under the assumption of a topological structure for the entity set. However, it is recognized that in real design, the overall organization of concepts and entities is hierarchical.

To address the aforementioned limitations, Braha and Maimon (1998) present a new modeling paradigm (incorporated within a theory called Formal Design Theory) by insisting on less restrictive assumptions: 1) the design process is a mapping of the desired functionality of a product onto the description of the final product *without the intervention of the entity set*; and 2) human designers use hierarchical knowledge structure for the overall organization of functional and structural properties. To this end, the new topological model attempts to cast these assumptions in the framework of *closure* spaces or *proximity* spaces (which include point–set topology as a *special case*), and uses this framework to define properties and prove theorems about the nature of design. Another useful property of the application of closure spaces to modeling design knowledge is an integration of formal logic representation of design knowledge (as used in knowledge-based and deductive design systems) and closure topological spaces.

The above topological concepts form the basis for the development of various AI-based intelligent computer-aided design systems (Veth, 1987; Tomiyama & Ten Hagen, 1990; Braha & Maimon, 1998). For example, when designers make an incremental change to the design problem, they expect that the resulting solution will be consistent with the beginning solution. After a design modification (redesign) has occurred, the designer should "honor" the initial design choice. When the specifications are modified, we wish not only to find a new satisfactory design solution, we wish to find the *intended* design solution (this is what is meant by *consistent design*). If there is only one possible solution to the new specifications, then it is easy to maintain a consistent design. It is much harder if there are multiple competing solutions, all of which satisfy the specifications. Fortunately, the continuity property of design as defined rigorously in Braha and Maimon (1998) directs us towards a principle of design consistency: *small changes in specifications should lead to small changes in design*. Furthermore, large changes in specifications can often be decomposed to a series of small changes, in which case the principle can still be applied. In Braha and Maimon (1998), the concept of design consistency in the area of parametric design is further formalized based on the continuity property; and a methodology is implemented for maintaining design consistency in those design areas where similarity between designs can be calculated.

3. MODELS OF DESIGN-MANUFACTURING MAPPING BASED ON SET–POINT TOPOLOGY

Topology has also been used to model the mapping of a design form into its corresponding manufacturing representation (Rosen & Peters, 1992; Peters et al., 1994). The topological essence of this research is to support reasoning about situations where there are close points in the design space for which their manufacturing representations are very far from each other (in relation to topological spaces, it is represented by a discontinuous mapping).

4. COMPUTATIONAL MODELS FOR MEASURING THE SIMILARITY OF FUNCTION CONCEPTS BASED ON METRIC SPACES

The above ideas have been extended by defining a metric/distance on the function space that measures the similarity between two design space entities based on the difference in functions performed by the entities (Taura & Yoshikawa, 1992). Thus, entities sufficiently close to one another in the function space perform very similar functions, whereas those entities that are far apart in metric space perform very different functions. Based on the theoretical work, a computational tool has been developed that measures the similarity of function concepts and directs the search toward components that meet the required functionality. The metric-based approach can be integrated into case-based reasoning techniques. It can also enable users to rigorously exploit notions such as "approximation" and "convergence" that arise in the context of manufacturing processes (e.g., material removal; see Allen, 1999).

5. COMPUTER-AIDED GEOMETRIC DESIGN MODELS BASED ON ALGEBRAIC TOPOLOGY

Another major area is geometric modeling. Topology has been developed in recent years as the unifying formal basis for both solid and nonmanifold modeling and for providing general and unified computational environments (Duffey & Dixon, 1988; Weiler, 1988; Hoffmann, 1989; Bohn & Wozny, 1990; Gadh et al., 1991; Desaulniers & Stewart, 1992; Lear, 1992; Zamanian et al., 1992; Sudhalkar et al., 1993). Within Computer-Aided Geometric Design (CAGD), topology has focused on adjacency relations amongst vertices, edges, and faces, where algebraic equations provide the defining relationship. The topology/algebraic interaction emerges as an important topic within CAGD (Lear, 1992). For example, the two dominant representation schemata used in solid modeling are constructive solid geometry (CSG) and boundary representation (B-rep). Algebraic topology is integrated into algorithms that test whether a given boundary representation is correct. This is based on a precise topological definition of what constitutes a valid solid, and deriving from it a validity check (Hoffmann, 1989).

The vast majority of geometric computations in solid modeling are performed in floating-point arithmetic. Because logical decisions are made based on these calculations, errors incurred by the limited precision to which the computations are performed should be of great concern (Hoffmann, 1989). A subject of considerable research has been to develop reasoning tools for dealing with the imprecision of floating-point arithmetic that results in approximate geometry, and may thus fail to accurately represent the topology of the object (for example, disconnected faces may be created that were previously connected).

Nonmanifold topology provides generalized data structures, algorithms, and a framework for geometric modeling representation. Several engineering research applications of nonmanifold modeling include feature recognition, feature-based design, and geometric abstractions for reasoning about shape (Rosen & Peters, 1996).

6. REPRESENTATION AND REASONING OF GEOMETRIC TOLERANCES BASED ON EUCLIDEAN METRIC TOPOLOGY

The role of topology within tolerance theory has been to develop topology-based computational tools to address the following fundamental issues (Requicha, 1983, 1993; Boyer & Stewart, 1991, 1992; Stewart, 1993; Rosen & Peters, 1996): 1) how to construct a locality around the boundary of the nominal object in which geometric variations are allowed; and 2) how "topologically" similar the geometry of the object within the tolerance is to that of the nominal object. Declarative and procedural algorithms to check various topological conditions have been incorporated into software systems.

7. INTEGRATING GEOMETRIC REASONING AND MODELS OF PHYSICAL BEHAVIOR BASED ON ALGEBRAIC TOPOLOGY

Many researchers have attempted to develop computational modeling that combines physical behavior with topological and geometrical properties (beyond finite elements, Bond graphs, and other lumped parameter representations for modeling energy exchanges, e.g. Ulrich & Seering, 1989; Cox & Anderson, 1991). An example of a computational model that combines physical behavior (function) and geometry (form) is Chain Models of Physical Behavior developed by Palmer (1995), Palmer and Shapiro (1993), and Shapiro and Voelcker (1989). Chain Models have been derived from algebraic topology based on Cell Complexes, Chains, and topological operations on Chains. Cell Complexes are much like finite elements, in that the geometry of an object is decomposed into a finite number of "Cells." Corresponding to each Cell is a distribution of a physical quantity represented by a "Chain." Physical laws can be modeled as constraints on the coefficients of Chains. Through the application of algebraic topology to physical behavior, Chain models can represent and compute with physical boundaries. While Bond graphs and other lumped parameter representations can model energy exchanges (Ulrich & Seering, 1989), they cannot represent the physical boundaries over which these energy exchanges occur. Algebraic-topological models translate directly into a computer language for engineering physics (Palmer & Shapiro, 1993). They enable the integration of much of the information that is currently assumed or missing in computer systems for analysis, simulation, and engineering design. Also, by associating Chains that represent specific behaviors with common engineering shapes, which are composed of Cell Complexes, primitive elements can be identified. Such primitive elements together with the ability to represent design specifications have been used by an automated synthesis of engineering designs.

In addition to the main application areas mentioned above, there are additional areas where topology has been exploited, such as in searching for an optimal topology during design synthesis and qualitative spatial reasoning using topology. Examples of the first area include truss design and graph-based optimization (Bremicker et al., 1991; Reddy & Cagan, 1994). Issues related to the second area include reasoning about properties of points or point sets in space, detecting intersection relations among combinations of point sets, developing methods where topological queries can be solved by topological computation without geometry, and topological-based reasoning for finding consistent *paths* through point–set combinations (such as in the "piano movers" problem; see Latombe, 1991).

In summary, this exciting new area of research involves the integration of topological properties in a wide variety of design-related issues and activities including descriptive and computational modeling of design knowledge, organization, and processes; geometric representation including reasoning about tolerances; design entity similarity measurement; geometric/topological/physical integrated modeling of physical behavior; design-to-manufacturing transformation modeling; topological optimization; and qualitative spatial reasoning. The three special issues on topological representation and reasoning in design and manufacturing will appear in the 2000 issue number 5 and the 2001 issues numbers 1 and 2 of ***AI**EDAM*. This series of special issues is oriented toward the exploration of recent advances in artificial intelligence related to topological design and manufacturing, and we hope that it will stimulate further research in this area as a unifying design abstraction.

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