If the reader can surmount these barriers and wade his way through the applications, he will enjoy some well written chapters like the ones dealing with molecular vibrations and molecular orbitals. It is to be regretted that the chapters on symmetric groups and their applications cannot be included in this class since the important technique of Young symmetrizers is not given an adequate treatment in existing books on the applications of group theory.

Long and fairly complete lists of references are given at the end of each chapter, but the papers and books listed (with titles translated into English!) are never referred to in the text.

It seems rather ironical that a book dealing with symmetry and structure should have so little use for either in its conception and execution.

Feza Gürsey, Princeton, N. J.

Studies in linear and non-linear programming, by K.J. Arrow, L. Hurwicz and H. Uzawa. Stanford University Press, 1958. 229 pages. $7.50.

The three main themes in this collection of papers are existence theorems (part I), the gradient method (part II) and the construction of special algorithms for specific programming problems (part III). The first chapter provides detailed summaries of the papers and connecting links between them.

In chapter 2 an explicit formula is obtained for a set of vectors spanning a convex polyhedral cone given as the intersection of half-spaces. This result is then used to prove the fundamental theorems on convex polyhedral cones. In a later chapter an elementary method for solving linear programming problems is based on this formula. As the method calculates all the extreme points of the feasible set, it is not as efficient as the simplex method.

Consider the problem: Find an n-vector x that maximizes $f(x)$ subject to the restriction that $x \geq 0$ and $g(x) \geq 0$, where $f(x)$ and $g(x) = \langle g_1(x), \ldots, g_m(x) \rangle$ are functions defined on the n-vector $x \geq 0$. The Lagrangian associated with this problem is

$$\phi(x,u) = f(x) + u \cdot g(x),$$

where $x = \langle x_1, \ldots, x_n \rangle$ and $u = \langle u_1, \ldots, u_m \rangle$. The case when $f(x)$ and $g(x)$ are concave functions in $x \geq 0$ (concave programming) is considered in chapter 3, and a new condition is given under which $\bar{x}$ is a solution of the maximizing problem if and only if there is a vector $\bar{u}$ such that $(\bar{x}, \bar{u})$ is a non-negative saddle-point of $\phi(x, u)$ (the Kuhn-Tucker theorem). Chapters 4 and 5 systematically develop a theory of programming in linear spaces. The basic concepts and needed theorems from the theory of linear topological spaces are explained at length, and the reader need have no previous familiarity with this material. Given a convex cone K in a linear space $\mathcal{W}$, an order relation on $\mathcal{W}$ is defined by putting $w' \geq w''$ if and only if $w' - w'' \in K$. (In the finite-dimensional case K is usually the non-negative orthant.) Let $\mathfrak{X}$, $\mathcal{Y}$ and $\mathfrak{Z}$ be linear spaces with order relations defined by convex cones, and let f and g be functions on $\mathfrak{X}$ into $\mathcal{Y}$ and $\mathfrak{Z}$ respectively. Then the problem of programming in linear spaces may be stated: Find $x \in \mathfrak{X}$ that maximizes f(x) subject to the restriction that $x \geq 0$ in $\mathfrak{X}$ and $g(x) \geq 0$ in $\mathfrak{Z}$. In connection with this problem one may define the Lagrangian

$$\phi(x, z^*, y^*) = y^*[f(x)] + z^*[g(x)],$$

where y^* and z^* are continuous linear functionals on $\mathcal{Y}$ and $\mathfrak{Z}$ respectively. Fundamental to the proof of the Kuhn-Tucker theorem is the Minkowski-Farkas lemma which states that if A is an $m \times n$ matrix and $Ax \geq 0$ implies $b'x \geq 0$ for all x, then there exists $t \geq 0$ such that $b = A't$. The main results of chapter 4 are generalizations of the Minkowski-Farkas lemma and the Kuhn-Tucker theorem to certain classes of linear spaces.

Let $\phi(x, y)$ be a real-valued function defined for $x = \langle x_1, \ldots, x_n \rangle \geq 0$ and $y = \langle y_1, \ldots, y_m \rangle \geq 0$, and let $(\bar{x}, \bar{y})$ be a saddle-point of $\phi(x, y)$ in $x \geq 0$ and $y \geq 0$. The gradient process is the system of differential equations

$$\frac{dx_i}{dt} = \begin{cases} 0 \text{ if } x_i = 0 \text{ and } \dfrac{\partial \phi}{\partial x_i} < 0 \\ \dfrac{\partial \phi}{\partial x_i} \text{ otherwise,} \end{cases} \qquad i = 1, \ldots, n,$$

$$\frac{dy_j}{dt} = \begin{cases} 0 \text{ if } y_j = 0 \text{ and } \dfrac{\partial \phi}{\partial y_j} > 0 \\ -\dfrac{\partial \phi}{\partial y_j} \text{ otherwise,} \end{cases} \qquad j = 1, \ldots, m.$$

The question is under what conditions do solutions $x_i(t, x(0), y(0))$, $y_j(t, x(0), y(0))$ of these equations converge to the saddle-point $(\bar{x}, \bar{y})$. The main result is:

If $\phi(x, u)$ is continuously twice differentiable, is convex in u for each x and is strictly concave in x for each u, then the gradient process converges in both x and u.

This result depends on the existence theorem for the above differential equations given in chapter 7. The reviewer doubts

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whether the proof given there is adequate. From this gradient method for approximating a saddle-point a gradient method for strictly concave programming is easily derived. A neat modification of this supplies an algorithm for all concave programming, which is illustrated by an example in linear programming. The gradient method is also formulated as a system of difference equations, and discussed in this form. In the final chapter of part II the possibilities of weakening the concavity assumption are investigated.

The three programming problems from mathematical economics discussed in part III show how great gains in simplicity and efficiency can be obtained by taking advantage of the special features of a problem, rather than making a mechanical application of some general programming algorithm.

A.M. Duguid, Brown University

Applications of Tensor Analysis, by A.J. McConnell. Dover, New York, 1957. xii + 318 pages. $1.85.

This is a paperback reprint of "Applications of the Absolute Differential Calculus". The book is still an excellent introduction to tensors from the classical "index pushing" viewpoint. It demonstrated, in 1931, the usefulness of tensor algebra for determinant theory and the analytic geometry of quadrics, a point which still needs emphasizing. It also contains sections on the differential geometry of curves and surfaces and on applied mathematics.

B.A. Rattray, McGill University

Elementary Statistical Methods, by W.A. Neiswanger. Macmillan, New York; Brett-Macmillan, Galt, Ont., revised edition, 1956. 749 pages. $7.25.

This is an excellent text for the student who wishes to employ statistical techniques without necessarily understanding their mathematical basis. Practical examples are used to present the techniques, simple intuitive approaches are given to the more mathematically difficult concepts and cautions are advanced in every topic to show the beginner the limitations of the techniques and to warn him of the more common misuses. Neither in the text nor in the summary is any attempt made to develop the formulae rigorously. For instance, when two formulae are given as applicable to the solution of some question, no attempt is made to show their equivalence. The strong point of this text is that motivation is given without mathematical argument, the emphasis being to enlighten the student as to what circumstances suggest the use of a particular technique.