Students will appreciate a large number of examples and exercises at the end of each section.

Hanna Schwerdtfeger, McGill University

Nomography, by Alexander S. Levens. 2nd edition, Wiley, New York, 1959. viii + 296 pages. $8.75.

An expanded version of the popular 1948 edition, this one includes three new chapters on circular nomograms, projective transformations, and the relationship between concurrency and alignment nomograms. Other chapters have been expanded to include methods for designing nomograms for four variables without the need for a turning axis, and material on nomograms consisting of two curved scales and a straight line, and three curved scales. The appendix has been expanded to 58 nomograms covering a wide variety of applications from statistics, engineering, biology, etc. The format has been considerably improved.

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Eléments d'algèbre, by Gaston Julia. Cours de l'Ecole Polytechnique. Gauthier-Villars, Paris, 1959. 209 pages. 38 NF = U.S. $7.93.

This is the reproduction of the lectures on algebra which the famous author presents to his first-year classes in the Ecole Polytechnique in Paris. Let us begin with a brief review of the content. Chapter I (pp. 1-25) "Groups, rings, fields" provides the reader with the terminology and the basic notions of abstract algebra, actually more than sufficient for the purposes of the book. Chapter II (pp. 27-89) "The finite-dimensional vector space E_n; bases" develops the foundations of linear algebra, mainly over the complex number field. The theory of linear equations with determinants is supposed to be known to the reader. The fact that h + 1 vectors in a subspace $E' \subset E_n$, generated by h linearly independent vectors, are always linearly dependent, is based upon non-trivial solubility of h linear homogeneous equations in h + 1 unknowns. While for a beginner this may be easier to understand than the usual procedure, a good deal of the motivation for a systematic theory of the linear vector space at this level is lost. There follows a theory of linear transformations and matrix algebra on more or less classical lines. However, the notion of rank of a matrix does not occur. For several simple facts as well as theorems of an advanced nature, mentioned without proof, reference is made to the author's two-volume work "Introduction mathématique aux théories quantiques" (Paris, 1958 and 1955). Similarity is discussed, but Jordan's canonical form is given without proof; reference is made to Jordan, to Schreier-Sperner and to MacDuffee. Finally we find some remarks on matrix functions (Liouville-Neumann series), on bilinear forms and on the dual space. Chapter III (pp. 91-128) deals with the unitary metric space and the unitary transformations, reduction of a matrix to triangular form, normal matrices. Chapter IV (pp. 129-157) "Etude particulière des matrices et operateurs hermitiens ou unitaires" gives a detailed reduction theory of the hermitian forms, a discussion of the eigenvalues of the hermitian and the unitary matrices, and Cayley's relation between these two classes of matrices. The reduction of a hermitian form is also dealt with by Courant's extremum method which has proved to be important in higher analysis.

The work ends with a chapter "Elements of tensor algebra" (pp. 159-207). The first part deals with tensors of the second order in real euclidean space E_3. Much space is wasted here with pure formalities and variations in the notation, involving repetitions of results of chapter II. The basic notions of tensor analysis (differential operations only) are discussed on 6 pages. In the second part of this chapter the space E_3 is supposed to be provided with a metric defined by an arbitrary positive definite quadratic form. After a few remarks on contravariant and covariant coordinates and vectors we are given an "explicit definition" of the tensor by the description of the behaviour of its coordinates under arbitrary regular linear substitution. Tensors subjected to the same transformation law are said to form a "corps de tenseurs", characterized by the degrees of contravariance and of covariance. Cartan's fundamental theorem on "corps de tenseurs" is then proved, and the rules of tensor algebra are established.

It is obvious that "Eléments d'algèbre linéaire" would be the adequate title of this volume. It is written in the serious, but interesting style of the French "Cours d'analyse". There are occasional exercises for the reader, however not sufficient and also not suitable for a systematic training; they are mostly hints concerning an alternate version of some development in the text, or statements of theorems provided with references (mainly to "Théories quantiques").

The book has been written in recognition of the fact that the elements of linear algebra are really fundamental for all further study of mathematics, pure and applied. Therefore they are to be presented to the students in their first year. To the reviewer's knowledge this has first been done systematically, if not to the same extent as in Julia's lectures, in the mathematics course of the university of Berlin since 1920, by Professor G. Feigl. His interesting lectures "Einführung in die höhere Mathematik" anticipating a good deal of the principles claimed as revolutionary in recent American introductory textbooks, have recently been edited by H. Rohrbach (Springer-Verlag, Berlin-Göttingen-Heidelberg, 1953).

Unfortunately the secondary school education on this continent is lagging far behind and we cannot hope to be successful in confronting our freshmen even with a much less sophisticated introduction to higher mathematics. In the third or the fourth year we offer courses on similar lines to the honours students of mathematics and physics referring for instance to the book by P. Halmos, Finite-dimensional Vector Spaces (or one of the many introductions to linear algebra) as a text, comparable in extent if not in actual execution, to Julia's book, but more specialized than Feigl's introduction to higher mathematics. Now, if we accept as an axiom that students of equal ages in different civilized countries are of equal intelligence, no reason can be seen why it should not be possible to raise the level of mathematical preparation of university freshmen to a degree such that also on this side of the Atlantic a reasonable course on linear algebra can be digested at the beginning, instead of the end of their mathematical courses.

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<u>The Theory of Matrices</u>, by F. R. Gantmacher. Chelsea, New York, 1959. Vol. I. x + 374 pages; vol. II. ix + 276 pages. $6.00 per volume.

Among all the books on Linear Algebra and Matrices that have been published during the last ten years in almost all languages, the present work, first published in Russian in 1954, is probably the most complete treatise on the subject. It has some of the features of a comprehensive handbook: its chapters are to a certain extent independent and the earlier ones can be read by anybody acquainted with determinants and the usual preliminaries from elementary algebra. The last three chapters, dealing with questions arising in probability, systems of differential equations, and the Routh-Hurwitz problem are of a more specialized nature.

Some eminent representatives of the abstract algebraic school have recently expressed their strong disapproval of the teaching of formal matrix theory, giving as reason that proofs of most theorems in linear algebra can be presented more briefly and more lucidly if the notion of matrix is avoided as long as there is no question of determinants. The present book does not attempt a development of linear algebra the abstract way. Its main object is finite matrices with number entries, real or complex, as they occur in ever so many problems of classical analysis, numerical analysis, mechanics, probability, etc. In the reviewer's opinion there can be no doubt that the methods used in Gantmacher's work are adequate and useful for all those who have to deal with problems of this kind. But also the casual reader, primarily interested in algebra, will find information not easily to be obtained elsewhere.