

demolishing this myth it is amusing to find another author propagating a different myth—the story that Newton's sole contribution to a parliamentary debate was to ask for one of the windows to be closed. At least, if this is not a myth, it is a story I have heard told about lack-lustre MPs in our own day.

To be fair, such faults as this book has seem minor in comparison with how greatly it enriches one's appreciation of the real Newton. It digests for the general reader a great range of recent scholarly studies, and is handsomely illustrated. Moreover, as with Newton's output itself, the whole is greater than the sum of its parts.

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HIGMAN, G. and SCOTT, E., *Existentially closed groups* (London Mathematical Society Monographs (New series) No. 3 Oxford University Press, 1988), 170 pp., 0 19 853543 0, £25.

Let  $G$  be a group. A system of equations and inequalities, involving elements of  $G$  and a collection of variables, is said to be soluble in  $G$  if they become true statements on replacing the variables by suitable elements of  $G$ . It is said to be soluble over  $G$  if it is soluble in some group  $H$  that contains  $G$  as a subgroup. A group  $G$  is said to be *existentially closed* if every finite system of equations and inequalities that is soluble over  $G$  is actually soluble in  $G$ .

This fairly innocuous definition opens up a rich and fascinating branch of group theory, closely related to logic in the form of model theory and recursive functions. The subject was first studied in the early 1950s by B. H. Neumann and W. R. Scott, who looked at the slightly weaker notion of *algebraically closed group* (the same definition as above, but without the inequalities: surprisingly, it turns out that the trivial group is the only algebraically closed group that is not existentially closed). Perhaps the most striking result in the subject is the theorem of Neumann, Simmons and Macintyre that a finitely generated group has soluble word problem if and only if it can be embedded as a subgroup of every existentially closed group. The subject developed further in the 1970s and 1980s through work of Hickin, Macintyre, Ziegler and others, and is still an area of current research activity.

The book under review is a welcome introduction to the subject, based on a course of advanced lectures given by Higman. It is aimed at group-theorists of at least research student level. No prior knowledge of logic is assumed, although it would be an advantage to have some. Certainly a fair amount of mathematical sophistication is required of the reader. Within these constraints, the book succeeds in making accessible a sizeable piece of mathematics, much of it highly non-trivial, that was previously available only in the form of research papers.

The first four chapters are fairly gentle. Various group-theoretic properties of an existentially closed group  $G$  are considered. For example,  $G$  is simple and infinitely generated, and there are strong restrictions on the centralizers and normalizers of subgroups of  $G$ . The existence of any existentially closed group at all is a non-trivial fact, which is not proved until Chapter 3. There follows a chapter on recursion theory. Again the treatment is fairly gentle, and does not assume a prior knowledge of logic.

The pace is stepped up in the second half of the book, which is largely based on work of Ziegler, together with some results of Hickin and Macintyre. Recursion theory, together with a subgroup theorem of Higman, are used to prove a number of results about existentially closed groups, including one half of the Neumann–Simmons–Macintyre result mentioned above. To prove the other half, along with a number of further results, the notion of “games” is introduced. This powerful device is used to show, among other things, that the set of isomorphism types of countable existentially closed groups has the cardinality of the continuum. The book ends with a substantial chapter on the first-order theory of existentially closed groups.

The material presented in this book is probably too specialized to be used for teaching purposes, except possibly for a very advanced graduate course. Its main use will be for specialists in group theory who are looking for a research topic, or merely interested in learning some new mathematics. To anyone in this category I recommend it as an interesting and informative source.

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