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SHORT-TERM RHEOLOGY OF POLYCRYSTALLINE ICE

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ABSTRACT. Deformation characteristics of polycrystalline ice and their dependence on time, temperature, and stress have been analysed on the basis of a phenomenological relation which describes creep in terms of initial elastic, delayed elastic, and permanent strain. It is shown that the effective modulus of ice observed in the laboratory or in the field can be examined on the basis of this model. The model also provides a basis on which the observed flow law of ice can be examined conveniently. Some apparent inconsistencies in the results of earlier investigations of the mechanical property of ice appear to be reconciled by the model.

RÉSUMÉ. *Modèle rhéologique à court terme de la glace polycristalline.* Les caractéristique de déformation de la glace polycristalline et leur dépendence du temps, de la température et de la contrainte ont été analysées en utilisant une relation phénoménologique décrivant le fluage en terme de déformation élastique, déformation viscoélastique et déformation plastique. On montre que le module effectif de la glace observé dans le laboratoire ou sur le terrain peut être considéré sur la base de ce modèle. Celui-ci fournit également les éléments permettant une analyse correcte de la loi d'écoulement expérimentale de la glace. Certaines contradictions apparentes dans les résultats des études antérieures concernant les propriétés mécaniques de la glace peuvent ainsi être dissipées.

ZUSAMMENFASSUNG. *Kurzzeit-Rheologie von polykristallinem Eis.* Die Verformungsmerkmale von polykristallinem Eis und ihre Abhängigkeit von der Zeit, Temperatur und Spannung wurden auf der Grundlage einer phänomenologischen Beziehung untersucht. Sie beschreibt das Kriechen in Ausdrücken von anfänglich elastischer, verzögert elastischer und bleibender Verformung. Es wird gezeigt, dass der wirksame Modul von Eis, der im Labor oder im Feld gefunden wird, auf der Grundlage dieses Modells untersucht werden kann. Das Modell liefert ebenso eine Grundlage für die bequeme Prüfung des beobachteten Fliessgesetzes für Eis. Einige augenscheinliche Widersprüche in den Ergebnissen früherer Untersuchungen der mechanischen Eigenschaften von Eis scheinen durch das Modell beseitigt zu sein.

INTRODUCTION

Studies of creep of polycrystalline ice have emphasized the steady-state or secondary creep stage. Little attention has been given to the initial or transient creep range, the range of particular importance to many engineering problems involving ice.

A programme of observation was undertaken on the initial uniaxial compressive creep and recovery of columnar-grained ice. From these observations a phenomenological viscoelastic relation was developed which appears to agree satisfactorily with actual measurements. This relation permits the static and dynamic elastic moduli of polycrystalline ice to be examined quantitatively; as well as allowing laboratory and field measurements of both short- and long-term deformation properties to be analysed in a rational manner. A critical examination of the relation indicates that the transition observed in the flow law of ice at low stresses may be due to the short duration of previous experiments. This paper reports the results of this work.

PHENOMENOLOGICAL VISCOELASTICITY OF ICE

Compressive creep tests were conducted in the temperature range -10 to $-45°C$ on columnar-grained type S-2 ice (Michel and Ramseier, 1969) with the [0001] axis of the grains normal to the columns. A load was applied in the direction normal to the columns with a simple lever system and measured by a load cell. Specimen deformation was measured by linear differential transformers mounted directly on the specimen, and recorded using a high-speed data-acquisition system. The loading duration was limited to a period of about 500 s. Fuller details of the experimental techniques and method of analysis are outside the scope of this paper and will be presented elsewhere.*

* Paper by N. K. Sinha entitled "Rheology of columnar-grained fresh water ice", to be published.

The ice showed an initial elastic deformation which was followed by a time-dependent, delayed elastic (or recoverable) strain and viscous flow. Both the delayed elastic strain and the viscous flow had the same activation energy. Thus, this ice could be considered as non-linearly stress dependent and rheologically simple. For such a material, creep curves for a given load at various temperatures can be reduced to a single master curve by means of a shift function $S_{1,2}$ given by

$$\ln(t_1/t_2) = \ln S_{1,2} = \frac{Q}{R}\left(\frac{1}{T_1}-\frac{1}{T_2}\right), \tag{1}$$

in which t_1 and t_2 are the times required to produce a given strain at temperatures T_1 and T_2, respectively, Q is the activation energy, and R is the gas content. The phenomenological relations can be described by

$$\epsilon_t^{\sigma} = (\sigma/E_0)[1+c\{1-\exp(-[a_T t]^b)\}]+\dot{\epsilon}_{v_1}|\sigma|^n t, \tag{2}$$

where ϵ_t^{σ} is the creep strain at time t after loading, the superscript denoting stress and the subscript indicating time, σ is the applied uniaxial stress, E_0 is Young's modulus which is relatively temperature independent, $\dot{\epsilon}_{v_1}$ is the steady-state strain-rate for unit stress, n is the usual stress exponent, b and c are constants, and a_T is a factor varying with temperature according to

$$a_T = \exp[-(Q/RT+d)], \tag{3}$$

in which d is a constant.

The steady-state strain-rate for unit stress $\dot{\epsilon}_{v_1}$ in Equation (2) varies with temperature as

$$\dot{\epsilon}_{v_1} = A\exp(-Q/RT), \tag{4}$$

where A is a stress-dependent constant (Glen, 1955; Gold, 1973).

It is known that the deformation behaviour of polycrystalline ice depends upon several factors, including crystallographic structure, purity, direction of application of load, and any previous strain history which might have introduced irreversible morphological changes into the microstructure. Equation (2) appears to satisfy the experimental results for previously undeformed S-2 ice of average grain size (3 mm) with the following values:

$$\left.\begin{aligned} E_0 &= 9.3 \text{ GN m}^{-2},\\ Q &= 66.9 \text{ kJ mol}^{-1} \ (= 16 \text{ kcal mol}^{-1} = 0.70 \text{ eV}),\\ c &= n = 3,\\ b &= 0.34 \approx 1/n,\\ d &= -22.34 \text{ giving } a = 2.5\times 10^{-4} \text{ s}^{-1} \text{ at } 263 \text{ K},\\ A &= 3.76\times 10^{3} \text{ for } \sigma \text{ in units of } 10^{5} \text{ N m}^{-2} \text{ (Gold (1973) for S-2 ice).} \end{aligned}\right\} \tag{5}$$

The stress exponent for viscous flow has been assumed to be equal to 3 in accordance with previous investigations.

Time-dependent modulus

Deformation in a viscoelastic material is time-dependent, and the quotient of the applied stress and corresponding strain can best be called the time-dependent or effective modulus. For a linear viscoelastic material the strain is linearly related to stress, and thus the effective modulus is only a function of time. The creep strain of ice is a function of both time and stress, it is therefore appropriate to use

$$E_t^{\sigma} = \sigma/\epsilon_t^{\sigma}, \tag{6}$$

for the time-dependent modulus of ice.

Substituting ϵ_t^{σ} in Equation (6) from Equation (2), the time-dependent modulus during creep of ice is

$$E_t^{\sigma} = \sigma \Big/ \left(\frac{\sigma}{E_0} [1 + c\{1 - \exp(-[a_T t]^b)\}] + \dot{\epsilon}_{v_1} t |\sigma|^n \right). \tag{7}$$

Its value reduces to E_0, the Young's modulus or the high-frequency modulus, for time t equal to zero. The relative contributions of the delayed elastic and viscous creep would determine E_t^{σ} in the intermediate time range, the long-term value is determined mainly by the viscous deformation.

Frequency response of effective modulus

The usual engineering practice is to record stress–strain curves at various applied strain-rates, from which initial tangent modulus, average tangent modulus, or secant modulus are determined (Hawkes and Mellor, 1972; Gold and Traetteberg, [1975]; Traetteberg and others, 1975).

The average strain-rate up to time t after loading is given, from Equation (2), by

$$\dot{\epsilon}_{t(\mathrm{av})}^{\sigma} = \frac{\epsilon_t^{\sigma}}{t} = \frac{\sigma}{E_0 t} [1 + c\{1 - \exp(-[a_T t]^b)\}] + \dot{\epsilon}_{v_1} |\sigma|^n. \tag{8}$$

Equation (7) in conjunction with Equation (8) gives an opportunity to compute the variation of the effective modulus as a function of strain-rate, provided the load amplitude is small, because the total strain-rate in a constant strain-rate experiment is the sum of the elastic and viscous strain-rates (Weertman, 1973). Comparisons were made of the experimental results of Gold and Traetteberg ([1975]) at -10°C and of Traetteberg and others (1975) at -39.5°C and -19.3°C with the corresponding calculated values. There was fair agreement for strain-rates ranging from 10^{-3} to 10^{-7} s^{-1} (Sinha, 1977). Gold (1976) summarized published results of the frequency dependence of the effective modulus (Fig. 1), showing that there is continuity between the effective modulus derived from strain-rate experiments and those derived from previous high-frequency measurements. Figure 1 also shows the dependence given by Equation (7), assuming that the relation between total time of loading and frequency is given by

$$f = \frac{1}{2t} \qquad (f \text{ in Hz}, t \text{ in s}). \tag{9}$$

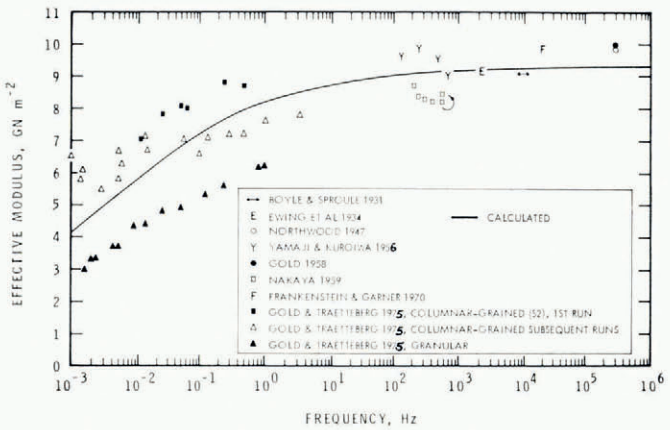

Fig. 1. Frequency dependence of effective modulus for polycrystalline ice at -10°C.

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As the low-frequency experimental data in Figure 1 were determined for a load amplitude of 0.3 MN m^{-2}, the calculated values shown are for this load level.

It may be noted in Figure 1 that values predicted on the basis of Equation (7), with creep parameters given in Equation (5) for S-2 ice, differ considerably from experimental data on granular ice. This indicates that the parameters for elasticity and viscous creep must be different for granular ice—a reasonable conclusion considering the difference in the structure of the two types of ice. Information on the short-term rheological response of granular ice is required to confirm this hypothesis.

This foregoing discussion indicates that the dependence of time-dependent modulus on the strain-rate is another manifestation of the creep behaviour of ice. The time-dependent modulus in creep, however, is highly dependent on stress level and temperature; giving the modulus only as a function of average strain-rate or frequency does not fully describe the material behaviour in general. A better representation of creep response is needed before more general conclusions can be drawn for engineering and material-science applications.

Creep compliance function

The creep compliance function D_t for a linear viscoelastic material is given by

$$D_t = \epsilon_t = E_t^{-1} \quad (\text{for } \sigma = 1). \tag{10}$$

This relation is particularly useful when using Boltzmann's superposition principle in the prediction of deformation processes in linear viscoelastic materials under conditions of relatively complicated stress or strain history. D_t cannot be described uniquely for non-linear viscoelastic materials such as ice because of the stress dependence of E_t, i.e.

$$D_t^{\sigma} = \epsilon_t^{\sigma}/\sigma = 1/E_t^{\sigma}. \tag{11}$$

Normalized creep compliance function

The normalized form of creep compliance is given by

$$D_0^{\sigma}/D_t^{\sigma} = \epsilon_0^{\sigma}/\epsilon_t^{\sigma} = E_t^{\sigma}/E_0, \tag{12}$$

where the subscript 0 indicates response at zero time.

This is a convenient method for expressing time-dependent values of creep compliance or effective modulus in terms of initial response. The rheological response of the same type of ice could vary from sample to sample owing to variations in the microstructure; experimental results are, therefore, always subject to a certain amount of unavoidable scatter. The normalization technique allows the experimental results to be compared in a rational way.

Normalized creep compliance functions for columnar-grained S-2 ice are presented in Figure 2 for a few stress levels at −10°C. For times less than one second and stress less than 2 MN m^{-2} the functions are essentially similar. For stress less than 0.5 MN m^{-2} this similarity continues up to about 20 s. The normalized creep compliance functions at other temperatures are obtained by shifting the curves in Figure 2 along the time scale by the shift function. For example, for stress less than 0.5 MN m^{-2}, the time-dependent modulus would be independent of stress for a period of about 2 000 s when the temperature is −45°C.

The first term of Equation (2) provides a model for temperature- and time-dependence of the elastic modulus after loading. The normalized elastic response, that is, the ratio of the initial elastic strain to the total elastic strain including the delayed elastic part, is also shown in Figure 2 as a function of time. This is independent of stress level as is evident from Equation (2). Deviation of the various creep compliance curves in Figure 2 is due to stress-dependent viscous flow. The time ranges discussed in the preceding paragraphs therefore indicate the period of time for which ice behaves in an essentially elastic manner for the given load levels.

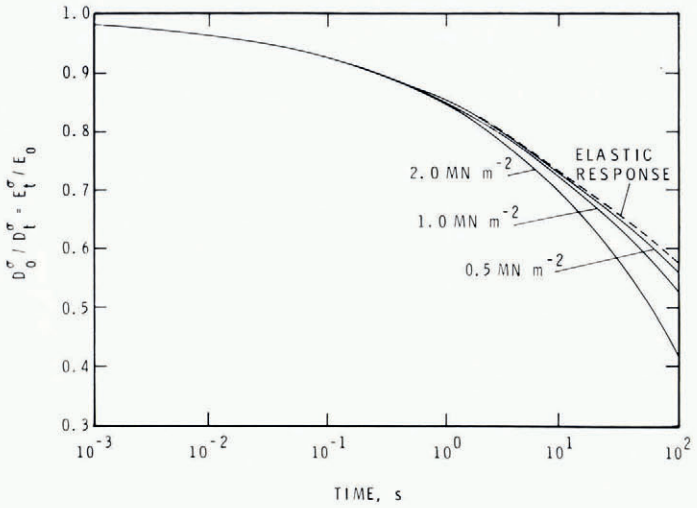

Fig. 2. Normalized creep compliance function for S-2 ice at −10°C for different loads.

Equation (12) in conjunction with Equation (2) could therefore be used to determine the time and strain at which permanent deformation is significant in relation to elastic and delayed elastic (total elastic) strain for a given load and temperature.

The theoretical analysis of most bearing-capacity problems involving moving loads and ice–structure interactions is based on the principles of linear elasticity. The foregoing presentation indicates the limits of time, temperature, and stress within which elastic theory may be applied to ice problems and the corresponding relaxed or effective elastic modulus may be used. In his discussion of the engineering properties of fresh-water ice, Gold (1976) stated that ice can be assumed to respond elastically to stress when the period of application of the load is less than 100 s for stresses less than 1 MN m^{-2} or if loaded to failure within about 2 s. The normalization technique adds greater quantitative precision to this conclusion regarding the applicability of elastic theory to ice problems.

Static and dynamic elastic modulus

The variation in the modulus of ice, determined experimentally by static and dynamic methods, can be explained using Equation (2) or its graphical presentation in Figure 2. Measurements made in times less than 10^{-3} s (in fact at 5×10^2 Hz) at −10°C should be very close to the true value of Young's modulus. Experimental data presented in Figure 1 support this observation. Static methods, however, will yield values that depend, for a given type of ice, on the time taken to load the specimen, the time at which the measurements are made, and temperature and stress, as is evident in Figure 2.

Static modulus

The normalized effective modulus as a function of temperature after times of 1, 5, and 30 s is plotted in Figure 3 for stress levels of 1 MN m^{-2} and 2 MN m^{-2} (loading time was assumed to be instantaneous for these curves). It can be shown (Fig. 3) that the curves at one second are essentially the same, assuming that full load is applied instantaneously. Complications as a result of pressure melting and impurities at grain boundaries could influence the activation energy for creep in the temperature range near to the melting point. Caution must be exercised, therefore, if the curves in Figure 3 are to be extrapolated to higher temperatures.

It has been pointed out that the effective modulus is practically independent of stress up to 2 MN m^{-2} for loading or measuring times less than 1 s at −10°C. Figure 3 reaffirms that ice can be considered to be an elastic material in the entire temperature range of practical interest providing that the loading time is less than one second and the load is less than 2 MN m^{-2}.

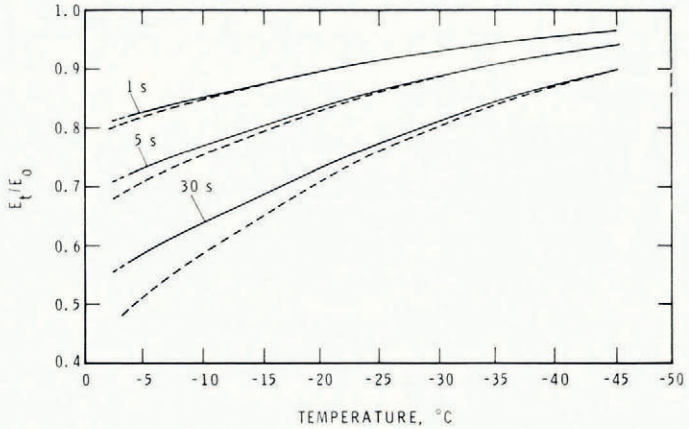

Fig. 3. Temperature dependence of normalized effective modulus of S-2 ice for loads of 1 MN m^{-2} (solid line) and 2 MN m^{-2} (broken line) at three different times.

Laboratory and field observations of the effective modulus

Static measurements of the effective modulus can be divided into two categories: those made in the laboratory, and those made in field tests. As both types usually differ greatly from each other, it seems appropriate to discuss the subject in some detail. Consider first the strain, and hence the effective modulus, if measurements are made five seconds after the application of full load for loads up to 1.0 MN m^{-2}. As shown in Figure 3, the effective value will be lower than the high-frequency modulus or Young's modulus by 6% at −45°C, 11% at −30°C, 23% at −10°C, and more than 30% near 0°C. Similarly, if strain is determined 30 s after load has been applied, values will be lower by 10% at −45°C, 19% at −30°C, 36% at −10°C, and more than 50% at temperatures just below 0°C. The increase in apparent activation energy observed for temperatures near the freezing point will introduce even larger discrepancies into the effective modulus in that temperature range. Higher loads will also introduce larger discrepancies (Fig. 3). Experimental observations by Voytkovskiy (1960) and field tests by Eyre and Hesterman (1976) support this conclusion. These workers observed the dependence of effective modulus on stress and found that modulus decreased as the load increased.

Typically, laboratory measurements are made within 5 s of load application and *in situ* field measurements within 30 s. Field values, which are subject to additional discrepancies owing to the temperature gradients inside the specimens, should therefore be lower than laboratory values. Both should show that effective modulus increases with decreasing temperature, and this has been verified by numerous field and laboratory measurements (e.g. Gold, 1958; Voytkovskiy, 1960; Tabata, 1967; Smirnov, 1971).

Further comments on experimentally determined modulus

It is common experimental practice to record strains for several load levels and determine the effective modulus by standard statistical methods. The finite time involved in loading or measuring strains, in conjunction with the statistical methods used, introduces yet another

uncertainty into the final result, an uncertainty that becomes more critical as higher load levels are used at higher temperatures. A specific example may clarify this.

Consider a hypothetical laboratory experiment in which loads are applied at a finite load rate of 1 MN m^{-2} s^{-1} and experiments are conducted for various load levels up to 2 MN m^{-2}. Figure 4 illustrates the stress–strain relation obtained from Equation (2) for four load levels. The points represent the calculated strains at −10°C for 0.5 MN m^{-2}, 1 MN m^{-2}, 1.5 MN m^{-2}, and 2 MN m^{-2} at times of 0.5, 1, 1.5, and 2 s respectively, to comply with the loading rate. These points do not lie strictly in a straight line. If they were *experimental* results, then the points would, in the ordinary way, be considered to be linearly related and the corresponding modulus would be determined from the slope of the line of best fit. The dashed line drawn by joining the second and the fourth points represents the line of best fit fairly well, giving a modulus of 7.2 GN m^{-2}.

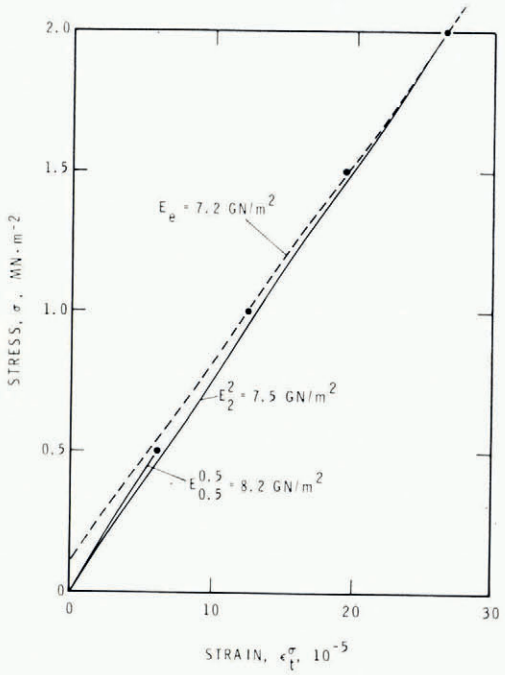

Fig. 4. Computed stress–strain relation for loading rate of 1 MN m^{-2} s^{-1} at −10°C. Solid lines indicate slope and expected effective modulus. Broken lines show slope and evaluated modulus Ee.

The evaluated result E_e is considerably lower than $E_{0.5}{}^{0.5}$, the maximum value, and is even lower than $E_2{}^2$, the lowest value in the series. Note also the intercept of the broken line with the stress axis. This intercept, if present in a real experiment, would be ascribed to the uncertainties of the measuring system.

Actual experimental data on S-2 ice at −10°C for an average load rate of 1 MN m^{-2} s^{-1} are shown in Figure 5. Results were obtained from a sequence of rapid loading up to full load, followed by rapid unloading. The conditions of the experiment were similar to those described for the creep tests. The average load rate was determined from recorded load histories. The line of best fit, obtained by the least-squares method, is represented by the solid line, and gives 7.0 GN m^{-2} for the evaluated modulus with a stress intercept of 0.07 MN m^{-2}. The example bears a close resemblance to the model presented in Figure 4.

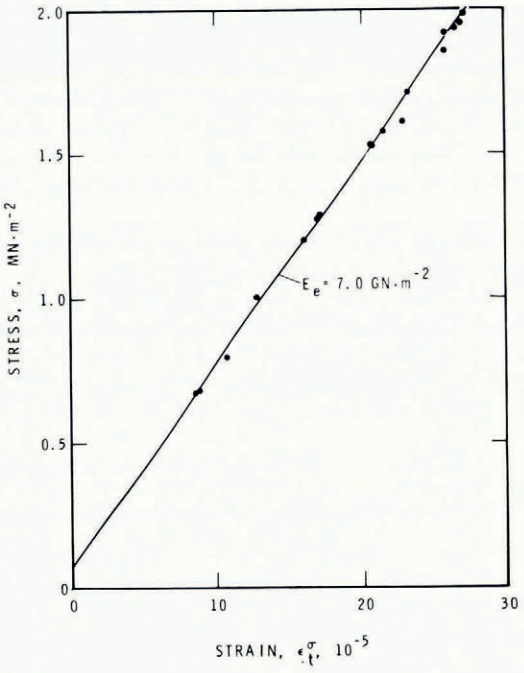

Fig. 5. Observations of stress–strain relation of S-2 ice at −10°C.

An example typical of experimental results obtained at −40°C is shown in Figure 6. Note the higher value of the evaluated modulus and a stress intercept which is lower than that for measurements at −10°C. The average load rate is, however, slightly more than that at the elevated temperature owing to increased stiffness in the loading system.

The variation of the experimentally-evaluated effective modulus as a function of temperature is shown in Figure 7. The solid line represents the variation calculated on the basis of the simple schedule illustrated in Figure 4, assuming a constant load rate. The agreement is fair in view of the method of calculation.

Although the maximum time taken to load the specimen to the highest load level was about two seconds, variation of the experimentally evaluated modulus with temperature is more like that of the five-second modulus presented in Figure 3. The load rate chosen was considered to be very rapid and the experimental conditions sufficiently precise; even under these conditions, however, the results show large discrepancies. The data obtained from laboratory experiments are not readily applicable to field conditions because of differences in the type of ice, grain size, temperature variation within the specimen, complex stress states occurring in the natural ice sheet, and the constraints on the conditions of performing tests. The examples show how even a well-performed laboratory experiment will give low values for the effective modulus. Extending the analysis to field conditions could demonstrate that under these conditions very low values (compared to the high-frequency elastic moduli) can be expected.

Flow law of polycrystalline ice

Problems encountered in the interpretation of the initial creep of ice seem to be repeated again, in a different form, for a time scale that is long for general experimental observations but short with respect to the period required to study the phenomenological flow laws of ice.

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Accurate determination of Young's modulus and variations of the effective modulus with temperature, stress, and time are the primary problems encountered at the very short end of the time scale. For long times we have the problem of how to determine the flow law and its variation with stress.

In the past, investigations have been made into the development of a relationship between "observed" steady-state strain-rate and applied stress. These studies demonstrated clearly

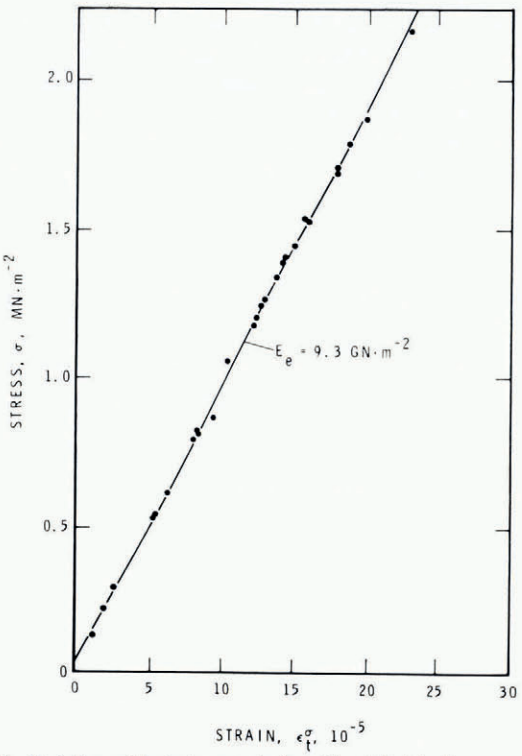

Fig. 6. Variation of instantaneous strain with applied load at −40°C.

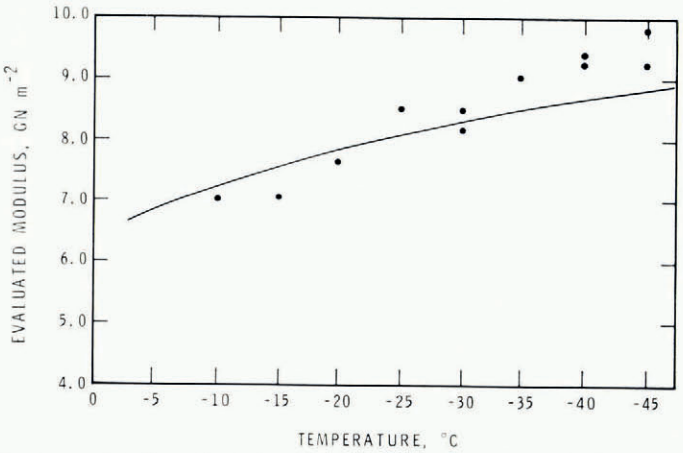

Fig. 7. Temperature dependence of experimentally evaluated modulus compared with predicted modulus.

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that there are three distinct zones in the load–creep-rate spectrum. A steady state was never reached for high stresses (above 1 MN m^{-2}) so that efforts were concentrated mainly on relating minimum creep rate to stress. In the intermediate range (0.1–1.0 MN m^{-2}) a steady or quasi-steady state has been observed. A definite flow law for this regime has been agreed upon (Glen, 1955; Steinemann, 1958; Voytkovskiy, 1960; Dillon and Andersland, 1967; Barnes and others, 1971; Gold, 1973). This may be called the zone of general agreement or a "no-conflict" zone. There are, however, controversies remaining concerning the mechanisms which obtain at the lower end of the stress range.

Several investigators (Butkovich and Landauer, 1960; Meier, 1960; Mellor and Smith, 1967; Bromer and Kingery, 1968; Colbeck and Evans, 1969, 1973; Mellor and Testa, 1969) have observed that creep rate changes from power law to a Newtonian viscous flow at low stress. There had been earlier suggestions of a transition in the flow pattern somewhere around 0.1 MN m^{-2}. An effort will be made in the following paragraphs to use the proposed viscoelastic relation to examine the early part of creep behaviour and its relation to stress, time, and temperature.

Creep rate

Creep rate at any time t after loading is given by differentiating Equation (2) with respect to time

$$\dot{\epsilon}_t^{\sigma} = (c\sigma b/E_0 t)(a_T t)^b \exp\{-(a_T t)^b\} + \dot{\epsilon}_{v_1}|\sigma|^n. \qquad (13)$$

The first term of Equation (13) is directly proportional to stress, whereas the second term varies as the cubic power of stress. Moreover, the term which represents recovery creep

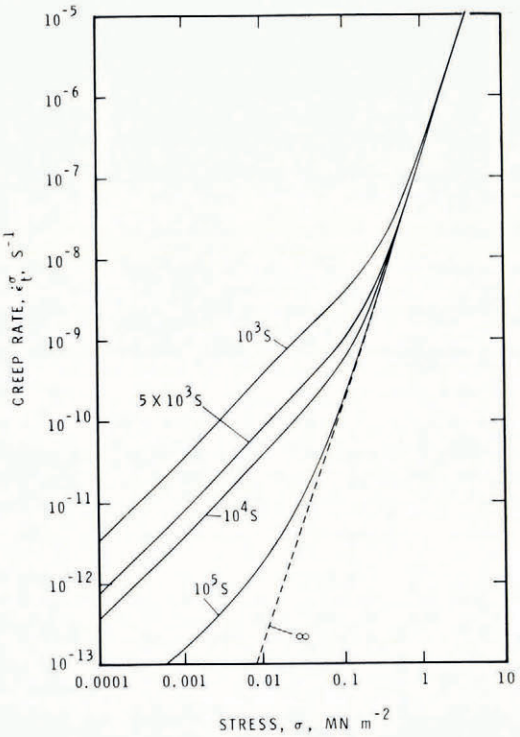

Fig. 8. Creep rate of S-2 ice in uniaxial compression against applied stress at −10°C for several loading times. Curves are calculated on the basis of proposed rheological relation.

approaches zero asymptotically at infinite time. Thus, a quasi-steady state in the total creep strain-rate would be attained in a shorter time with increasing stress. This state may not be observed at low stresses, except for prolonged creep, provided the morphological changes in the microstructure of ice do not alter the creep behaviour. Observed steady state would, in any case, depend largely on the conditions and accuracy of measurements.

The computed stress dependence of strain-rate for a number of creep times is presented in Figure 8. Calculations were made on the basis of Equation (13) and information given in Equation (5). It may be seen from the graph that 15 min is considered sufficient (depending on the accuracy of measurement) to reach a quasi-steady state with a load of 2 MN m^{-2} at $-10°C$, whereas to reach about the same state with a load of 0.2 MN m^{-2} would take more than a day.

The effect of temperature may be seen by translating the curves in Figure 8 along the ordinate by the amount given by the shift function. Thus, for a load of 0.2 MN m^{-2} it would take more than 130 d at $-45°C$ to reach the same creep rate as can be reached in 1 d at $-10°C$. It will be shown that determining an assumed steady-state strain-rate from plotted creep data might not be sufficient for low stress levels. An alternative method is suggested for the treatment of experimental results.

Evaluated stress exponent

It is customary to determine the stress exponent for ice from the observed creep rate using the power law relation first proposed by Glen (1955). Consider the following relation in which $\dot{\epsilon}_t^{\sigma}$ of Equation (13) is restated as

$$\dot{\epsilon}_t^{\sigma} = \dot{\epsilon}_{v_1 e}|\sigma|^{n_e}, \tag{14}$$

where $\dot{\epsilon}_{v1e}$ is a constant and subscript e is used to distinguish values calculated in the normal way from the ideal steady-state value; n_e will be called the effective stress exponent. Equation (14) can be re-arranged to give n_e in terms of strain-rates and stresses,

$$n_e = \log (\dot{\epsilon}_t^{\sigma_1}/\dot{\epsilon}_t^{\sigma_0})/\log (\sigma_1/\sigma_0), \tag{15}$$

where σ_0 is the base stress and σ_1 applied stress. A graphical presentation of the variation of n_e as a function of time and stress is presented in Figure 9. The solid curves shown correspond to those in Figure 8. Calculations are based on $\sigma_0 = 0.1$ MN m^{-2}.

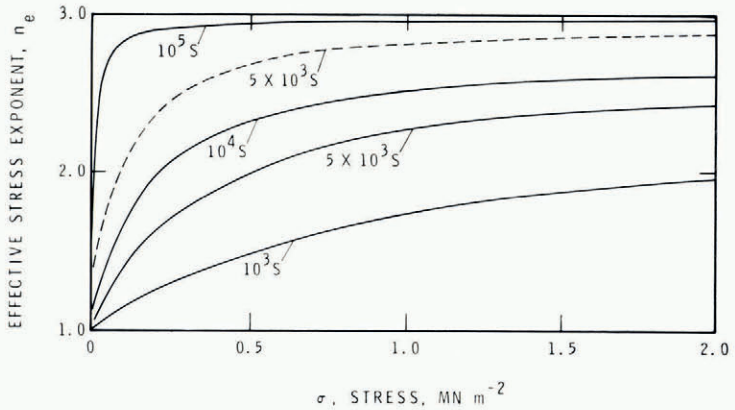

Fig. 9. Evaluated stress exponent of S-2 ice in uniaxial compression at $-10°C$ as a function of stress for several loading times. Solid curves are calculated on the basis of the proposed rheological model using 0.1 MN m^{-2} as the base stress level and the broken line for base stress of 0.5 MN m^{-2}.

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Experiments are usually performed in a narrow range of stress. It is conceivable that the lowest stress would be chosen as the base stress for calculating the stress exponent, unless one tried to fit a straight line to a log–log plot of stress versus strain-rate. Figure 9 (broken line) illustrates the change in the effective stress exponent for 5×10^3 s when σ_0 is changed to 0.5 MN m^{-2} from 0.1 MN m^{-2}.

A few observations should be made regarding the characteristics of n_e. The effective stress exponent would be a function of time for a given stress and temperature. It would be a function of stress level, resulting in an "apparent" transition at low stresses, and would also depend upon the choice of the base stress. These dependences would disappear as the steady state is reached and n_e approaches n. In fact, this suggests an alternative method of examining the quality of the experimental results, provided the existence of a true steady state is real.

Gold (1965) studied the initial creep rate of S-2 ice between 60 s and 2×10^4 s at -10°C for stresses of from 0.4 to 1.4 MN m^{-2}. This is the only study, to the author's knowledge, of initial creep in the intermediate stress range that can be compared directly with Figure 9. Gold observed that creep, at a given time, for previously undeformed ice could be approximated by a power-law function. He found that the stress exponent increases from unity to about 2.2 in the first 5×10^3 s and thereafter remains almost steady up to 2×10^4 s. This agrees well with the prediction.

Experimental results of long-term creep-rate investigations (mentioned earlier) of the flow law of polycrystalline ice show remarkable similarity to the curves in Figure 8. The compilations of laboratory studies by Dillon and Andersland (1967), the separate compilations for laboratory and field data shown by Langdon (1973), and recent compilations by Roggensack (unpublished) and Shumskiy (1974) on both field and laboratory data (see also the review by Hobbs, 1974) corroborate the convergence at high stresses and the time of measurement-dependent flare in strain-rates obtained at low stresses (Fig. 10) which are predicted by the present analysis. Figure 10 gives the bounds for calculated results and realistic ranges of temperature, stress, and time.

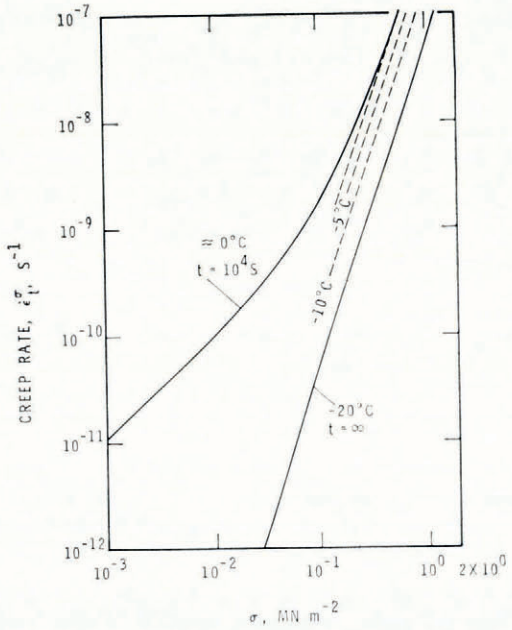

Fig. 10. Zone of strain-rate versus stress for S-2 ice between measuring times of 10^4 s and ∞ for a temperature range 0 to -20°C. Calculations are based on Equation (13) assuming no activation energy change near 0°C.

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It is also worth noting that previous observations (Glen, 1955; Steinemann, 1958; Voytkovskiy, 1960; Dillon and Andersland, 1967; Barnes and others, 1971; Gold, 1973) are more or less consistent with each other in the intermediate stress range for which a quasi-steady state occurs in a reasonable length of time. Agreement could, in fact, be predicted by the convergence of various curves of widely separated time and temperature ranges in Figures 8 and 10. That different workers obtained a stress exponent of about 3 in this stress range indicates that the choice of the exponent in Equation (2) may not be far from reality.

Internal crack formation at high stresses (Gold, unpublished) and dynamic recrystallization at still higher stresses (Barnes and others, 1971) influence creep rate. The present model is not valid at higher stresses unless the third term in Equation (2) is modified.

Creep data at low stresses need further analysis in examining the applicability of the proposed rheological model. Butkovich and Landauer (1960) observed the stress exponent to be between 0.86 and 1.15 in the stress range of 0.002 to 0.02 MN m^{-2}. Colbeck and Evans (1969) found the power to be 1.9 over a range of stresses 0.01 to 0.1 MN m^{-2}. Mellor and Testa (1969) observed the stress exponent to be 1.8 in the range from 0.009 to 0.04 MN m^{-2}. Colbeck and Evans (1973) noticed that their results fitted a power of 1.3 at stresses ranging from 0.006 to 0.1 MN m^{-2}. Thus there are a variety of stress exponents at low stresses. But these discrepancies are to be expected, according to Figures 8 and 9, if creep times are short in comparison with the time required to reach a steady state. The experimental results probably reflect inadequate time of loading and inaccuracy of measurements; this has been suggested by Weertman (1969, 1973).

Some direct support of this hypothesis comes from Thomas (1973), who extended the power law with the stress exponent of about 3 to the stress range 0.04 to 0.1 MN m^{-2}. Thomas based this extension of the generalized flow law on his interpretation of the available data on ice-shelf deformation. Although his results support the present hypothesis, they do not fully clarify the reported discrepancies because the ice shelves studied had average temperatures of -6 to -16°C, whereas some of the laboratory creep data were obtained at temperatures just below 0°C. An indirect approach, although not fully convincing, lends some support to the hypothesis.

Meier (1960) has suggested that the appropriate flow law for stresses less than about 0.05 MN m^{-2} is a two-term equation given by

$$\dot{\epsilon}_s = A\sigma + B\sigma^n. \tag{16}$$

Here, A and B are constants and n is about 4.5. A similar relation was tried by Butkovich and Landauer (1960), who assumed n to be 3. Mellor and Smith (1967) also described their creep data in the form of Equation (16) with $n = 3.5$.

It is of considerable interest that Equation (16) bears a close resemblance to Equation (13), which also contains a linear stress-dependent term. If t in Equation (13) is fixed at some convenient value, then the two equations are identical.

Lliboutry (1969) proposed a polynomial relation of the type

$$\dot{\epsilon}_s = A\sigma + B\sigma^3 + C\sigma^5, \tag{17}$$

this is an extension of Equation (16). Colbeck and Evans (1973) used Equation (17) to fit their experimental data, which are rather scattered. Nonetheless, they found that

$$\dot{\epsilon} = 0.21\sigma + 0.14\sigma^3 + 0.055\sigma^5 \tag{18}$$

satisfied their experimental results in the range of stress from 0.006 to 0.1 MN m^{-2}.

It may be seen that the first term of Equation (18) is the major contributor to strain-rate in the low stress range. This is exactly the same situation as for the first term of Equation (13) in relation to the second term during the first part of the deformation.

Discussion

The technique (Sinha, 1978) developed for observing the microstructure of ice shows that grain-boundary sliding is the major contributor to strain during early creep. Preliminary investigations indicate that delayed elasticity is primarily controlled by sliding and resistance to sliding brought about by the interaction between grain boundaries, accommodation at triple points, and grain deformation and re-arrangement. Microstructure observations of deformed ice showed the formation of jogs or steps in the grain boundaries, development of small-angle boundaries, vein-like and cellular structures due to basal and non-basal dislocations, and the pile-up of basal dislocations against grain boundaries and internal grain obstructions. The deformation process is further complicated by the initiation of fracture at jogs in the grain boundaries and at triple points. The formation of these microstructural features depends on stress, temperature, and time, and these structural changes cause changes in the macroscopic flow properties.

The rheological relationship proposed here was developed primarily to relate the total deformation to the measurable elastic, recoverable, and permanent deformation, and its representation, given in Equation (2), was the result of both necessity and desired convenience. Observations indicate that the contribution of delayed elasticity to the total strain, however, depends on microstructure, crystallographic orientation, and previous strain history. Equation (2) provides a reasonable method for analysing the measurements of deformation behaviour and the results presented here indicate its general applicability.

Analysis suggests that there is continuity of behaviour in spite of the fact that previous investigations sometimes appear to be inconsistent, if not irrational. Analysis has further confirmed that the near-linear relation observed between creep rate and stress at low stresses may be due to measurements being made in the transient region. If this hypothesis is correct, then the permanent creep strain that remains after complete relaxation following prolonged creep could be used to calculate the average strain-rate for the steady-state flow condition. This method was used by the author on S-2 ice in the intermediate stress range and the results agree well with those of Gold (1973) on the same type of ice. Available data obtained at low stresses show creep but not recovery, and this makes it difficult to judge the validity of the proposed method of analysis for this condition.

Conclusion

A clear distinction has been made between the Young's modulus of ice, which can be measured only at high frequencies, and the effective modulus, which is determined by static methods. The effective modulus represents a combination of truly elastic (recoverable) and mixed viscoelastic response that depends upon load, time, and temperature.

There is a limited period after loading during which ice can be considered to be an elastic material, irrespective of its temperature. For S-2 ice loaded perpendicular to the columns, this time is found to be about 1 s for loads up to 2 MN m^{-2} and about 20 s for stresses less than 0.5 MN m^{-2}. Analysis shows that the effective modulus will decrease with increased temperature and stress level, although the magnitude of its variation will depend upon experimental conditions.

There is a strong indication that the failure of previous investigators to observe power-law creep reported at low stresses could be the result of either inaccurate measurement or terminating the tests too early. A reported stress dependence which is nearly linear at low stresses and an associated increase in the stress exponent with stress are both consistent with the rheological model now introduced. The proposed relation to describe this model (Equation (2)) seems to be consistent with the observed behaviour of ice, and its first derivative (Equation (13)) seems appropriate to describe creep-rate.

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DISCUSSION

D. TABOR: As I understand it, this paper applies the ideas of polymer rheology to the flow of ice. In polymers there are two very distinct mechanisms responsible for the elastic deformation of the polymer chains. But with a crystalline material such as ice I do not see how there can be the equivalent of a relaxed and non-relaxed elastic modulus. Could the author explain the physical basis for these two elastic moduli?

N. K. SINHA: Creep of polycrystalline ice is a part of a larger field of study—creep of materials. The ideas applied here brings the analysis of the transient creep in line with the larger subject of rheology of materials, and the physical bases of this model are similar to what has been developed to explain the rheology of other polycrystalline materials. It is hoped that our further studies of simultaneous deformation and microstructure will lead us to a clear explanation of the various mechanisms.

R. W. BAKER: What was the average crystal size used in your tests on columnar-grained ice? Also you mentioned in your talk that you made studies of the microstructure after testing samples. Did you find any microstructural evidence for diffusional creep in your studies?

SINHA: The average grain size (cross-sectional) of the columnar grained specimens tested was about 3 mm in diameter. Examination of the microstructure after deformation, by the etching and replicating technique developed, showed extreme grain-boundary migration when the load was small (less than 0.5 MN m^{-2}) without any pile-up for creep strains of about 10^{-2}. It appears that the deformation of the grain boundaries are related to slip in the neighbouring grains and some sort of diffusional accommodation processes in these regions.

P. DUVAL: Have you verified over long time periods that the recovery creep represents the whole transient creep? I am thinking about the Andrade creep found in granular ices.

SINHA: The rheological model presented was developed mainly by examining the recorded creep and recovery curves for several loading times at various loads and temperatures. The transient creep in this model represents the recoverable creep. The experiments were, however, not conducted for very long times—say, days or so. The morphological changes in microstructure (we are beginning to see, after long times, creep conditions) are expected to have profound influence on the recoverable amount of the creep. This is precisely what we are examining now and I hope we will be able to understand the phenomena better in the near future.

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