Correspondence

DEAR EDITOR,

D. V. Lindley offers a rigorous 'proof' of Stirlng's formula [1], but a more experimental demonstration may be appropriate, e.g. with physics and chemistry students (for whom the formula is important in deriving the Boltzmann distribution):

$$\ln n! = \ln a! + \sum_{a+1}^{n} \ln x \qquad (a < n)$$

$$\approx \ln a! + \int_{a+\frac{1}{2}}^{n+\frac{1}{2}} \ln x \, dx = \ln a! + \left[x \ln x - x\right]_{a+\frac{1}{2}}^{n+\frac{1}{2}}.$$

Whence $\quad n! \approx a!\left(\left(n + \frac{1}{2}\right)/e\right)^{n+\frac{1}{2}}\left(e/\left(a + \frac{1}{2}\right)\right)^{a+\frac{1}{2}} = K\left(\left(n + \frac{1}{2}\right)/e\right)^{n+\frac{1}{2}}$ where $K = a!\left(e/\left(a + \frac{1}{2}\right)\right)^{a+\frac{1}{2}}$. Now $\left(\left(n + \frac{1}{2}\right)/e\right)^{n+\frac{1}{2}} = \sqrt{n + \frac{1}{2}}\left(\frac{n}{e}\right)^n\left(1 + \frac{1}{2n}\right)e^{-1/2}$ $\approx \sqrt{n}\left(\frac{n}{e}\right)^n$ for large n.

Investigation of the value of K^2 shows that it rapidly approaches 2π as a increases (e.g. $a = 100$, $K^2 = 6.278$; $a = 200$, $K^2 = 6.281$), and the usual formula for $n!$ immediately follows.

Reference

1. D. V. Lindley, More on Stirling's formula, *Math. Gaz.* **82** (November 1998) pp. 484-485.

Yours sincerely,

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DEAR EDITOR,

Tony Gardiner [1] hit the nail firmly on the head: the emperor has long been walking around naked and nobody has had the courage to remark on the fact. How can we virtually eliminate proof from the material we teach and call what is left mathematics? Surely it is proof that sets mathematics aside from other enterprises.

I would agree with his analysis that elementary Euclidean geometry remains the most effective vehicle for teaching the ideas of proof. It is accessible to the pupils *who can understand proof*, and the discipline of setting the steps of the proof out logically can have the desirable by-product of teaching pupils how to write mathematics properly. It is also a topic where that universal good luck charm of the mathematics classroom, the calculator, can be rendered impotent.

However I question Tony Gardiner's analysis that a substantial fraction of each cohort is capable of understanding proof. Certainly my experience in teaching mathematics (GCSE, A level and Oxbridge entry) in a selective school leads me to believe that it is probably no more than the fraction that sat the old O level examination. Surely proof virtually vanished from the GCSE syllabus because there is only a small fraction of the cohort who can understand proof, and the examination was designed for a much larger

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