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ON FUNCTIONAL CENTRAL LIMIT THEOREMS FOR LINEAR RANDOM FIELDS WITH DEPENDENT INNOVATIONS

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Abstract

For a linear random field (linear p-parameter stochastic process) generated by a dependent random field with zero mean and finite qth moments ($q > 2p$), we give sufficient conditions that the linear random field converges weakly to a multiparameter standard Brownian motion if the corresponding dependent random field does so.

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1. Introduction and preliminary results

Define a linear random field by

$$u(t_1, \ldots, t_p) = \sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} a(i_1, \ldots, i_p)\xi(t_1 - i_1, \ldots, t_p - i_p),$$
$$(t_1, \ldots, t_p) \in \mathbb{Z}^p, \qquad (1.1)$$

where $\{\xi(t_1, \ldots, t_p)\}$ is a random field with $E\xi(t_1, \ldots, t_p) = 0$ and $E|\xi(t_1, \ldots, t_p)|^q < \infty$ for $q > 2p$, and $\sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} \sum_{k_1=i+1}^{\infty} \cdots \sum_{k_p=i_p+1}^{\infty} |a(k_1, \ldots, k_p)| < \infty$. Functional central limit theorems for mixing and martingale-difference fields were presented in [5, 7] and [10]; in [2, 4, 6], functional central limit theorems for associated random fields were also proved.

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Marinucci and Poghosyan [8] generalized a result for $p = 1$, known as the Beveridge–Nelson decomposition (see [9]), to the $p \geq 2$ case. A functional central limit theorem was then derived for a linear random field generated by the independent and identically distributed innovations $\{\xi(t_1, \ldots, t_p)\}$; this was done by exploiting the generalized decomposition result to decompose a partial sum of linear fields into a partial sum of independent components, together with a remainder which is shown to be uniformly of smaller order on $\mathbb{Z}_+^p$. By applying this technique, we shall derive sufficient conditions for $\sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} u(t_1 \ldots, t_p)$ to converge weakly to a multiparameter standard Brownian motion if $\sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} \xi(t_1 \ldots, t_p)$ does so. We will also consider functional central limit theorems for linear random fields those are generated by dependent random fields such as associated random fields and martingale-difference random fields.

We now introduce the decomposition of multivariate polynomials presented in [8] as the main tool in our subsequent arguments: consider the multivariate polynomial

$$A(x_1, \ldots, x_p) = \sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} a(i_1, \ldots, i_p) x_1^{i_1} \cdots x_p^{i_p}, \quad (x_1, \ldots, x_p) \in \mathbb{R}^p, \tag{1.2}$$

where it is assumed that $|x_i| \leq 1$ for $i = 1, \ldots, p$ and

$$\sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} \sum_{k_1=i+1}^{\infty} \cdots \sum_{k_p=i_p+1}^{\infty} |a(k_1, \ldots, k_p)| < \infty. \tag{1.3}$$

Assumption (1.3) implies that

$$A(1, \ldots, 1) = \sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} |a(i_1, \ldots, i_p)| < \infty.$$

In [8], a result known as the “Beveridge–Nelson decomposition” in the $p = 1$ case (see [9]) was generalized as follows.

Lemma 1.1. *Let Γ_p be the class of all 2^p subsets γ of $\{1, 2, \ldots, p\}$. Let $y_i = x_i$ if $j \in \gamma$ and $y_i = 1$ if $j \notin \gamma$. Then*

$$A(x_1, \ldots, x_p) = \sum_{\gamma \in \Gamma_p} \Big\{ \prod_{j \in \gamma} (x_j - 1) \Big\} A_\gamma(y_1, \ldots, y_p), \tag{1.4}$$

where it is assumed that the product over $j \in \phi$ is 1, and

$$A_\gamma(y_1, \ldots, y_p) = \sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} a_\gamma(i_1, \ldots, i_p) y_1^{i_1} \cdots y_p^{i_p}, \tag{1.5}$$

$$a_\gamma(i_1, \ldots, i_p) = \sum_{s_1=i_1+1}^{\infty} \cdots \sum_{s_p=i_p+1}^{\infty} a(s_1, \ldots, s_p), \tag{1.6}$$

with the sums being taken over indices s_j such that $j \in \gamma$, whereas $s_j = i_j$ if $j \notin \gamma$.

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Marcinucci and Poghosyan [8] also introduced the partial back shift operator which satisfies

$$B_i\xi(t_1, \ldots, t_i, \ldots, t_p) = \xi(t_1, \ldots, t_i - 1, \ldots, t_p), \quad i = 1, 2, \ldots, p.$$

This enables us to write (1.1) more compactly as

$$\begin{aligned} u(t_1, \ldots, t_p) &= \sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} a(i_1, \ldots, i_p) B_1^{i_1} \cdots B_p^{i_p} \xi(t_1, \ldots, t_p) \\ &= A(B_1, \ldots, B_p)\xi(t_1, \ldots, t_p), \end{aligned} \tag{1.7}$$

where

$$A(B_1, \ldots, B_p) = \sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} a(i_1, \ldots, i_p) B_1^{i_1} \cdots B_p^{i_p}.$$

The above ideas will be exploited in this paper to establish functional central limit theorems for linear random fields. With this goal in mind, we write

$$\xi_\gamma(t_1, \ldots, t_p) = A_\gamma(L_1, \ldots, L_p)\xi(t_1, \ldots, t_p), \tag{1.8}$$

where the operator L_i is defined as $L_i = B_i$ for $i \in \gamma$, and $L_i = 1$ otherwise; for instance, when $p = 2$,

$$\xi_1(t_1, t_2) = A_1(B_1, 1)\xi(t_1, t_2), \quad \xi_2(t_1, t_2) = A_2(1, B_2)\xi(t_1, t_2)$$

and

$$\xi_{12}(t_1, t_2) = A_{12}(B_1, B_2)\xi(t_1, t_2).$$

Before proving the theorems, let us introduce some notation. Let $\mathcal{A}$ be a family of parallelepipeds in $\mathbb{R}_+^p$ of the form $V = (a, b]$; that is, $V = (a_1, b_1] \times \cdots \times (a_p, b_p]$, where $a_i, b_i \in N \cup \{0\}$ with $0 \le a_i \le b_i < \infty$ for $i = 1, \ldots, p$. For $V \in \mathcal{A}$, we write $|V| = \prod_{i=1}^{p}(b_i - a_i)$ and

$$S(V) = \sum_{(t_1,\ldots,t_p)\in V} \xi(t_1, \ldots, t_p), \quad M(V) = \max\{|S(Q)| : Q = (a, q] \subset V\}. \tag{1.9}$$

Let C denote a positive constant which may vary from line to line, and let $[\,\cdot\,]$ denote the integer part of a real number.

2. Functional central limit theorems

Let $W(\cdot, \ldots, \cdot)$ denote multiparameter standard Brownian motion; that is, a zero-mean Gaussian process with covariance function satisfying

$$EW(t_1, \ldots, t_p)W(s_1, \ldots, s_p) = \prod_{j=1}^{p} \min(t_j, s_j). \tag{2.1}$$

Also, let D_p be the space of "cadlag" functions from $[0, 1]^p$ to $\mathbb{R}$. It is possible to endow D_p with a metric topology which makes it complete and separable; indeed, D_p is the multi-dimensional analogue of the Skorohod space $D[0, 1]$; see [11] or [1] for details.

We are now in a position to prove the main result of this section.

THEOREM 2.1. *Let $u(t_1, \ldots, t_p)$ satisfy model (1.1), where*

$$\sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} \sum_{k_1=i_1+1}^{\infty} \cdots \sum_{k_p=i_p+1}^{\infty} |a(k_1, \ldots, k_p)| < \infty \tag{2.2}$$

and $\{\xi(t_1, \ldots, t_p), (t_1, \ldots, t_p) \in \mathbb{Z}^p\}$ is any stationary random field such that $E\xi(t_1, \ldots, t_p) = 0$, $E|\xi(t_1, \ldots, t_p)|^q < \infty$ for $q > 2p$, and

$$0 < \sigma^2 = \sum_{(t_1,\ldots,t_p)\in\mathbb{Z}^p} \operatorname{Cov}(\xi(0, \ldots, 0), \xi(t_1, \ldots, t_p)) < \infty. \tag{2.3}$$

Assume that

$$E|\xi_\gamma(t_1, \ldots, t_p)|^q < \infty \quad \text{for } \gamma \in \Gamma_p \tag{2.4}$$

and

$$E(M(V))^q \le C|V|^{q/2} \text{ for some constant } C \text{ and all } V \in \mathcal{A}. \tag{2.5}$$

Then

$$\frac{1}{\sigma n^{p/2}} \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} \xi(t_1, \ldots, t_p) \quad \Rightarrow \quad W(r_1, \ldots, r_p)$$

implies

$$\frac{1}{\sigma n^{p/2}} \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} u(t_1, \ldots, t_p) \quad \Rightarrow \quad A(1, \ldots, 1)W(r_1, \ldots, r_p) \quad \text{as } n \to \infty,$$

where $\Rightarrow$ denotes weak convergence in D_p.

PROOF. We start with the case $p = 2$, for which we give full details; the extension to $p > 2$ will be discussed later.

If we apply Lemma 1.1 to the back shift polynomial $A(B_1, \ldots, B_p)$, we find that the following almost-sure equality holds:

$$\begin{aligned} u(t_1, t_2) = {} & A(1.1)\xi(t_1, t_2) + (B_1 - 1)A_1(B_1, 1)\xi(t_1, t_2) \\ & + (B_2 - 1)A_2(1, B_2)\xi(t_1, t_2) + (B_1 - 1)(B_2 - 1)A_{12}\xi(t_1, t_2). \end{aligned}$$

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This equality implies that

$$\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} u(t_1, t_2) = \sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} A(1, 1)\xi(t_1, t_2) - \sum_{t_2=1}^{[nr_2]} \xi_1([nr_1], t_2) + \sum_{t_2=1}^{[nr_2]} \xi_1(0, t_2)$$
$$- \sum_{t_1=1}^{[nr_1]} \xi_2(t_1, [nr_2]) + \sum_{t_1=1}^{[nr_1]} \xi_2(t_1, 0) - \xi_{12}(0, [nr_2]) + \xi_{12}(0, 0)$$
$$- \xi_{12}([nr_1], 0) + \xi_{12}([nr_1], [nr_2])$$
$$= \sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} A(1, 1)\xi(t_1, t_2) + R_n(t_1, t_2). \tag{2.6}$$

From Markov's inequality and assumption (2.5), we find that for $0 \le r_1, r_2 \le 1$ and $q > 2$,

$$P\left\{\max_{0\le r_1,r_2\le 1} n^{-1} \sum_{t_2=1}^{[nr_2]} \xi_1([nr_1], t_2) > \delta\right\} \le \frac{E \max_{0\le r_1,r_2\le 1}\left|\sum_{t_2=1}^{[nr_2]} \xi_1([nr_1], t_2)\right|^q}{n^q \delta^q}$$
$$\le Cn^{-q/2} = o(1) \tag{2.7}$$

as $n \to \infty$. We can apply exactly the same argument to establish also that

$$P\left\{\max_{0\le r_1,r_2\le 1} n^{-1} \sum_{t_1=1}^{[nr_1]} \xi_2(t_1, [nr_2]) > \delta\right\} = o(1) \quad \text{as } n \to \infty. \tag{2.8}$$

From assumption (2.4) it follows that for $0 \le r_1, r_2 \le 1$,

$$E|\xi_{12}([nr_1], [nr_2])|^q < \infty$$

and hence

$$P\left\{\max_{0\le r_1,r_2\le 1} n^{-1}\xi_{12}([nr_1], [nr_2]) > \delta\right\} = o(1) \quad \text{as } n \to \infty. \tag{2.9}$$

Thus,

$$n^{-1}\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} u(t_1, t_2) = n^{-1}\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} A(1, 1)\xi(t_1, t_2) + n^{-1}R_n(t_1, t_2)$$
$$\text{where} \quad \sup_{0\le r_1,r_2\le 1} |n^{-1}R_n(t_1, t_2)| = o_p(1),$$

which implies that

$$(\sigma n)^{-1}\sum_{t_1=1}^{[nr_1]}\sum_{t_2=1}^{[nr_2]} u(t_1, t_2) \quad \Rightarrow \quad A(1, 1)W(r_1, r_2) \quad \text{as } n \to \infty.$$

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When $p > 2$, the argument is analogous. In this case,

$$\sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} u(t_1, \ldots, t_p) \\ = A(1, \ldots, 1) \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} \xi(t_1, \ldots, t_p) + R_n(t_1, \ldots, t_p) \tag{2.10}$$

where

$$R_n(r_1, \ldots, r_p) \\ = \sum_{\gamma \in \Gamma_p, \gamma \neq \phi} \left\{ \prod_{j \in \gamma} (B_j - 1) \right\} \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} A_\gamma(L_1, \ldots, L_p) \xi(t_1, \ldots, t_p), \tag{2.11}$$

with L_i defined as in (2.2). Note that for $j \in \gamma$,

$$\begin{aligned} &\sum_{t_j=1}^{[nr_j]} (B_j - 1) A_\gamma(L_1, \ldots, L_p) \xi(t_1, \ldots, t_p) \\ &\quad = \sum_{t_j=1}^{[nr_j]} A_\gamma(L_1, \ldots, L_p) \xi(t_1, \ldots, t_j - 1, \ldots, t_p) \\ &\qquad - \sum_{t_j=1}^{[nr_j]} A_\gamma(L_1, \ldots, L_p) \xi(t_1, \ldots, t_p) + R_n(t_1, \ldots, t_p) \\ &\quad = A_\gamma(L_1, \ldots, L_p) \xi(t_1, \ldots, 0, \ldots, t_p) \\ &\qquad - A_\gamma(L_1, \ldots, L_p) \xi(t_1, \ldots, [nr_j], \ldots, t_p). \end{aligned} \tag{2.12}$$

Thus the right-hand side of (2.11) can be written more explicitly as

$$\begin{aligned} &\sum_{t_2=1}^{[nr_2]} \sum_{t_3=1}^{[nr_3]} \cdots \sum_{t_p=1}^{[nr_p]} A_1(B_1, \ldots, 1) \xi(0, \ldots, t_p) \\ &\quad - \sum_{t_2=1}^{[nr_2]} \sum_{t_3=1}^{[nr_3]} \cdots \sum_{t_p=1}^{[nr_p]} A_1(B_1, \ldots, 1) \xi(n_1, \ldots, t_p) \\ &\quad + \sum_{t_1=1}^{[nr_1]} \sum_{t_3=1}^{[nr_3]} \cdots \sum_{t_p=1}^{[nr_p]} A_2(1, B_2, \ldots, 1) \xi(0, \ldots, t_p) \end{aligned}$$

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$$
\begin{aligned}
&- \sum_{t_1=1}^{[nr_1]} \sum_{t_3=1}^{[nr_3]} \cdots \sum_{t_p=1}^{[nr_p]} A_2(1, B_2, \ldots, 1)\xi(n_1, \ldots, t_p) + \cdots \\
&+ A_{12\cdots p}(B_1, \ldots, B_p)\xi(0, \ldots, 0) - A_{12\cdots p}(B_1, \ldots, B_p)\xi(0, \ldots, n_p) \\
&- A_{12\cdots p}(B_1, \ldots, B_p)\xi(n_1, \ldots, 0) + \cdots \\
&+ A_{12\cdots p}(B_1, \ldots, B_p)\xi(n_1, \ldots, n_p)
\end{aligned} \tag{2.13}
$$

where, in view of (2.12), the sums corresponding to each $A_\gamma(\cdot, \ldots, \cdot)$ range over t_i such that $i \notin \gamma$. Now

$$
\frac{1}{\sigma n^{p/2}} A(1, \ldots, 1) \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} \xi(t_1, \ldots, t_p) \quad \Rightarrow \quad A(1, 1, \ldots, 1) W(r_1, \ldots, r_p)
$$

as in [1], so it is sufficient to prove that

$$
\sup_{0 \le r_1, \ldots, r_p \le 1} |n^{-p/2} R_n(r_1, \ldots, r_p)| = o_p(1). \tag{2.14}
$$

Let us consider, for instance, the first term on the right-hand side of (2.13) for $0 \le r_1, \ldots, r_p \le 1$ and $q > 2p$; then assumption (2.5) and the same argument as for $p = 2$ give

$$
\begin{aligned}
&P\left\{ \max_{0 \le r_1, \ldots, r_p \le 1} n^{-p/2} \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} \xi([nr_1], \ldots, t_p) > \delta \right\} \\
&\qquad \le C n^{-pq/2} n^{(p-1)q/2} = n^{-q/2} = o(1) \quad \text{as } n \to \infty.
\end{aligned} \tag{2.15}
$$

More generally, let $\sharp(\gamma)$ denote the cardinality of γ; every other term in (2.14) is $n^{-p/2}$ times a partial sum of $n^{p-\sharp(\gamma)}$ elements, and we can apply the same argument iteratively to complete the proof. □

Corollary 2.2. *Let $u(t_1, \ldots, t_p)$ satisfy model (1.1). Assume that $a(i_1, \ldots, i_p) > 0$, $\sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} \sum_{k_1=i_1+1}^{\infty} \cdots \sum_{k_p=i_p+1}^{\infty} a(i_1, \ldots, i_p) < \infty$, and $\{\xi(t_1, \ldots, t_p), (t_1, \ldots, t_p) \in \mathbb{Z}^p\}$ is an associated wide-sense stationary mean-zero random field such that:*

(a) *$M_q = \sup_{(t_1,\ldots,t_p)\in\mathbb{Z}^p} E\xi(t_1, \ldots, t_p)^q < \infty$ for some $q > 2p$;*
(b) *$u(m) = \sup_t \sum_{s:\|t-s\|\ge m} \mathrm{Cov}(\xi(t), \xi(s)) = o(m^{-\nu})$ for some $\nu > 0$, where $t = (t_1, \ldots, t_p)$, $s = (s_1, \ldots, s_p)$, and $\|\cdot\|$ is defined by $\|a\| = \max_{1\le i\le p} |a_i|$ for any $a = (a_1, \ldots, a_p) \in \mathbb{R}^p$;*
(c) *$E|\xi_\gamma(t_1, \ldots, t_p)|^q < \infty$ for $\gamma \in \Gamma_p$.*

Then

$$
\frac{1}{\sigma n^{p/2}} \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} u(t_1, \ldots, t_p) \quad \Rightarrow \quad A(1, \ldots, 1) W(r_1, \ldots, r_p), \tag{2.16}
$$

where $\sigma^2 = \sum_{(t_1,\ldots,t_p)\in\mathbb{Z}^p} \mathrm{Cov}(\xi(0, \ldots, 0), \xi(t_1, \ldots, t_p)) < \infty$.

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PROOF. From a theorem in [2] and assumptions (a) and (b), it follows that

$$\frac{1}{\sigma n^{p/2}} \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} \xi(t_1, \ldots, t_p) \quad \Rightarrow \quad W(r_1, \ldots, r_p), \tag{2.17}$$

which yields (2.16) by Theorem 2.1. □

LEMMA 2.3. *Let $u(t_1, \ldots, t_p)$ satisfy model (1.1). Assume that*

$$\sum_{i_1=0}^{\infty} \cdots \sum_{i_p=0}^{\infty} \sum_{k_1=i_1+1}^{\infty} \cdots \sum_{k_p=i_p+1}^{\infty} |a(i_1, \ldots, i_p)| < \infty$$

and $\{\xi(t_1, \ldots, t_p), (t_1, \ldots, t_p) \in \mathbb{Z}^p\}$ is a translation-invariant, ergodic, martingale-difference random field with $\sigma^2 = E\xi(t_1, \ldots, t_p)^2 < \infty$ and $E|\xi(t_1, \ldots, t_p)|^q < \infty$ for $q > 2p$. Then, for $\gamma \in \Gamma_p$, we have $E|\xi_\gamma(t_1, \ldots, t_p)|^q < \infty$.

PROOF. First, note that because $\mathbb{Z}^p$ is countable, there exists a one-to-one correspondence $\phi : \mathbb{Z} \to \mathbb{Z}^p$. Hence,

$$\begin{aligned}\xi_\gamma(0, \ldots, 0) &= \sum_{i_1=0}^{\infty} a_\gamma(i_1, \ldots, i_p)\xi(-i_1, \ldots, -i_p) \\ &= \sum_{i_1=0}^{\infty} a_\gamma(\phi(i))\xi(-\phi(i)),\end{aligned}$$

where $\xi(-\phi(i))$ is a sequence of translation-invariant, ergodic, martingale-difference variables. Now

$$E|\xi_\gamma(t_1, \ldots, t_p)|^q = E|\xi_\gamma(0, \ldots, 0)|^q$$

and therefore

$$\begin{aligned}E|\xi_\gamma(t_1, \ldots, t_p)|^q &= E\left|\sum_{i_1=0}^{\infty} a_\gamma(\phi(i))\xi(-\phi(i))\right|^q \\ &\le CE\left|\sum_{i_1=0}^{\infty}[a_\gamma(\phi(i))\xi(-\phi(i))]^2\right|^{q/2} \\ &\le C\left\{\sum_{i_1=0}^{\infty} a_\gamma^2(\phi(i))[E|\xi(-\phi(i))|^q]^{2/q}\right\}^{q/2} < \infty,\end{aligned}$$

where the first bound follows from Burkholder's inequality [3] and the second from Minkowski's inequality. □

COROLLARY 2.4. *Under the conditions of Lemma 2.3, (2.16) holds.*

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PROOF. By [10, Theorem 3],

$$\frac{1}{\sigma n^{p/2}} \sum_{t_1=1}^{[nr_1]} \cdots \sum_{t_p=1}^{[nr_p]} \xi(t_1, \ldots, t_p) \quad \Rightarrow \quad W(r_1, \ldots, r_p).$$

Then (2.5) follows from the definition of martingale difference (see [10]). Hence, by Theorem 2.1, we obtain the desired result. □

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