Journal of Glaciology, Vol. 22, No. 86, 1979

FLEXURAL WAVE STUDIES ON THE BASIS OF SINGLE-SENSOR RECORDINGS

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Abstract. The method described here provides a means of measuring flexural-wave dispersion and attenuation appropriate to a selected area, on the basis of single-sensor recordings made remote from the area of interest. The measured dispersion may then be inverted in the conventional manner to obtain thickness and elastic properties of the ice within the selected area. The procedure eliminates the need for equipment within the study area and minimizes the need for personnel within the area. As only a single sensor is necessary, the requirements for recording equipment and sensor cabling are also minimized. The method is suited to any study of flexural-wave dispersion from artificial sources and is ideally suited to the study of areas where personnel and equipment safety are a major concern.

Résumé. *Enregistrement de mesures d'ondulations de flexions par télédetection à un seul capteur.* La méthode décrite permet de mesurer la dispersion et l'atténuation des ondulations de flexion propre à un secteur choisi, à l'aide des enregistrements issus d'un capteur unique et réalisé à distance. La dispersion mesurée peut être traitée de manière conventionnelle pour en déduire l'épaisseur et les propriétés élastiques de la glace dans le secteur choisi. Cette méthode élimine la nécessité d'équiper la zone d'étude et réduit les besoins en personnels sur le terrain. Comme il n'est besoin que d'un seul capteur, le budget en matériel d'enregistrement et en câblage est également réduit. La méthode est adaptée à toute étude de dispersion d'ondulations de flexion depuis une source artificielle, et est parfaite pour l'étude de régions où la sécurité des personnels et du matériel constitue un souci majeur.

Zusammenfassung. *Messung von Biegewellen durch Aufzeichnungen mit einem entfernten Einzelsensor.* Die hier beschriebene Methode bietet ein Mittel zur Messung der Dispersion und Abschwächung von Biegewellen in einem ausgewählten Gebiet; sie beruht auf Aufzeichnungen mit einem Einzelsensor aus grösserer Entfernung vom Untersuchungsgebiet. Die gemessene Dispersion kann dann auf dem üblichen Wege zur Bestimmung der Dicke und der elastischen Eigenschaften des Eises im Untersuchungsgebiet verwendet werden. Dieses Vorgehen erspart den Einsatz von Geräten und bringt den Bedarf an Bodenpersonal auf ein Minimum. Da nur ein Einzelsensor benötigt wird, sind auch die Anforderungen an Registriergerät und Verkabelung äusserst gering. Die Methode eignet sich für jede Untersuchung der Dispersion von Biegewellen aus künstlichen Quellen, besonders in Gebieten, wo die Sicherheit von Personal und Gerät gefährdet ist.

Introduction

The dispersion associated with flexural-wave propagation in floating ice sheets is related to the elastic properties and thickness of the ice, and measured dispersion may be inverted to yield the elastic properties and ice thickness. The use of dispersion measurements in this manner has been summarized by Anderson (1963) and the measurements have received fairly wide application (Clements and others, 1958; Crary, 1954; Oliver and others, 1954; Press and Ewing, 1951; Richter, unpublished).

The conventional practice used for the determination of flexural-wave dispersion is to establish two recording sites and determine the frequency-dependent phase shifts occurring as a result of propagation between the sites. The end result is the determination of average elastic properties and ice thickness along the path separating the recording sites. While yielding satisfactory results, this procedure requires the transport of equipment over the ice sheet and the presence of both personnel and equipment in the area of interest. In addition, the study of azimuth dependence in the area of interest requires the movement of at least one recording site or the presence of more than two sites.

A method proposed by Alexander and Taylor (1969) and employed by Taylor (1972) for the study of Rayleigh-wave propagation, provides an alternative to the conventional method of determining flexural-wave dispersion. A version of this procedure modified for use in ice studies is presented here.

https://doi.org/10.3189/S0022143000014155 Published online by Cambridge University Press

THEORY

The frequency-domain representation of a propagating flexural wave may be written as,

$$R(\omega) = I(\omega)\, S(\omega)\, P(\omega), \tag{1}$$

where the transfer function of the recording system

$$I(\omega) = |I(\omega)| \exp\,[\,\mathrm{j}\phi(\omega)],$$

the Fourier transform of the source excitation function

$$S(\omega) = |S(\omega)| \exp\,[\,\mathrm{j}\theta(\omega)],$$

the transfer function of the ice sheet

$$P(\omega) = \frac{|P_1(\omega)| \exp\,[\,\mathrm{j}kx] \exp\,[-\alpha x]}{\sqrt{x}},$$

k being the wave number, α the material attenuation of the ice, and x the distance from the source. Thus, for the geometry shown in Figure 1, the ratio of $R_2(\omega)$ to $R_1(\omega)$, assuming identical sources, may be written as,

$$\frac{R_2(\omega)}{R_1(\omega)} = |A(\omega)| \exp\,[\,\mathrm{j}\psi(\omega)] = \frac{\sqrt{x_1}}{\sqrt{x_2}} \exp\,(+\mathrm{j}k\Delta x - \alpha\Delta x). \tag{2}$$

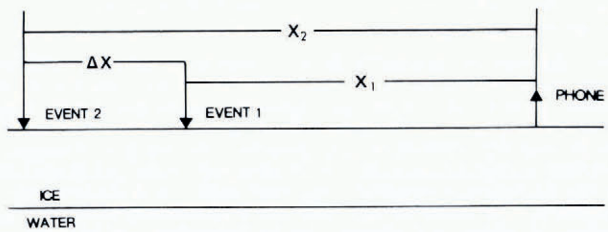

Fig. 1. The source–sensor geometry required for single-sensor measurements.

Equating the phase spectra and solving for the phase velocity yields,

$$C(\omega) = \frac{\omega\Delta x}{\psi(\omega) \pm 2N\pi}, \tag{3}$$

where $C(\omega)$ is the phase velocity for propagation across path Δx and $\psi(\omega)$ is calculated from the ratio $R_2(\omega)/R_1(\omega)$. The term $2N\pi$ represents the ambiguity present in all phase spectra. Equating the amplitude spectra and solving for the attenuation yields,

$$\alpha = -\frac{1}{\Delta x} \ln\left\{\frac{\sqrt{x_2}}{\sqrt{x_1}} |A(\omega)|\right\}, \tag{4}$$

and $|A(\omega)|$ is known on the basis of the ratio $R_2(\omega)/R_1(\omega)$.

Thus, the phase velocity and attenuation appropriate to the path Δx, may be determined from a single phone measurement made colinear with but remote from the path. The results are independent of propagation effects along the common path x_1, and the response of the recording system $I(\omega)$, and no assumptions of uniform ice thickness or elastic properties are necessary. The only necessary assumptions are that the sources are identical to within the desired accuracy of the measurements. It should also be noted that it is not necessary to know the sensor–source distances x_1 or x_2; only the source separation Δx is required.

The assumption of identical source excitation functions is reasonable for any underwater explosive source. However, even this assumption would not be necessary if another sensor were available at an arbitrary distance on the opposite side of the area of interest. The response of this sensor would again cancel and there would be no need to match the characteristics of the sensors. The use of this second sensor would also eliminate any need for origin time of the source.

Results and conclusions

The single-phone measurements described here were taken on Pike Lake, Washington County, Wisconsin, U.S.A., and on Green Bay, 10 km east of Marinette, Wisconsin, U.S.A. The sources in both studies were blasting caps detonated beneath the ice. A typical recording obtained at each of the sites is shown in Figure 2. The flexural wave is easily identified by its amplitude and inverse dispersion. There is little indication of any contamination by other modes and both recordings are fairly broad-band. The lack of contamination and broad-band nature of the flexural waves generated by sources located under the ice is expected on the basis of the results of Ewing and others (1957, p. 306).

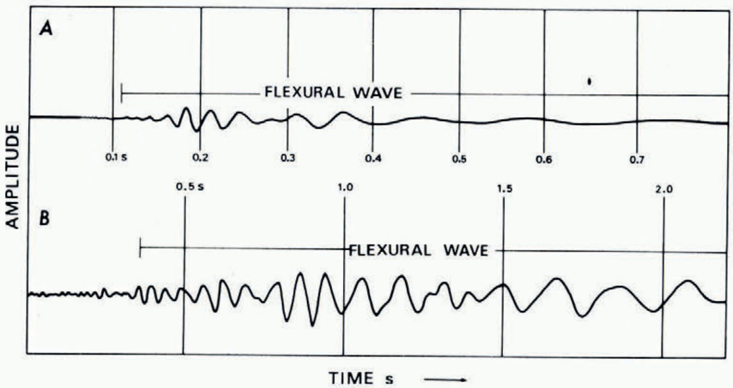

Fig. 2. Typical recordings resulting from the underwater detonation of blasting caps at A, the Pike Lake site and B, the Green Bay site.

The measured phase velocities and attenuations for each site, calculated on the basis of the procedure described here, are shown in Figures 3 and 4 respectively. To demonstrate the validity of the single-phone phase velocities the data of Figure 3 were used to derive the thickness and Young's modulus appropriate to the ice at each site. This was accomplished by assuming a Young's modulus of 5×10^{10} dyn cm^{-2} (5×10^{9} N m^{-2}) and then calculating theoretical phase velocities in the pass band from 1 Hz to 50 Hz for ice thicknesses from 0.1 m to 1 m in 1 cm increments. The thickness of the model for which calculated phase velocities best represented the measured phase velocities was then assumed to be the ice thickness. Values of Young's modulus for this model were then varied from the original value to improve the correlation between measured and calculated velocities. The final results of this procedure are summarized in Table I.

The results shown in Figures 3 and 4 and Table I establish the validity of the single-sensor method for flexural-wave dispersion measurements. The procedure as used required at most one individual in the area of interest and, thus, minimizes the problems associated with work on ice sheets. Alternately, no personnel would be required in the area of interest if the sources

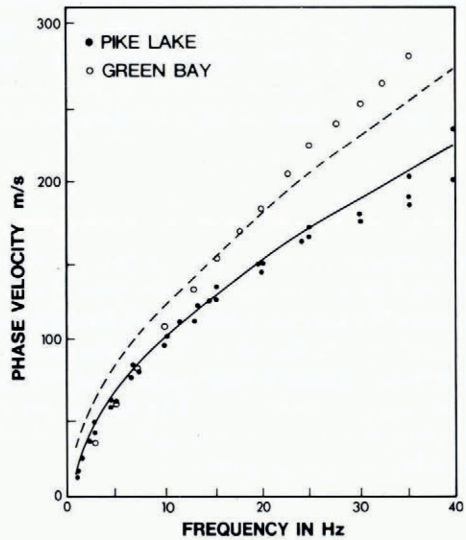

Fig. 3. Flexural-wave dispersion as determined from single-sensor recordings.

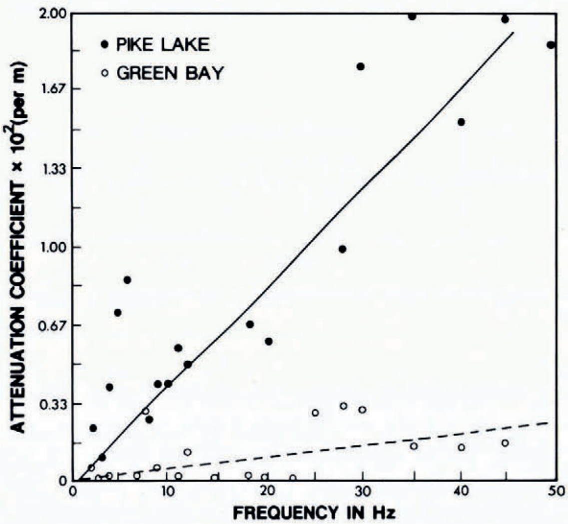

Fig. 4. Flexural-wave attenuation as determined from single-sensor recordings.

TABLE I. ICE THICKNESS, YOUNG'S MODULUS, AND ATTENUATION VALUES

Location	*Ice thickness measured in field* cm	*Ice thickness estimated from phase velocity* cm	*Young's modulus from phase velocity* dyn cm^{-2}	*Attenuation* cm^{-1} Hz^{-1}
Pike Lake	35–48	45	5.5×10^{10}	4×10^{-6}
Green Bay	45–55	55	7.8×10^{10}	0.24×10^{-6}

https://doi.org/10.3189/S0022143000014155 Published online by Cambridge University Press

were projected into the area from some outside point. The procedure also minimizes the required instrumentation, with consequent economic and logistical benefits, and eliminates the need for matched sensors.

ACKNOWLEDGEMENT

The effort of Mike Richter, Department of Geological Sciences, University of Wisconsin–Milwaukee, in conducting the field measurements was greatly appreciated. The research which employed the single-sensor dispersion measurements was supported by National Science Foundation Grant No. KG-41011 for the study of elastic properties of fresh-water ice.

MS. received 7 February 1978 and in revised form 11 May 1978

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