Environment and Development Economics (2023), **28**, 353–367
doi:10.1017/S1355770X22000328

RESEARCH ARTICLE

Does idiosyncratic risk matter for climate policy?

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(Submitted 15 March 2021; revised 21 February 2022, 4 July 2022; accepted 25 August 2022; first published online 2 November 2022)

Abstract

This paper studies the implications of distortions in intertemporal margins for the conduct of climate policy. We do so by introducing a framework that combines a standard two-period overlapping generations (OLG) model with a tractable model of household heterogeneity, in which over-accumulation of capital arises from uninsurable idiosyncratic labor income risk. We illustrate that market-based climate policies must be adjusted when the government cannot provide full insurance to households by taxing only capital and is constrained to transfer resources across generations for risk-sharing. In a numerical exercise, we find that idiosyncratic risk leads to an optimal capital income tax rate of 35 per cent and a carbon price 7.5 per cent lower than its first best.

Keywords: externalities; environmental policies; fiscal policy; optimal taxation

JEL classification: E62; H21; H23; Q58

1. Introduction

Climate change is a clear and current threat to our societies, especially in developing countries, due to their vulnerability and low adaptive capacity. Yet, only 28 nations have implemented carbon taxes. For example, according to the World Bank database of Carbon Pricing, in 2022 these policies would cover 3 GtCO2e, approximately 5 per cent of global greenhouse gas (GHG) emissions.[1] If we want to achieve sustainable development and encourage a global and efficient energy transition, we need to price carbon, but we should do it right.

A central idea in the macro-climate literature for the setting of carbon prices is that we should discount the future marginal social costs of current emissions using the market interest rate because it represents the real opportunity cost of capital (Nordhaus, 2008). Nonetheless, what if the aggregate capital is not at its first-best level? How should climate policy be implemented? To answer these questions, we layout an overlapping generations (OLG) model with idiosyncratic labor income risk and a climate externality, from using

[1]The Wold Bank also reports that in 2022 alternative initiatives such as cap-and-trade schemes have been implement in 9 nations and would cover 7GtCO2, representing 11 per cent of global GHG emissions.

fossil energy sources, to examine the implications of capital over-accumulation, due to precautionary savings, on climate policy, i.e., carbon prices.[2]

The objective of this paper is thus to study the implications of distortions in intertemporal margins for the conduct of climate policy. We do so by introducing a framework that combines a standard two-period OLG model with a tractable model of household heterogeneity, in which over-accumulation of capital arises from idiosyncratic risk. Our objective is not to claim that the only motive for capital taxes is because people engage in precautionary savings, and the government decides to tax capital at a positive rate to move the economy closer to the first best. Rather, we want to put forward some analytical macroeconomic tools to think about capital taxation as a form of intertemporal distortions and examine the implications of this idea for the implementation of carbon prices.

Specifically, this paper considers a global economy with two different externalities. First, households face uninsurable idiosyncratic labor income risk when they get older.[3] The government would like to provide social insurance to people facing labor income risk by means of individual transfers but it is constrained to do so because markets are incomplete.[4] Notwithstanding, given that people respond to such a risk by increasing their precautionary savings when they are young, there is over-accumulation of capital. This creates an incentive for the government to implement capital income taxes to avoid the aggregate capital becoming too high. The government then uses such fiscal revenues for partial risk-sharing. The intuition for this mechanism is as follows. When people face uninsurable labor productivity shocks, the presence of a precautionary motive for saving then creates a pecuniary externality, i.e., households do not internalize the effect of a higher saving rate on current and future wages and interest rates. The government thus would like to tax capital in order to move the economy closer to its efficient level (Aiyagari, 1995; Krueger *et al.*, 2021).

Second, the production of the final good generates carbon dioxide (CO_2) emissions by using fossil fuel energy as an input. Emissions accumulate in the atmosphere, increase the pollutant stock over time, and raise the level of atmospheric concentrations of CO_2, causing global warming. It turns out that higher temperature levels decrease future output through a multiplicative damage function. When the representative firm maximizes its profits, it does not internalize the future marginal social costs associated with the use of energy, creating a negative externality. To correct this externality, the government can implement a tax on CO_2 emissions, the so-called carbon price. To set the carbon price, a government would calculate the net present value of future marginal damages from climate change using the market interest rate.[5]

[2] Despite physical capital being instrumental for economic growth and development, too much capital in an economy would generate allocations that are not dynamically efficient. For an in-depth discussion about dynamic efficiency see, for example, de la Croix and Michel (2002). This paper shows that a capital income tax can help the government to restore efficiency. Indeed, recent empirical evidence supports the idea that capital taxation is not growth-deterring and points out heterogeneous effects across countries depending on their stage of development (ten Kate and Milionis, 2019).

[3] In order to make the model tractable, as in Krueger *et al.* (2021) and Harenberg and Ludwig (2015), we assume that people supply labor inelastically in both periods.

[4] For simplicity, we also assume that the government only has access to capital income taxes and it does not have additional public spending requirements. Note that to focus only on taxes that distort intertemporal margins, we ruled out labor income taxes and lump-sum transfers.

[5] Notice that we follow the positive (or descriptive) approach as in Nordhaus (2008, 2017) for setting the appropriate discount rate. For a normative (or prescriptive) approach see, for instance, Stern (2007).

How are then these two externalities related? If the aggregate level of capital is too high relative to the first best, due to the precautionary saving motive, it would imply a lower market interest rate and higher carbon prices, since the future becomes more important. From the perspective of a social planner, these allocations are inefficient. Given the set of instruments available to the government, hence, it can use a capital income tax to avoid the over-accumulation of capital. However, since that policy cannot replicated completely the first best because of incomplete markets, the economy ends up with a lower aggregate level of capital and a higher market interest rate. It follows that if the government uses the market interest rate to evaluate the marginal social costs of current emissions, it would yield a lower carbon price relative to the first best.

To derive a simple formula for the carbon price, as in recent macro-climate literature, we do rely on some crucial assumptions in spirit of Iverson and Karp (2021), Gerlagh and Liski (2018b), and Golosov *et al.* (2014): (i) a logarithmic utility function; (ii) multiplicative climate damages which are proportional to output; (iii) carbon stocks being linear in emissions, and (iv) a constant saving rate.[6] In principle, a constant saving rate implicitly requires additional assumptions: (i) full capital depreciation, (ii) exogenous labor supply, and (iii) income for the young is a constant fraction of total output. Finally, to introduce idiosyncratic labor income risk in a tractable manner, we follow closely Krueger *et al.* (2021), Harenberg and Ludwig (2015) and Hiraguchi and Shibata (2015).

Our key findings are then as follows. Firstly, we show that in the first best, when the government can provide full insurance to the households, the saving rate is constant, there is not capital over-accumulation, capital income taxes are thus zero, and the optimal carbon price equals the Pigouvian tax (the so-called social costs of carbon). In particular, we show that the carbon price is then proportional to output, it grows at the same rate of the economy, and only depends on the intergenerational discount factor and a reduced-form multi-box representation of the carbon cycle and temperature adjustments, quite similar to the ones derived in Iverson and Karp (2021), Gerlagh and Liski (2018a, 2018b), and Golosov *et al.* (2014).

Secondly, we show that in the second best, the saving rate is also constant but increases with the level of idiosyncratic labor income risk, it depends positively on the intergenerational and private discount factor, and, as expected, decreases with capital income taxes. We then show that the optimal carbon price is again proportional to output, but it now depends on both the intergenerational and private discount factor, productivity level for the old, the capital share and the parameters associated to the climate module. Finally, exploiting these analytical results, in a numerical exercise, we find that uninsurable labor productivity shocks lead to a capital income tax rate of 35 per cent and a carbon price of $24.33e/tCO_2$, approximately 7.5 per cent lower than its first best. The intuition is straightforward as pointed out above. Since the capital tax distorts the saving decisions for the households, it alters the aggregate level of capital and the implied market interest rate, and the carbon price should be adjusted accordingly.

This paper contributes to several strands of the literature. First, we contribute to the emerging literature on simple formulae for the carbon price. For instance, Golosov *et al.* (2014) derive a simple formula for the social cost of carbon using a standard infinitely-lived representative agent model depending on a few parameters. Gerlagh and Liski (2018b) also derive a simple formula bearing in mind time-inconsistency issues in social

[6]See, van den Bijgaart *et al.* (2016), Rezai and van der Ploeg (2015), and Barrage (2014), for details of the robustness of simple formulae for carbon prices.

https://doi.org/10.1017/S1355770X22000328 Published online by Cambridge University Press

preferences. Gerlagh *et al.* (2017) present an analytical formula for understanding the implications of demographic change on climate policy. We add to those an analytical simple formula in a model that features idiosyncratic labor income risk and capital income taxation.

Second, our analysis is also related to the extensive literature about pricing externalities and how those prices should be adjusted in the presence of other distortions in the economy. For example, (Barrage, 2020) studies the role of exogenous income taxation for climate policy using an infinitely-lived representative agent model. Jaimes (2021) examines the joint design of climate and fiscal policy in an OLG economy. Likewise, Barrage (2018) and Belfiori (2017) consider the implications of differences between private and social discounting for the setting of carbon prices. We contribute to the analysis by looking specifically at the effects of capital taxation due to precautionary saving motives and idiosyncratic risk on climate policy.

Third, notice that other studies also point out to adjustment in policies when, for instance, distributional concerns are taken into consideration (Chiroleu-Assouline and Fodha, 2011, 2014), firms face financial frictions or productivity shocks (van den Bijgaart and Smulders, 2017; Hoffmann *et al.*, 2017), there is asymmetric information (Tideman and Plassmann, 2010; Kaplow, 2012; Jacobs and de Mooij, 2015), or when polluting sources location matters for policy (Marrouch and Sinclair-Desgagné, 2012). We add to this literature by considering an analytical macro-climate framework with idiosyncratic labor income risk, precautionary savings and capital income taxes, along the lines of Krueger *et al.* (2021), Gottardi *et al.* (2015), Hiraguchi and Shibata (2015), and Aiyagari (1995).

The remainder of this paper is structured as follows. Section 2 describes the basic model and deals with optimal policies under idiosyncratic risk and incomplete markets. Section 3 presents a basic quantitative exercise. Section 4 concludes.

2. The model

We consider climate policy in an otherwise standard two-period OLG model with exogenous labor supply extended to include both idiosyncratic labor income risk, along the lines of Krueger *et al.* (2021) and Harenberg and Ludwig (2015), and multiplicative damages to future output coming from climate change in spirit of Golosov *et al.* (2014) and Gerlagh and Liski (2018b). Households only live for two periods, young and old age, and make decisions for consumption in each period and individual savings. They face uninsurable idiosyncratic risk for their productivity when they get older and engage in precautionary savings against such a risk. A representative firm produces the final good using capital, labor and energy. It generates CO_2 emissions as a by-product of output. Emissions accumulate into the atmosphere and change the climate. The government thus uses capital income taxes to reduce the inefficient level of aggregate capital and implements a carbon tax for internalizing the marginal social costs of current emissions on future output.

2.1 Households

We assume a stationary population.[7] Labor supply is inelastic and only depends upon productivity levels for young, $1 - \phi_2$, and older people, ϕ_2, respectively. Along the lines

[7] As in standard OLG models, we ruled out population growth and within-generation heterogeneity.

https://doi.org/10.1017/S1355770X22000328 Published online by Cambridge University Press

of Krueger *et al.* (2021) and Hiraguchi and Shibata (2015), we allow for uninsurable idiosyncratic productivity shocks, χ_{t+1}, at old-age, which are independently and identically distributed across agents. Let $\Psi(\chi_{t+1})$ be the cumulative distribution function and $\psi(\chi_{t+1})$ the probability density function. Assuming that $\int \chi d\Psi = 1$, it turns out that the effective labor supply is equal to 1,

$$L_t = 1 - \phi_2 + \phi_2 \int \chi \, \mathrm{d}\Psi = 1 \tag{1}$$

In order to obtain a closed-form solution for both the savings rate and the optimal carbon prices, we follow tradition in recent analytical macro-climate models, e.g, Golosov *et al.* (2014), van den Bijgaart *et al.* (2016), Gerlagh and Liski (2018a, 2018b), Iverson and Karp (2021), and let preferences be represented by a logarithmic function,[8]

$$W_t \equiv \log C_{1,t} + \beta \int \log C_{2,t+1}(\chi_{t+1}) \, \mathrm{d}\Psi(\chi) \tag{2}$$

This utility function, (2), implies that the households receive utility only from consumption over their life-cycle, since we assume that labor supply is inelastic. Importantly, the future is discounted through the parameter, β, and consumption when they are old is uncertain due to the presence of idiosyncratic labor income risk in the second period of their lives. The sequence of budget constraints, when they are young and old, respectively, are as follows,

$$C_{1,t} + K_{t+1} = (1 - \phi_2)(w_t + T^E_{1,t}) \tag{3}$$

$$C_{2,t+1} = \phi_2 \chi_{t+1}(w_{t+1} + T^E_{2,t+1}) + (1 - \tau^K_{t+1}) r_{t+1} K_{t+1} + T^K_{2,t+1} \tag{4}$$

where $\{C_{1,t}, C_{2,t+1}\}$ are consumption levels for young and old people, $r_{t+1}K_{t+1}$ represents capital income, non-capital income is the sum of wages, $\{w_t, w_{t+1}\}$, and lump-sum transfers to households coming from climate policy revenues, $\{T^E_{1,t}, T^E_{2,t+1}\}$, respectively. Likewise, $T^K_{2,t+1}$ denotes lump-sum transfers to the old households coming from taxes on capital income, τ^K_{t+1}, as in Krueger *et al.* (2021). Since there is no labor choice, by solving this problem for capital holdings after replacing the budget constraints into the utility function, the first-order condition yields an usual Euler equation,

$$U_{C_{1,t}} = \beta(1 - \tau^K_{t+1}) r_{t+1} \mathbb{E}[U_{C_{2,t+1}}] \tag{5}$$

This expression describes the trade-off between current and future consumption. Using the assumption of logarithmic preferences over consumption, the Euler equation can be rewritten as follows,

$$\frac{1}{C_{1,t}} = \beta(1 - \tau^K_{t+1}) r_{t+1} \int \left(\frac{1}{C_{2,t+1}(\chi_{t+1})} \right) \mathrm{d}\Psi(\chi_{t+1}) \tag{6}$$

[8]It is also important to note that assuming a unitary elasticity of intertemporal substitution is not uncommon in two-period OLG models since each period in the model spans approximately 30 years.

2.2 Firms

The representative firm uses a Cobb-Douglas technology, $F(\cdot) = \Omega(Z_t)K_t^{\alpha}[H_t(E_t, L_t)]^{1-\alpha}$, and profit maximization yields standard first-order conditions for capital K_t, aggregate labor L_t, and energy E_t, where the function $H_t(\cdot)$ is a composite labor-energy input in the spirit of Iverson and Karp (2021) and Gerlagh and Liski (2018b). There is full capital depreciation.[9] As in Gerlagh and Liski (2018a, 2018b), production leads to CO_2 emissions (one-to-one with the use of energy) that increase the stock of pollution over time, $Z_t = \sum_{i=1}^{\infty} \theta_i E_{t-i}$, according to a reduced-form multi-box representation of the carbon cycle and temperature adjustments, θ_i. High levels of atmospheric concentrations of CO_2 change the climate and reduce future output through a damage function, $\Omega_t = \exp(-Z_t)$.[10]

The representative firm does not internalize these external marginal costs. Hence, the government would like to implement some sort of climate policy: for instance, a carbon price would aim to reduce the impacts of current emissions on carbon concentrations, changes in temperature, and future production losses due to global warming. The first-order conditions are thus given by,

$$r_t = F_{K_t} \tag{7}$$

$$w_t = F_{L_t} \tag{8}$$

$$\tau_t^E = F_{E_t} \tag{9}$$

where τ_t^E denotes the carbon price and $F_{i,t}$ for $i \in \{K, L, E\}$ represents the marginal productivity of capital, labor, and energy, respectively. At the optimal allocation, all the inputs will be paid the value of their marginal productivity.

2.3 Government

The government has two main goals in this economy. First, due to uninsurable idiosyncratic labor income risk in the second period, households have the incentive to increase their precautionary savings but fail to internalize the general equilibrium effects on wages and capital returns of such decisions. The government can then use capital taxes to move the economy closer to the first best by reducing the level of aggregate capital. As in Krueger *et al.* (2021), we assume that the government uses the revenues from capital taxation to finance transfers to the same generation to provide partial insurance against labor income risk.[11]

$$T_{2,t}^K = \tau_t^K r_t K_t \tag{10}$$

Second, CO_2 emissions generate a climate externality in the economy by reducing future output. As an available instrument, the government can levy an excise tax on energy use to internalize the associated future marginal economic costs into firm's decisions. Since the government is constrained in the way it distributes resources across generations, we also assume that all tax revenues coming from carbon taxation are

[9] This assumption is typical in two-period OLG models since each period is equivalent to 30 years.

[10] Multiplicative damages factors are standard in integrated assessment models of climate and the economy. The assumption of an exponential function follows from its simplicity, see, for instance, Golosov *et al.* (2014), Gerlagh and Liski (2018b) and Iverson and Karp (2021).

[11] For simplicity, distribution of resources across generations and government debt are ruled out.

https://doi.org/10.1017/S1355770X22000328 Published online by Cambridge University Press

rebated to households as lump-sum transfers period-by-period as in Gerlagh *et al.* (2017).[12] Here, since older people also supply labor, the revenues are allocated bearing in mind age-specific productivity levels, that is, transfers for the young and old people are constant fractions of total environmental revenues, formally,

$$\tau_t^E E_t = (1-\phi_2)T_{1,t}^E + \phi_2 T_{2,t}^E \tag{11}$$

2.4 Equilibrium

A competitive equilibrium for this economy can be written as follows,

Definition 1. *Given a set of policies* $\{\tau_t^K, \tau_t^E\}_{t=0}^{\infty}$, *initial capital holdings* K_0, *an initial stock of pollution* Z_0, *a competitive equilibrium in this economy consists of relative prices* $\{r_t, w_t\}_{t=0}^{\infty}$, *transfers* $\{T_{1,t}^E, T_{2,t}^E, T_{2,t}^K\}_{t=0}^{\infty}$, *allocations for the households* $\{C_{1,t}, C_{2,t+1}(\chi_{t+1}), K_{t+1}\}_{t=0}^{\infty}$ *and allocations for the firm* $\{K_t, E_t\}_{t=0}^{\infty}$ *such that,*

1. *The allocations for the households solve, (6), for each realization of* χ_{t+1},
2. *The allocations for the firm solve (7-9),*
3. *The budget constraints for the government,* $\tau_t^E E_t = (1-\phi_2)T_{1,t}^E + \phi_2 T_{2,t}^E$, *and* $T_{2,t}^K = \tau_t^K r_t K_t$ *are satisfied period-by-period,*
4. *And markets clear,*

$$C_{1,t} + \int C_{2,t}(\chi_{t+1})\,\mathrm{d}\Psi + K_{t+1} = \Omega(Z_t)K_t^{\alpha}[H_t(E_t, L_t)]^{1-\alpha} \tag{12}$$

$$L_t = 1 \tag{13}$$

2.5 The saving rate

We follow Krueger *et al.* (2021) closely to derive an implementability condition for relaxing the government problem and solving for allocations instead of tax rates using the well-known primal approach in the optimal taxation literature (Lucas and Stokey, 1983; Erosa and Gervais, 2002). Since labor supply is exogenous and preferences are logarithmic, the government can determine allocations by choosing only saving rates. Below, we show that the optimal saving rate can be decentralized as a competitive equilibrium using capital income taxes. Let s_t be the saving rate, which is a fraction of total income for the young, i.e., labor income and transfers from carbon taxation revenues,

$$s_t = \frac{K_{t+1}}{(1-\phi_2)(w_t + T_{1,t}^E)} \tag{14}$$

Using the sequence of budget constraints described above and the definition for the saving rate, we can rewrite the Euler equation as,

$$1 = \beta(1-\tau_{t+1}^K)\int \frac{1-s_t}{\frac{\phi_2\chi_{t+1}\hat{w}_{2,t+1}}{r_{t+1}(1-\phi_2)\hat{w}_{1,t}} + (1-\tau_{t+1}^K)s_t + \frac{T_{2,t+1}^K}{r_{t+1}(1-\phi_2)\hat{w}_{1,t}}}\,\mathrm{d}\Psi(\chi_{t+1}) \tag{15}$$

[12]Otherwise, the government could use these revenues to provide additional insurance to old people facing labor income shocks. We do not consider the implications of this channel on optimal policies, but it is an interesting area for future research.

https://doi.org/10.1017/S1355770X22000328 Published online by Cambridge University Press

where $\hat{w}_{i,t} = w_t + T^E_{i,t}$ for $i = \{1, 2\}$ is the non-capital income for households as in Iverson and Karp (2021). Given the assumption about technology and the energy–labor composite input, it can be shown that factor compensations are then a constant fraction of output, that is,

$$w_t L_t + \tau^E_t E_t = (1-\alpha) Y_t \tag{16}$$

$$r_t K_t = \alpha Y_t \tag{17}$$

Hence, by substituting prices and transfers in terms of aggregate variables into the Euler equation, assuming that the government distributes carbon tax revenues bearing in mind productivity levels, and given an initial capital stock, K_0, one can obtain an expression for the dynamics of capital as follows, which also hinges on the level of idiosyncratic labor income risk in old age, Φ,

$$1 = \alpha\beta(1-\tau^K_{t+1})\left[\frac{(1-\phi_2)(1-\alpha)Y_t - K_{t+1}}{K_{t+1}}\right]\underbrace{\int \frac{1}{\phi_2 \chi_{t+1}(1-\alpha)+\alpha}\,\mathrm{d}\Psi(\chi_{t+1})}_{\Phi=\Phi(\phi_2,\alpha;\Psi)} \tag{18}$$

Notice that since the term in the integral is convex in χ_{t+1}, Φ is increasing in the level of idiosyncratic risk. For vanishingly small spread, then it converges to $\frac{1}{\phi_2 \chi_{t+1}(1-\alpha)+\alpha}$. Finally, along the lines of Krueger *et al.* (2021), for a given tax policy $\tau^K_{t+1} \in (-\infty, 1]$, we can solve for the saving rate in competitive equilibrium to obtain,

Proposition 1. *The saving rate in competitive equilibrium is given by,*

$$s_t = \frac{1}{1 + \frac{1}{\alpha\beta(1-\tau^K_{t+1})\Phi(\phi_2,\alpha;\Psi)}} \tag{19}$$

Proof : In appendix A1. □

This constant saving rate is unique and independent of climate policies and capital stocks, but dependent of the capital income tax policy and the level of idiosyncratic risk. Specifically, a positive tax on capital reduces the incentives for saving but labor income risk increases it. As one may expect, a higher subjective discount factor, β, and a larger capital share, α, also increase the saving rate. This will be crucial for the setting of fiscal and climate policies as discussed below. Indeed, as shown in Krueger *et al.* (2021), and following the primal approach, the government can then solve directly for the saving rate and then decentralize it by choosing capital income taxes.

2.6 First-best allocations

To begin with, we solve for the first-best allocations under the case of idiosyncratic labor income risk and global warming. We then perform a comparative analysis about the implications of constraints on policy instruments. In the first best, the government would like to: (i) internalize future climate damages by means of a carbon tax, and (ii) provide full insurance to households facing productivity shocks, and in order to do so, it should be able to transfer resources across generations. By solving the social planner's problem, it follows that,

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Proposition 2. *The unconstrained government finds it optimal to implement a constant saving rate,*

$$s^* = \frac{\alpha\gamma}{(1-\phi_2)(1-\alpha)} \tag{20}$$

and a carbon price given by,

$$\tau_t^E = Y_t \sum_{i=1}^{\infty} \gamma^i \theta_i \tag{21}$$

where γ is the intergenerational discount factor used for the social planner in its welfare maximization problem.

Proof : In appendix A2. □

As expected, given the assumptions about technology and preferences, the saving rate, (20), is constant over time and increasing in the intergenerational discount factor, γ, the capital share, α, and the productivity for the old people, ϕ_2. As in recent literature related to simple formulae for the social costs of carbon, i.e., as in Golosov *et al.* (2014), the optimal carbon price, (21), is proportional to output (and grows at the same rate) and rises with higher future weights and damages.

In the first best, the carbon price thus equals both the market costs of carbon, i.e., the present value of future marginal damages discounted using the market interest rate, and the Pigouvian tax. Since the government would like to avoid intertemporal distortions, by providing full insurance to households through transfers, optimal policies yield a zero capital income tax rate and a carbon price that fully internalizes the future marginal damages of one additional unit of CO_2 emissions today. The key difference here is the use of a distinct discount factor which depends upon government preferences and not on the households' discount factor. If we assume that those discount factors coincide, we are back into the seminal simple formula for the carbon price derived by Golosov *et al.* (2014) using an infinitely-lived representative agent model and a particular climate module, that is,

Corollary 1. *If the government uses the same discount factor as the households, $\gamma = \beta$, then the optimal carbon price resembles the one in* Golosov *et al.* (2014),

$$\tau_t^E = Y_t \sum_{i=1}^{\infty} \beta^i \theta_i \tag{22}$$

2.7 Second-best allocations

When the government cannot either implement a carbon tax or transfer resources across generations for risk-sharing the first-best allocation could not be attained. In a second-best scenario, the problem then is to choose optimal policies given the set of policy instruments available to the government. In particular, we assume that the government can implement carbon taxes to internalize climate externalities and rebate such fiscal revenues to household via lump-sum transfers, and tax capital income in order to make lump-sum transfers to members of the same generation as a means of partially insuring people against idiosyncratic labor income risk.

https://doi.org/10.1017/S1355770X22000328 Published online by Cambridge University Press

In a second-best world, the government now chooses carbon taxes and capital income taxes such that the resulting allocation maximizes the welfare of current and future generations. From the previous section, we can solve a relaxed problem by choosing the saving rate directly instead of tax rates. By solving the government maximization problem the saving rate, the optimal capital tax, and the optimal carbon tax are given by,

Proposition 3. *The constrained government finds it optimal to implement a constant saving rate,*

$$s^{*} = \frac{\alpha(\beta+\gamma)}{1+\alpha\beta} \tag{23}$$

a constant capital income tax,

$$\tau^{K} = 1 - \frac{\beta+\gamma}{(1-\alpha\gamma)\beta\Phi(\phi_2,\alpha;\Psi)} \tag{24}$$

and a carbon price given by,

$$\tau_t^{E} = Y_t \sum_{i=1}^{\infty} \hat{\gamma}^{i}\theta_i, \text{ with } \hat{\gamma} \equiv \frac{(\beta+\gamma)(1-\phi_2)(1-\alpha)}{1+\alpha\beta} \tag{25}$$

Proof : In appendix A3. □

From proposition 3, it turns out that the optimal saving rate, (23), is constant over time, it depends on the capital share as before, and positively on both the subjective and intergenerational discount factors, but it is independent of idiosyncratic risk. The government can decentralize this saving rate using a constant capital income tax rate (24), which is increasing in the level of idiosyncratic risk, Φ and decreasing concerning the discount factors. The intuition for this expression is as follows. If households face a higher level of idiosyncratic labor income risk in old age, they would like to save more to insure against that risk. Hence, the aggregate capital would be higher than the efficient level. The government can correct this inefficiency by taxing capital income.

Furthermore, the carbon price in the second best, (25), takes a similar form as before, but now it uses a modified discount factor, $\hat{\gamma}$, for the Ramsey planner. It happens because capital income taxes distort an intertemporal margin, i.e., how much to consume in each period. Recall also that climate externalities also distort an intertemporal margin. Hence, the government finds it optimal to implement a lower carbon price to move the economy closer to its efficient level. Importantly, in the second best, the adjusted-discount factor, $\hat{\gamma}$, depends on the capital share, the productivity of the old, and both the subjective and intergenerational discount factors. As in the first best, note that we can get a simple formula for the carbon price because the saving rate is constant and we obtain a closed-form solution for the market interest rate.

3. A numerical example

Contributing to the recent literature that proposes simple formulae for the social costs of carbon following the work of Golosov *et al.* (2014), we exploit the closed-form solutions for the optimal carbon prices derived in the previous sections for an OLG economy with idiosyncratic labor income risk. To see the implications of different saving rules in the

Table 1. Median parameter values for the climate system

Carbon cycle							
a_0	a_1	a_2	a_3	η_0	η_1	η_2	η_3
0.220	0.279	0.278	0.222	0	0.0035	0.0507	0.2892
Temperature adjustment							
b_0	b_1	b_2	ε_0	ε_1	ε_2		
0.2218	0.3306	0.4476	0.9787	0.1980	0.0036		

setting of carbon taxes, we proceed to calculate the path of optimal carbon taxes in both a first-best scenario, when the government can provide full insurance to households, and a second-best world, where the presence of idiosyncratic risk and incomplete markets lead to over-accumulation of capital and the government implements a capital income tax to correct for pecuniary externalities.

3.1 Climate module

We follow Gerlagh and Liski (2018b) and van den Bijgaart *et al.* (2016) closely in modeling climate change and use a multi-box representation for the carbon cycle and temperature adjustments. As shown in Gerlagh *et al.* (2017), the response function to current emissions, θ_i, can be written as,

$$\theta_i = \sum_j \sum_k a_j b_k \pi \varepsilon_k \frac{(1-\eta_j)^i - (1-\varepsilon_k)^i}{\varepsilon_k - \eta_j}, \tag{26}$$

where η_j presents the rates for atmospheric depreciation factors, ε_k denotes the temperature adjustment speeds, a_j sets the shares of carbon emissions entering the reservoirs considered i.e., atmosphere and upper ocean, biomass and deep oceans, b_k designates how temperature responds to changes in carbon stocks, and $\pi = 0.0167$ is the climate sensitivity which comes from a linear approximation to the relationship between carbon concentrations, temperature changes, and damages. Below, we determine the paths of this response function for the remainder of this century, by using the procedure explained in Gerlagh *et al.* (2017), when the saving rate depends only on a few parameters, i.e., it is constant over time, and the parameter values for the climate system as reported in table 1. Calibrated parameters come from van den Bijgaart *et al.* (2016).[13]

3.2 Optimal carbon and capital taxes

This subsection describes the calibration procedure and presents the numerical exercise using the simple formulae derived above for the capital income tax and the carbon price in both a first-best and a second-best world. Nonetheless, notice that the purpose is not to provide a completed calibration and characterization of policies but to present a

[13]For more details see Joos *et al.* (2013) and Caldeira and Myhrvold (2013).

https://doi.org/10.1017/S1355770X22000328 Published online by Cambridge University Press

Table 2. Calibrated parameters for the benchmark model

Parameter	Value	Description
σ	1	Elasticity of intertemporal substitution
β	0.985^{30}	Sub. discount factor
γ	0.985^{30}	Intergenerational discount factor
α	0.2071	Capital share
ϕ_2	0.3117	Old-age productivity
$\sigma_{\ln \chi}$	0.250	Variance of idiosyncratic risk
$\mu_{\ln \chi}$	−0.125	Mean of idiosyncratic risk
L	1	Total labor supply
δ	100%	Depreciation rate

Note: The value for $\mu_{\ln \chi}$ is set such that $\mathbb{E}\chi = 1$.

quantitative illustration of the possible impacts of the given externalities on climate and fiscal policy in our simplified model.[14]

As a benchmark, we only look at the situation when the government uses the same discount factor as the households, $\gamma = \beta$, and we set $\beta = 0.985$, as in Eggertsson *et al.* (2019), which is a standard value for OLG models.[15] Following Krueger *et al.* (2021), we also assume a value for old age productivity of $\phi_2 = 0.3117$, a capital share, $\alpha = 0.2071$, a total labor supply normalized to 1, full depreciation of capital, a constant elasticity of intertemporal substitution equal to one, and the other calibrated parameters for the mean and variance of idiosyncratic labor income risk as described in the table 2.

Figure 1 shows the path for carbon prices in the first-best and second-best world for the remainder of the century. Two features stand out. On the one hand, carbon taxes grow at 2 per cent per year. That is, at the same growth rate we assume for the economy, because carbon prices are proportional to output. On the other hand, in contrast to Barrage (2018), even if the government resembles the same level of impatience than the households, the optimal carbon taxes in a second-best world are approximately 7.5 per cent lower than those derived in the first-best scenario. The intuition for this is straightforward. Since the government cannot provide full insurance to households and the capital income tax is not able to replicate completely the first best, the government finds it optimal to adjust downwards the carbon price accordingly.

A closer look at capital and carbon taxes across different scenarios yields interesting insights. Under the baseline case in which $\beta = \gamma$, table 3 reports saving rates for the competitive equilibrium without fiscal and climate policy, the first-best allocation and the constrained efficient allocation or second-best with both capital and carbon taxes.[16]

In the first case, as one may expect, the presence of idiosyncratic labor income risk creates a motive for precautionary savings and therefore a saving rate of 0.319, a rate

[14] A fully-fledged climate-economy model with heterogeneity within and across generations, an elastic labor supply, more than two periods, a more realistic depreciation rate for capital, and a general functional form for the utility function would be computationally intensive. We leave it for future research.

[15] Typically, in infinitely-lived representative agent models, this parameter is calibrated to match a particular interest rate.

[16] To calculate the constant $\Phi(\phi_2, \alpha; \Psi)$, we assume that χ is log-normally distributed and approximate it using a Gaussian quadrature with 20 nodes along the lines of Krueger *et al.* (2021).

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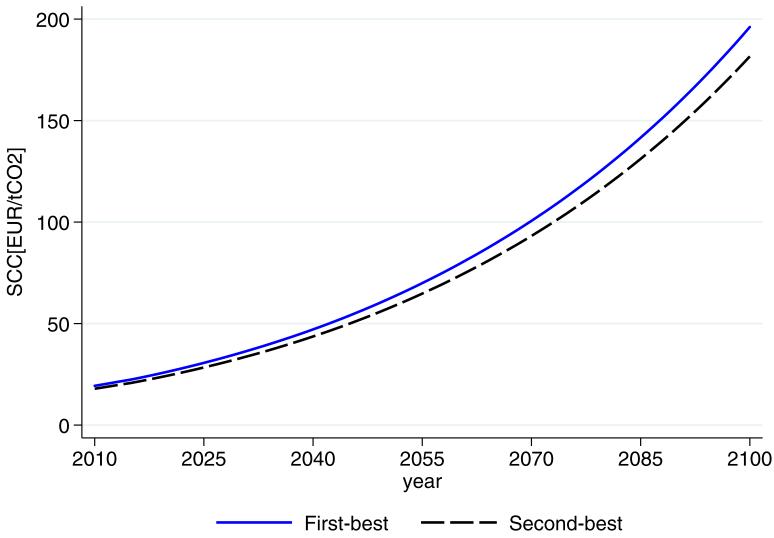

Figure 1. Optimal carbon taxes, 2010–2100.
Notes: Optimal carbon taxes in a first-best world vs. second-best policy. The value of standard (private) discounting is 0.985.

Table 3. Optimal climate and fiscal policy, 2020

	Saving rate	Investment share	Carbon tax	Capital tax
Competitive eq.	0.319	0.174	–	–
First-best	0.243	0.132	26.27	–
Second-best	0.234	0.127	24.33	0.35

Notes: Values for $\beta = 0.985^{30}$, $\gamma = 0.985^{30}$, $\alpha = 0.2082$, $\phi_2 = 0.3117$, and $\Phi(\phi_2, \alpha; \Psi(\chi)) = 3.55$ where $\chi \sim LN(-0.125, 0.25)$. Carbon taxes in €/$tCO_2$.

that is too high from the perspective of a social planner. In contrast, the optimal saving rate in a world where the government can provide full insurance to households is just 0.243, while at the same time being able to implement an optimal carbon tax (at its Pigouvian level, since there are not intertemporal distortions coming from taxing capital) of 26.27€/tCO_2 at 2020. However, when the government can provide only partial insurance, due to the presence of incomplete markets, it finds it optimal to tax capital at a rate of 35 per cent, yielding a saving rate of 0.234, and to set a lower carbon tax of 24.33€/tCO_2 relative to the first best.

Despite of being an analytical climate-economy model, this framework is able to provide reasonable values for macroeconomic policies. For example, according to empirical estimates for a set of countries, in her analysis, Barrage (2020) uses as baseline for the capital tax a value of 33.40 per cent. Similar values for baseline capital taxes, 41.1 per cent for US and 36.8 per cent for Europe, can be found in Trabandt and Uhlig (2012) and Trabandt and Uhlig (2011). Likewise, the level of the carbon prices (26.27€/tCO_2 is approximately equal to 110 USD/tC, bearing in mind the fact that 1 tCO_2 = 3.67 tC and 1€ is about 1.142 USD) reported in table 3 are in line, for instance, with some

https://doi.org/10.1017/S1355770X22000328 Published online by Cambridge University Press

of the estimations made by Barrage (2020) and Nordhaus (2017) using infinitely-lived representative agent models.

4. Discussion

In this paper we characterize the effects of precautionary saving, due to uninsurable idiosyncratic labor income risk and incomplete markets, for the conduct of climate policy and the setting of carbon prices. We do so in a standard two-period OLG model extended to include income risk and climate change. We find that the optimal capital tax cannot replicate completely the first best, because the government can only use transfers to partially insure people in old age. It turns out that the economy ends up with a lower aggregate level of capital and a higher market interest rate relative to the first best, and the government finds it optimal to adjust downwards the carbon price.

Supplementary material. The supplementary material for this article can be found at https://doi.org/10.1017/S1355770X22000328

Acknowledgments. I am grateful to the Editor as well as to the Associate Editor and two anonymous referees for careful readings and helpful recommendations. I also thank Reyer Gerlagh and Sjak Smulders for their continuous guidance and support. All errors are my own.

Conflict of interest. The author(s) declare none.

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Cite this article: Jaimes R (2023). Does idiosyncratic risk matter for climate policy?. *Environment and Development Economics* **28**, 353–367. https://doi.org/10.1017/S1355770X22000328