SESSION 1

Observations of Rotating Stars

Symposium participants enjoying a visit to Chichen Itza.

Nolan Walborn and Son with other participants touring at Chichen Itza.

Stellar Rotation
Proceedings IAU Symposium No. 215, © *2003 IAU*
André Maeder & Philippe Eenens, eds.

The Determination and Interpretation of the Rotation Parameter $v \sin i$

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Abstract. In this paper we review the development of the concept of the stellar rotation parameter commonly known as $v \sin i$. We emphasize that the interpretation of the parameter in terms of physical characteristics of the star always depends on comparison with a model that is intended to represent the physical properties of the star. To that end we will trace the development of such models along with the observational means of determining the parameter. Emphasis will be place on the traditional methods involving stellar spectroscopy, but some attention will be place on indirect methods involving direct measurement of the rotation period and recent interferometric determination of stellar oblatness. In addition we will comment on recent techniques involving the simultaneous measurement of many spectral lines and synthetic spectra to improve the accuracy of rotational half-widths.

The natural desire for simplicity of such models has often resulted in erroneous values for stellar parameters. This is particularly the case for the most rapidly rotating stars generally of early spectral type, but may also be present in some giants and supergiants where rapid rotation is difficult to detect. Finally, we will comment on the possibilities of improving the quality of both the measurement and interpretation of this important stellar rotation parameter.

1. Introduction

Ever since Galileo proclaimed, in the Sidereal Messenger (1610), that some celestial objects both rotated and revolved those who observe the heavens have enquired into the rotation of stars. Thomas Wright (1750) noted that stars were like the sun and vice verse and, by noting that the sun rotated on its axis, he may be credited with being the first to speculate that stars do so as well. But the actual observation of such phenomenon had to wait until stellar spectroscopy improved sufficiently to quantitatively observe the shape of stellar spectra lines. While Otto Struve is generally credited with being the "father of stellar rotation", seminal work by a number of astronomers early in the 20th century laid a broad foundation for the quantitative determination of stellar rotation. Although Shapley & Nicholsen (1919) were the first to formalize the concept of rotational broadening, Carroll (1928, 1933) and Carroll & Ingram (1933) developed the first workable model and applied it to stellar systems. Collins & Truax (1995) refer to this as the "Classical Model of a Rotating Star" (CMRS). However, the unambiguous demonstration of the existence of stellar rotation awaited

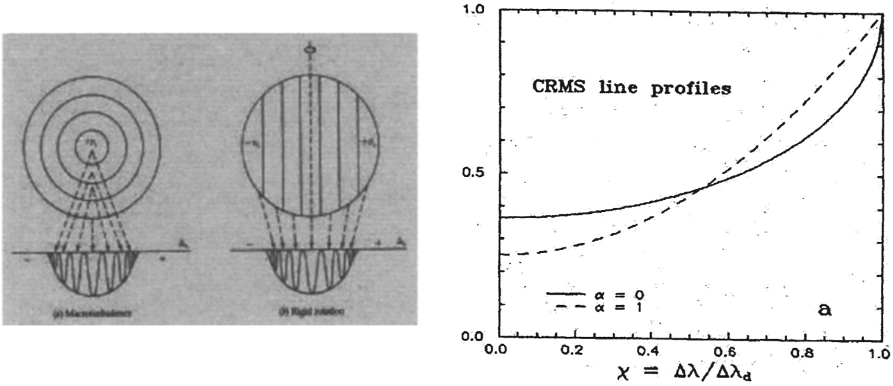

Figure 1. The figure on the left shows schematic models for turbulence on the left and rotation on the right. Both models yield the same profile. The figure on the right shows the effect of limb-darkening on the line profile of the CMRS. α is the normal limb-darkening coefficient where zero is an undarkened star and unity implies complete darkening at the limb. The increase in darkening weakens the contribution of the limb in favor of the center of the disk driving the profile from an ellipse to a parabola.

Rossiter's (1924) explanation of an anomaly in the radial velocity curve β Lyrae as resulting from the axial rotation of the eclipsed star during the eclipse. The systematic covering of the eclipsed disk yields shifts in the center of spectral lines during the eclipse resulting from apparent changes in the mean line-of-sight motion of the rotating eclipsed star. This effect, which is yet to be described for any star by an *ab initio* model, is still generally known as the Rossiter Effect and is considered to be first direct confirmation that stars do rotate. It was almost immediately confirmed by, Rossiter's doctoral advisor, McLaughlin (1924) in Algol. It seems likely that Rossiter's (1924) explanation was a major impetus for the subsequent development of rotating stellar models.

In spite of the elegant formulation by Caroll (1928, 1933), it was the simple graphical presentation of the model by Shajn & Struve (1929), shown at the right of Figure 1, that laid the foundation for much of the work done in the 20th century. This classical model led to the term $v \sin i$ for it had a very specific meaning within its context and as Collins & Truax (1995) point out should probably be denoted $v_e \sin i$. For a spherical uniformly rotating star, the parameter is simply the equatorial speed multiplied by the sine of the inclination of the rotation axis to the line of sight. This is still the interpretation of the parameter most widely held in the astronomical community. However, it is doubtful that such a simple single number can represent the projected velocity field of a stellar photosphere that is distorted by rapid rotation and may exhibit differential rotation as well. An additional problem in uniqueness is demonstrated in Figure 1 in that a simple model for radial macroturbulence yields the same profile as that of an undarkened star modeled by the CMRS. This non-uniqueness is present to some extent in more complicated models and should be considered when investigating stars where various types of large scale motions are likely to be present.

The early model of Shajn & Struve (1929), (i.e. the CMRS) considered the Doppler broadening of an arbitrarily sharp spectral line, having finite equivalent width, by the Doppler motion of a rotating spherical star with its axis inclined at an angle i to the line of sight. This has the decided advantage that there is no astrophysics associated with the line formation involved in the model. Although Shajn & Struve's (1929) graphical approach employed a uniformly bright star, Carroll (1933) extended it to included limb-darkening. Carroll (1933) did realize that limb-darkening would affect the resultant line shape by reducing the contribution from the stellar limb where the greatest projected rotational velocities are found. It remained for Unsöld (1955) to provide a clear extension of Carroll's (1933) work, but even here the limb-darkening used was a "Combined law of darkening" where the single value was meant to characterize the different darkening in the line and continuum. Gray (1976) took the limb-darkening of the line to be that of the adjacent continuum, and it is this approach that has been widely used with the early classical model. Figure 1 shows the classical results of this approach.

In an effort to make the classical model more appropriate for actual stars, Slettebak (1949) added an empirical aspect by using the line profile of a "sharp-lined" star instead of the arbitrarily sharp line of the simple model. When combined with the Shajn & Struve (1929) graphical approach some sort of 'convolution' of a Doppler profile with the intrinsic profile results. Slettebak's inclusion of gravity-darkening first described by von Zeipel (1924a,b) as well as limb-darkening resulted in a first-approximation improvement over the classical model that was widely used for the determination of $v \sin i$. However, the model being a theoretical-empirical hybrid model was difficult to relate to the stellar rotational velocity.

Thus it is difficult to understand the extent to which published values of $v \sin i$ actually represent aspects of the velocity field of the stellar photosphere as presented to the observer. Collins & Truax (1995) attempted to evaluate the extent to which stellar rotation can be correctly understood in terms of the classical model and its empirical modifications. In the next section, we shall review the important aspects of their results and extend them to dealing with stars, which may contain more complex photospheric velocity fields than those implied by the classical model. Following this, we shall consider some of the common systematic difficulties encountered in the spectroscopic determination of $v \sin i$. We shall then discuss some of the more sophisticated problems encountered in relating $v \sin i$ to the intrinsic properties of the stars. Finally we will conclude with some speculation as to how $v \sin i$ may be more accurately determined and used to understand the nature of the stars themselves.

2. Stellar Models for the Determination of $v \sin i$

In this day of relatively convenient numerical computation, it is easy to criticize the simple model of Shajn & Struve (1929) as being unreasonably simplistic and ignore the extremely clever graphical technique that allows such a model to yield an analytical result for a line profile of an intrinsically sharp line in terms of a single parameter $v_e \sin i$ (see Collins 1989 for derivation). While Carroll, Struve and others realized that atmospheric limb-darkening and von Zeipel's

(1924a,b) arguments concerning "gravity-darkening" would result in the simple interpretation yielding only a lower limit on the equatorial speed where $v_e \sin i$ itself only yields an upper limit, the model could be regarded as an excellent first step for investigating stellar rotation.

The seductive simplicity of the CMRS, along with later modifications, would appear to allow for excellent approximations for very slowly rotating stars. However, since a simple model of radial turbulence yields the same line shape as uniform rotation, (e.g. Collins 1989), we should perhaps have some concern about the uniqueness of the rotational interpretation of line profiles showing dish-shaped broadening. In addition, it is not possible to determine whether or not a star is an intrinsically slowly rotating star or rather a rapid-rotator seen at small inclination. In any event, it is now possible to move beyond the CMRS. But before doing so, let us review the potential nature and size of errors introduced by its use.

In an attempt to evaluate the range of applicability of the CMRS to real stars Collins & Truax (1995) compared the application of the CMRS to the line profiles for specific lines of *ab initio* models with known characteristics. They also investigated the effects of limb-darkening on the lines in question to ascertain the impact of using a "Combined law of darkening". Finally they considered the impact of the empirical effects of using a sharp-line profile along with the CMRS. As might be expected, when there are many approximations made in comparing the observations to a model, many sources of systematic error may be introduced.

Difficulties with application of the CMRS to real stars can be traced to two main areas. First, to what extent do the assumptions regarding the formation and broadening of the spectral lines to be analyzed actually represent the situation in actual stars ? Secondly, and more difficult to asses, is to what extent can the actual velocity field of the stellar photosphere be represented by a single parameter. The first area can again be divided into two general areas. First, to what extent is the physics of line formation properly represented by the model and secondly, to what extent is the radiative transfer yielding the observed line properly incorporated in the analysis. While these two affect one another to some extent, we shall consider them separately for purposes of discussion.

The CMRS largely avoids these problems by using an arbitrarily sharp line of finite equivalent width as the line to be rotationally broadened. Applying some type of limb-darkening probably improves the situation slightly as it does correctly reduce the contribution from the equatorial limb regions which show large Doppler-shifts. Using a flux profile of a sharp-lined star as the underlying line profile to be rotationally broadened would appear to be an improvement as it empirically contains all the physics of the line formation, but it introduces a host of additional systematic effects whose cumulative impact on the resulting line is difficult to evaluate. Even for slowly rotating stars, the underlying profile which is appropriate locally on the stellar photosphere is an intensity profile, not the flux profile of a sharp-lined star. Although integration of an intensity profile will yield a flux profile for a non-rotating spherical star, such is not the case for a distorted rotating star. A subtler effect results from differences between properties of the star used for the empirical flux-profile and the star under investigation. These effects are seriously exacerbated for rapidly rotating

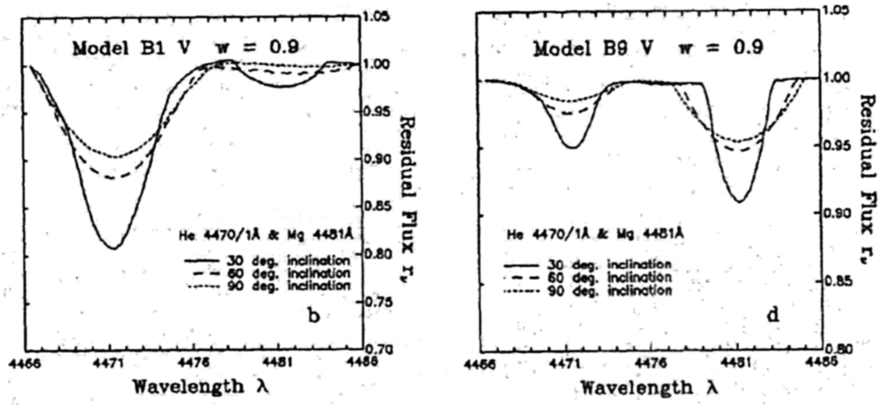

Figure 2. This figure shows rotational broadening of the line typically used to determine $v \sin i$ for stars near the limits of the B spectral type and seen at various angles of inclination. The asymmetry in the HeI $\lambda\lambda 4471/4470$ line is largely removed in the spectrum of the B1V model by the high degree of rotation while the presence of the MgII $\lambda 4481$ line vanishes for the equator-on model.

stars where the appropriate profile on the stellar photosphere varies with latitude due to changes in temperature and gravity associated with rotationally induced distortion.

Collins & Truax (1995) attempted to estimate the cumulative impact of all these effects by comparing the flux profiles of non-rotating models which were classically broadened via the CMRS to *ab initio* rotational line profiles for models having the mass, luminosity and polar radius of the non-rotating model. Figure 2 shows the results for two rapidly rotating stars near the extreme range of spectral type B.

While to some extent this avoids systematic errors associated with an inappropriate comparison between the sharp line and target stars, it does beg the issue of the correct values for the mass, luminosity and polar radius for any given rotating star. Never-the-less the remaining effects are adequately dealt with and can be very large.

The important results of the theoretical model-star comparison is that systematic under estimates of the true value of $v \sin i$ will be obtained for the most rapidly rotating stars throughout the B-spectral type as is shown in Figures 3a and b. This is true regardless of whether line half-widths or Fourier Transforms are employed in the determination of $v \sin i$. This results from the fact that the CMRS does not allow for shape distortion expected for rapid rotation and the variability of the line strengths over the surface resulting from variations in the local effective temperature and gravity.

Collins & Truax (1995) list three assumptions implicit in the use of the CMRS. They are:

1. The observational aspect of a rotating star may be approximated by a circular disk subject to a linear limb-darkening law which is applicable to all parts the stellar disk.

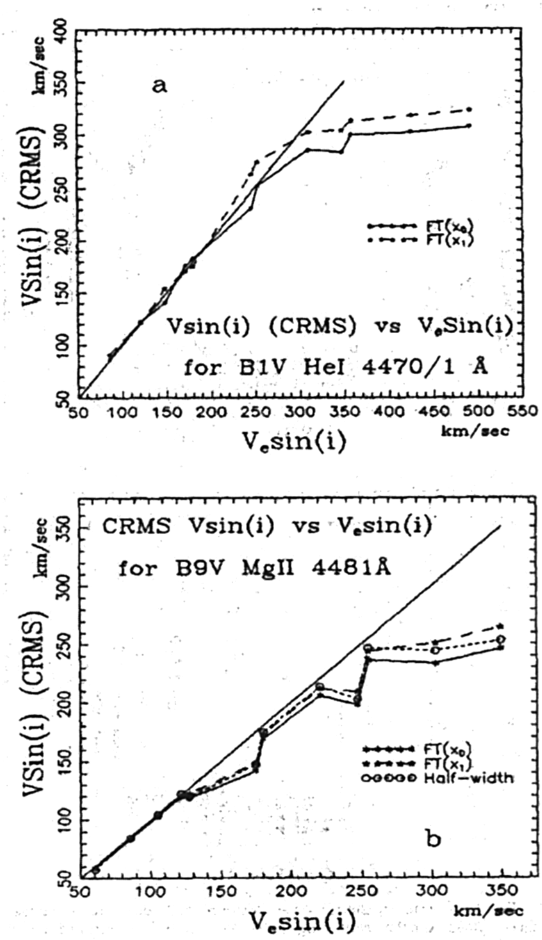

Figure 3. Figures a and b show the systematic departure of "measured" $v \sin i$ from $v_{\mathrm{e}} \sin i$ as determined by comparing observed properties of the line profiles with the CMRS for the spectral types shown in Figure 2.

2. The limb-darkening law appropriate for the continuum is the same as that in the spectral line.

3. The form or shape of the line does not change over the apparent disk; it is simply uniformly Doppler shifted by the motion along the line of sight.

For rapidly rotating stars assumption 1 fails due to the rotationally induced distortion and the associated "gravity-darkening" yielding a reduced contribution from the darker and cooler equatorial regions. This is true for the upper Main Sequence stars and particularly the Be stars. However, it may well also be true for late-type giants and supergiants where the critical velocity is small and of the order of estimated turbulent motions. While distortion in these stars is yet demonstrated, it is likely to be present. The issue is further complicated by lack of knowledge of the appropriate "gravity-darkening" in such stars.

Assumption 2 fails for strong lines where the saturated line core reflects the temperature high in the atmosphere which is also appropriate for the limb of the star. Thus there is likely to be little change in the central intensity of the line core over the disk while the continuum may show significant limb-darkening. Unfortunately only strong lines are present in the atmospheres of the early-type stars and rotation only exacerbates the problem by 'washing out' weak lines. Since many of the lines in late type giants are also saturated, there may be similar problems with assumption 2 for these stars as well.

Assumption 3 while helpful for the calculation of rotational broadening is incorrect even for slow-rotators. Even for weak lines, the intensity line profile appropriate for the center of the disk is qualitatively different from that appropriate at the stellar limb. The center-limb variations will be significantly different for pure absorption lines as compared to that for lines having a strong resonance scattering component. For slowly rotating stars, this may be related to the use of incorrect limb-darkening specified in assumption 2. However, for the rapidly rotating stars, the situation is made far worse by the variation of gravity and local effective temperature resulting from the rotational distortion. This leads to variation in the local ionization equilibrium which can drastically affect the local contribution to the line profile.

Collins & Truax (1995) estimated the extent to which these effects produce systematic errors in the resultant determination of $v \sin i$. They considered two of the strongest spectral lines (i.e. MgII $\lambda\lambda$4481 and HeI $\lambda\lambda$4470/4471) most commonly used to determine $v \sin i$ in B stars. They found that analysis employing the CMRS worked reasonably well for the slowly rotating stars but seriously under estimated $v_e \sin i$ for the rapidly rotating stars. For the later-type B stars where MgII $\lambda\lambda$4481 is commonly used (i.e. B9V) they noted that significant errors began to occur for $v \sin i > 150$ km s^{-1}. For the early-type B stars where Fourier Transforms of He I line are often used, systematic errors would begin to appear for $v \sin i > 250$ km s^{-1} and were indicated by difference between values determined from the first two zeros of the transforms. They also found that it would be unlikely to obtain a $v \sin i > 300$ km s^{-1} regardless of the actual value. A similar result proved to be the case for late-B stars where the maximum value of $v \sin i$ in approximately 230 km s^{-1} regardless of the method of determination. This could lead serious problems with the Be Stars and any study relying on the statistical distribution of $v \sin i$.

Limitations of the CMRS suggest that it is time to improve the underlying model used for the determination of $v \sin i$. Advances in model atmosphere theory and computational facilities have made this relatively easy to accomplish. For a star of given mass, luminosity, and polar radius the basic approach was laid out by Collins (1974) and Collins & Sonneborn (1977), expanded by Collins, Cranmer & Truax (1991) and basically involves creating a grid of model atmospheres at specific latitudes on the model star and using them to calculate specific intensity line profiles appropriate for those locations and the direction to the observer. These profiles are then Doppler shifted to the appropriate reference wavelengths by an amount dictated by the location on the model and its assumed rotational velocity. Quadrature formulae are then used to determine the integrated profile that would be seen by the observer.

This procedure automatically exceeds the limitations listed above by Collins & Truax (1995) by using Roche model shapes appropriate for the given mass and angular velocity, specific intensity profiles instead of convolved flux profiles of sharp line stars, and gravity darkening determined by the luminosity and polar radius. The use of the specific intensity profile eliminates any concerns regarding limb-darkening. The appropriate values for rotating interior models are becoming available (e.g. Maeder 2003 and Deupree 2003) which removes the last major theoretical barrier facing the development of physically based *ab initio* rotating model atmospheres.

3. The Observational Determination of $v \sin i$

The difficulties associated with obtaining interpretable values for $v \sin i$ are not limited to comparison of observation parameters with models alone. In addition to the obvious sources of instrumental error generally associated with the signal-to-noise ratio, there are a number of systematic sources that are sometime overlooked or incorrectly included. Perhaps the most well known of these is the choice of the location of the "continuum" when normalizing the profile for comparison to model calculations. Particular problems may result when reducing data taken on echelle spectrographs where the continuum may vary dramatically over broad lines. The situation is made even worse when dealing with the wide, shallow lines of the fast rotating early-type stars. The central depths of these line profiles may only be a few percent so that a one percent error in the location of the continuum may result in huge and unacceptable errors in the normalized profile.

Since the determination of $v \sin i$ is the determination of a single parameter, many observers choose to use a single property of the observed line for its determination. The most common of these is the half-width at half-depth (HWHD), which is relatively easy to measure, but is particularly subject to systematic errors. In addition, it is not obvious how to obtain the "best" value for this parameter in the maximum-likelihood sense for a given line. Least-square comparison of the entire line with a full model line profile will avoid this problem, but raises others.

There is always a problem deciding what kind of star has been observed when comparing to a model that purports to be a certain kind of star. To what extent can one be sure the two physical types are actually the same ? While it is true that Morgan & Keenan (1943) chose the classification dispersion or 80 mm to minimize the effects of rotational and turbulent broadening on the line "strengths", such compensation is rarely the case for photometric standards. Thus some very fast rotators such as αLeo and ηUMa are included in many photometric standards. The most accurate photometry is relative photometry, but when the standards contain the effect being investigated, that accuracy is illusory. The model computation has a similar problem. As noted above, all *ab initio* models rely on values of M, L, and R. The extent to which these correspond to values appropriate for the observed stars is rarely considered. Even the realm of spectral classification doesn't provide safety from these concerns. Slettebak, Kuzma & Collins (1980) showed that there could be significant changes of the spectral type assigned to a star of given M, L, and R as a result of axial rotation. For example, Kuzma (1981) has shown that spectral types of close binaries may be influenced by the illumination of the companion resulting in the incorrect assignment of stellar mass to stars of that spectral type. This has obvious implications for comparisons of observation with models. Fortunately most of the systematic errors are most prominent for stars showing extreme rotation. Collins (1987) went to some length to discuss the systematic errors associated with the loss of spherical symmetry associated with extreme rotation. One cannot be too circumspect when dealing with such stars.

While advances in technology have enabled astronomers to analyze multiple lines in the same spectrum to compare to synthetic spectra improving the determination of $v \sin i$, the results have been limited to mid to late spectral types simply due to a lack of available lines in the spectra of early-type stars. Nevertheless, "improvements" in the value of $v \sin i$ may be illusory for modest to large values since the lines may systematically arise at different latitudes due to variations in the local values of T_e and g. However, for small values of $v \sin i$, this is not a problem as long as the investigators can be convinced that they are not dealing with a pole-on rotator.

The problems introduced by shape distortion and the resultant gravity-darkening can be viewed as an opportunity. Improved computation allows the generation of model atmospheres to almost be reduced to that of a "subroutine" allowing for the generation of rotating atmospheres of unprecedented computational accuracy. Using such atmospheres as a basis for generating synthetic atmospheres as is currently done for abundance determinations suggests the ability to probe the variation in the structure of atmosphere of rapidly rotating stars. In principle, this could even be done for a rapid rotating early type a star providing great care is taken in the radiative transfer for the strong lines that remain visible in these stars.

4. Further Complications

We have seen how difficulties are encountered with fitting any model to observation with the intent of obtaining a single parameter to represent the axial rotation of a star. Unfortunately there are further problems to be dealt with if we are to associate the value of $v \sin i$ with a star of given mass, radius and luminosity. As pointed out by Collins (1987) the radius and luminosity of rapidly rotating stars are not directly observable properties. Extreme rotation may lead to a 50% increase in the equatorial radius over that of the pole, while the associated distortion and "gravity-darkening" will assure that the star will not radiate isotropically. Thus the "luminosity" associated with a star's distance and apparent brightness will not reflect the total energy production of the star. Years ago George Rybicki (1969) suggested the term "specific luminosity" be used to denote the observed value so as to make clear the difference between the energy emitted in the direction of the observer and the total energy produced by the star associated with stellar interior models. Sadly this distinction has never caught on and the resulting confusion persists. In a similar manner the radius of a rotating star will depend on latitude suggesting that any single number must represent some sort of mean value, which is obscurely related to any actual physical value on the star. Since the mass, luminosity, and polar radius are required to produce a model of the stellar photosphere, an absence of directly observed values for these properties make it impossible to assign a particular rotating model to any specific star. Fortunately these effects are minimal for slow to modest values of the rotation. But they are decisive for any analysis of rapidly rotating stars. Here the effects may result in shifts of several spectral sub-types (e.g. Slettebak et al. 1980) and changes of several photometric magnitudes (e.g. Collins & Sonneborn 1977; Collins et al. 1991). Such effects are not small. Since even rapidly rotating stars may be seen nearly pole-on and thus mistaken for slowly rotating stars, odd results may occasionally be found for stars otherwise presumed to be 'normal' (e.g. Collins 1987).

Such arguments suggest that that the rotating model might be directly linked to a specific mass, polar-radius and luminosity by using *ab initio* stellar interior models. However, again in spite of many efforts, there are significant problems associated with generating credible rapidly rotating stellar interior models. The distortion for the rapid rotators is so large that perturbation methods converge slowly if at all. As pointed out by Moss (1968), even for stars with modest rotation, the Coriolis forces will destroy a convecting element in the equatorial regions long before it has traveled a mixing length thereby reducing the efficiency of convective energy transport. Sadly it is not at all clear how to include this in rotating stellar model interiors. Since convection is so incredibly efficient at transporting energy, it is unlikely that it will make a great difference for specific models. However, it is less clear that the systematic nature of the effect will not have an impact on evolving models.

In any event, even should the rotating photosphere be related to an *ab initio* stellar interior model, there would still be the problem of comparing the model to a specific star. Perhaps, this could be done if the entire spectrum would be utilized in the comparison, but there is no obvious guarantee that such a comparison would be unique. Indeed the work of Collins & Smith (1985) on rotation in A-stars suggests that such a full comparison would not be unique. They considered the effects that differential rotation would have on the determination of $v \sin i$ by considering the most extreme case where, contrary to the sun, the rotation increases toward the pole. They found that for the most extreme rotation cases, there was no uniqueness of the line profile for stars with rather different values of $v \sin i$ when different values of v and $\sin i$ were examined. For large values of v the rapid rotation made all profiles look about the same even for moderate ranges of the inclination i. In addition they found that changes in the colors and magnitudes induced by rotation result in biases that invalidated the standard relation of Chandrasekhar & Münch (1950)

$$\langle V \rangle = \frac{4}{\pi} \langle v \sin i \rangle .$$

These biases which also result from rotation in B stars should be considered in any statistical study of these stars.

5. Additional Determinations of $v \sin i$

To this point I have emphasized the difficulties in obtaining and interpreting $v \sin i$ by spectroscopic means. The cautionary view results from extensive experience with overly optimistic determinations and interpretations of the parameter throughout the 20th century. However, I should emphasize that there is hope for a clearer understanding of the rotation phenomena. Recently there have been attempts to improve the spectroscopic accuracy by using spectrographs which simultaneously measure the properties of many lines (e.g. Doppmann, Jaffe & White 2000). The results for slowly rotating stars certainly do reduce the errors associated with continuum and line-width determinations. Problems associated with comparison to a model still remain, but may be reduced if the spectral range is chosen to minimize the effects of gravity- and limb-darkening. Indeed variation of the results over the spectral range may provide a basis for estimating the magnitude of these effects for specific cases. Unfortunately the problems of a paucity of lines available for the fast rotating early type B stars cannot be solved by this approach. However, it does seem that it would be an excellent tool for investigating the extent of rotation in late-type giants and supergiants. Careful comparison with models may even yield new information about the atmospheres of these stars.

Although rotationally induced variations in the light of Ap stars were detected in the early 20th century (i.e. Guthnick & Prager 1914), it remained for Deutsch (1956) to connect these variations with the rotational period of the star. Since then numerous investigators have used this information together with a value for $v_{\mathrm{e}} \sin i$ to obtain a relationship between the equatorial radius and the inclination. Specifically,

$$R_{\mathrm{e}} \sin i = P(v \sin i)_{\mathrm{obs}},$$

where P is the period of the light variations and $(v \sin i)_{\mathrm{obs}}$ is the observed value for $v \sin i$ obtained from analysis of the line profiles usually based on the CMRS.

Another approach that seems to hold promise particularly for dealing with the rapid rotating early-type B and Be stars is the use of interferometric measurement of the oblateness of these stars. The recent work of van Belle, Ciardi & Thompson (2001) (see also van Belle, Ciardi & Thompson 2003) shows that a consistent, and presumably accurate, value of $v \sin i$ has been determined for Altair. While it is true that the technique is limited to relatively nearby stars, this is also the class of stars for which Hipparcos has provided parallaxes that yield useful constraints on the specific luminosity. In addition the method becomes more accurate with increasing rotational velocity and even holds the promise for separating v and $\sin i$. Since the class also represents relatively bright stars, accurate photometry is available for them and consistent monitoring of them will allow for photometry of the Be-stars when they are not in the emission phase so that the colors may be unambiguously compared to models. In addition, the interferometry, while yielding oblatness, also yields values, when combined with the parallax, for the projected radii. That, and the specific luminosity, should allow for a determination of the mass from interior models which in turn allow for the construction of credible rotating model atmospheres for the star. This happy combination of events will allow for the investigation of the effects of rapid rotation on the structure of a modest set of relatively nearby rapid rotating early-type stars. Then, in the long standing tradition of astronomy of extending knowledge of the nearby to remote stars, we have the possibility of understanding the detailed structure and evolution of the most massive stars.

6. Summary and Conclusions

In preparing this paper I conducted a 'modern' library search of the most recent work in the field. My initial efforts suggested more than 100000 references. Somewhat more careful use of the search engines significantly reduced that number. However, the result may well be that I have overlooked a 'landmark' work of seminal importance and for that I apologize. Nevertheless, it is my general overall impression that a significant number of investigators are measuring a parameter they designate as "$v \sin i$" with increasing precision. In the vast majority of cases it would appear that the observational nature of the measurements are related to the determination of line 'half-widths' and the connection to actual properties of the stars still relies on a usually unstated model so that the results are subject to may of the problems I have raised in this discussion. However, the rapid improvements in the collection of data that have characterized the beginning of this century suggest that improvements in the determination and interpretation of $v \sin i$ will be forth coming. Parallaxes of greater accuracy are providing more accurate values of L and R particularly for the early-type stars. This will enable the construction of more accurate stellar interior models upon which definitive rotating model atmospheres rely. Line profiles resulting from such models will provide a far better foundation for the study of rotating stars than the CMRS.

The simultaneous determination of multiple line widths, in principle, offers the possibility of testing the extent of distortion in rapidly rotating stars. Lines arising from different excitation potentials will preferentially be formed at different latitudes in regimes with different temperatures and gravities. The conditions depend critically on the extent of the distortion in rapidly rotating stars. We have already seen that knowledge of the distortion, in principle, enables the determination of $v \sin i$. Unfortunately the lack of visible lines in the most rapidly rotating stars may make this extremely difficult to carry out in practice. In addition it would appear that the concern of Collins (1970) that spectral line profiles are a poor way of determining the extent of rotation in the most rapid rotators remains in effect today.

What is equally intriguing is the possibility that similar studies may enable one to probe the extent of differential rotation in later-type more slowly rotating stars where lines are more abundant. Again, success may be frustrated by the lack of distortion and hence variation of the temperature and gravity over the surface. Nevertheless, stars such as Altair, already having a measured oblatness (see van Belle, Ciardi & Thompson 2001, 2003), may provide such an opportunity. The difficulties are great, but the rewards in understanding of the stellar atmospheres of such stars are profound.

It is important that opening addresses to conferences such as this emphasize the positive aspects of the discipline. By emphasizing areas deserving of caution, I fear I may well have failed in this mission. However, I take comfort that Isaac Newton once observed that:

> "To explain all nature is too difficult for any one man or even for any age. T'is much better to do a little with certainty, and leave the rest for others that come after you, than to explain all things" (see Bronowski 1973).

In spite of this word of caution, I would also observe that we stand on the threshold of an era that will see our knowledge of stars increased beyond the imagination of any twentieth century astronomer. The potential exists to obtain knowledge of stars rivaling our present knowledge of the sun and thereby increasing the former immeasurably. It is indeed, an exciting time for stellar astrophysics.

In closing, I would note that we are currently in the midst of an amazing transition from the 'data-starved' twentieth century of astronomy to an era where vast amounts of data on ever more widely ranging topics is becoming available. It will be the task of twenty-first century astronomers to "sift and winnow" this data in search of useful knowledge. This is an awesome task and we can only hope that they are fortunate enough to be able to apply that knowledge with the same level of wisdom as has been shown by their predecessors.

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