Black Holes from Stars to Galaxies – Across the Range of Masses
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Kozai resonance model for Sagittarius A* stellar orbits

Ladislav Šubr,[1,2] Vladimír Karas,[3] and Jaroslav Haas[2]

[1]Argelander Institut für Astronomie, University of Bonn, Bonn, Germany
[2]Charles University, Astronomical Institute, Prague, Czech Republic
[3]Astronomical Institute, Academy of Sciences, Prague, Czech Republic
email: subr@sirrah.troja.mff.cuni.cz

Abstract. We study a possibility of tidal disruptions of stars orbiting a supermassive black hole due to eccentricity oscillations driven by Kozai's mechanism. We apply the model to conditions relevant for the Galactic Centre where we consider two different sources of the perturbation to the central potential, which trigger the resonance mechanism. Firstly, it is a disc of young massive stars orbiting Sgr A* at $r \gtrsim 0.08\,$pc, and, secondly, a molecular circumnuclear disc. Each of the two possibilities appears to be capable of exciting eccentricities to values sufficient for the tidal disruption of ~ 100 stars from the nuclear stellar cluster on a time-scale of 0.1–10 Myrs. Tidally disrupted stars may cause periods of increased accretion activity of Sgr A* .

Keywords. Galaxy: center – stellar dynamics – celestial mechanics

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1. Introduction

Tidal disruptions of stars from a gravitationally bound dense cluster may represent one of the channels of the supermassive black holes (SMBH) growth. This process has a relatively small cross-section as the radius at which the tidal disruption of a solar type star occurs is of the order of several tens gravitational radii of the SMBH, while a characteristic radius of the stellar cluster under the dominance of the massive centre is of the order of $10^6 - 10^7 R_{\rm g}$. Hence, only stars on nearly radial orbits, i.e. those with very small angular momenta can be a subject of this process. In the case of a star cluster with semi-major axis distribution $\propto a^{1/4}$ (Bahcall & Wolf 1976) and eccentricity distribution $\propto e$ the loss-cone contains only a few ($\lesssim 10$) stars and it is emptied on a very short, orbital time-scale. Therefore, the tidal disruptions channel can be effective only if there are some physical processes that refill or enlarge the loss-cone.

Kozai resonance (Kozai 1962, Lidov 1961) is a process that leads to secular evolution of the orbital elements of a particle moving in the central Keplerian potential perturbed by an axisymmetric perturbation. In such a system a component $L_{\rm z}$ of the angular momentum vector parallel to the symmetry axis is conserved. The total angular momentum, evolves in time with the lower limit $\geqslant L_{\rm z}$. According to the analysis of Kozai (1962) and Lidov (1961) there exists an additional integral of motion in the system, which poses further constraint on the angular momentum. This integral appears to be an average of the perturbing (non-Keplerian) part of the Hamiltonian over one orbital revolution, $\bar{V}_{\rm p}$.

The maximum eccentricity attained during the orbital evolution depends on the form of the (axisymmetric) perturbing potential. An additional, spherical perturbation of the Keplerian potential decreases the eccentricity oscillations. In this paper we discuss two different kinds of perturbations that damp the Kozai oscillations, preventing the stars from being tidally disrupted: (i) smoothed potential of the stellar cluster gravitationally bound to the central BH and (ii) relativistic pericentre advance.

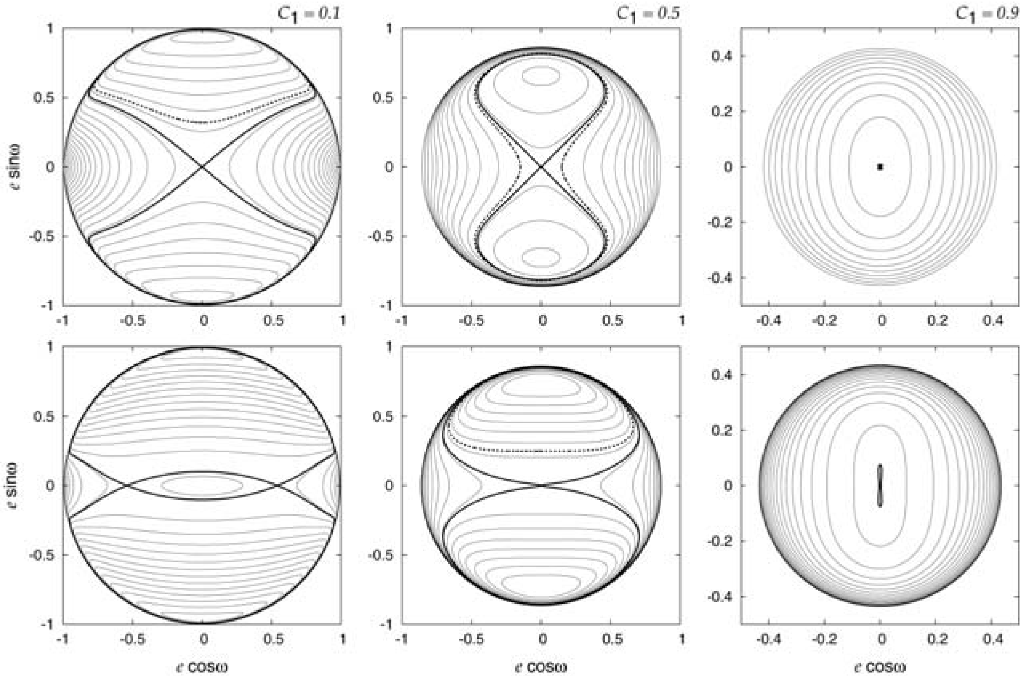

Figure 1. Iso-contours of time-averaged perturbing potential of a ring (top) and a disc of constant surface density (bottom) are plotted with solid lines; thick solid line emphasizes a separatrix. Dotted lines show tracks of numerically integrated orbits evolving in the respective potential from arbitrarily selected initial conditions. Both axisymmetric perturbations are characterised by radius $R_\mathrm{d} = 5 \times 10^4 R_\mathrm{g}$; value of semi-major axis $a = 2.4 \times 10^4 R_\mathrm{g}$ is common for all panels. Value of z-component of angular momentum (expressed by means of C_1) differs in individual panels in order to demonstrate dependence of the topology of contours on this quantity.

2. Evolutionary diagrams

Motion in the axially symmetric static potential obeys two 'trivial' integrals of motion — semimajor axis a and z-component of angular momentum (expressed by means of $C_1 \equiv \sqrt{1-e^2}\cos i$; $C_1 = 1$ corresponds to a circular orbit in the equatorial plane) — that reduce the number of free orbital parameters. Additional integral of motion, $\bar{V}_\mathrm{p}$, further reduces the independent variables to three and enables us to analyse topology of orbital tracks in the space of eccentricity e and argument of pericentre ω.

Figure 1 shows examples of the iso-contours of the averaged perturbing potential for two different sources: (i) an infinitesimally narrow ring and (ii) a razor-thin disc of constant surface density. These cases were used as convenient basic examples with analytic form of potential (see e.g. Lass & Blitzer 1983). Both sources are characterised by mass M_d and (outer) radius R_d. We see that topology of the curves in the (e, ω) space qualitatively differs for the two sources. In both cases, increase of C_1 increases fraction of the plots covered with 'rotating' curves ($\omega \in \langle 0, \pi \rangle$) with decreasing amplitude of the eccentricity oscillations along them. In addition, in some panels we plotted examples of tracks of numerically integrated orbits evolving in the respective potential. These tracks fit perfectly in the congruences of iso-contours, confirming that the quantity $\bar{V}_\mathrm{p}$ is an integral of motion. This allows us to interpret the plots as 'evolutionary diagrams'.

In Figure 2 we plot evolutionary diagrams for the case of ring-like axisymmetric perturbation accompanied by a smoothed potential of a spherical stellar cusp with radial

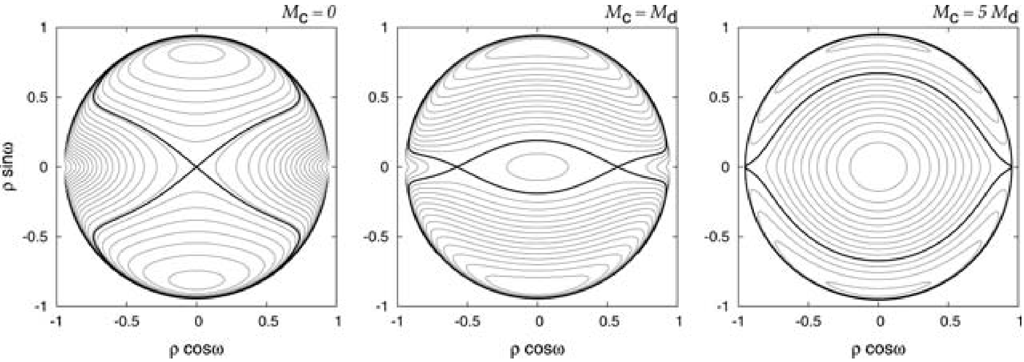

Figure 2. Evolutionary diagrams of orbits in a compound potential of a ring and a spherically symmetric cusp. Ratio of their masses is indicated above each panel. Value of the integral $C_1 = 0.1$; other parameters are identical to Fig. 1. Left panel corresponds to the upper left panel in Fig. 1; here we use variable ρ; $\rho^2 = 1 - (1-e^2)^{1/2}$ as the radial coordinate, which allows us to resolve the structure of the contours at large eccentricities (i.e. on the perimeter).

density profile $\propto r^{-7/4}$ which implies potential of the form $V_c \propto r^{1/4}$. The stellar cusp is characterised by its mass M_c enclosed within the radius R_h (we set $R_h = R_d$ in this case). The ratio of M_c/M_d clearly plays an important role. While the topology of the evolutionary diagrams, in general, becomes more complex when the star cluster is considered, the inner zone of rotation gradually increases with M_c increasing and the eccentricity oscillations are damped in the whole parameter space.

General relativistic pericentre advance stands as another effect that damps the Kozai oscillations. In order to visualise it by means of the iso-contours of the perturbing potential we employed an additional spherically symmetric term, $V_{PW} = -2GM_\bullet r^{-1}(r - R_g)^{-1}$, which represents a non-Keplerian part of the Paczyński–Wiita (1980) pseudo-Newtonian potential. Examples of the evolutionary diagrams including this perturbation are shown in Figure 3. By comparison of the left and middle panels we clearly see that the relativistic pericentre advance tends to smear the structure of the curves and bring the libration point to lower eccentricities, i.e. it enlarges the outer rotation zone. On the other hand, the precession due to the stellar cluster pushes the libration point to higher eccentricities and enlarges inner rotation zone. The axisymmetric source in the form of the disc competes more successfully with the damping perturbations.

3. Capture rates

Due to eccentricity oscillations the stars may get close enough to the central BH to be tidally disrupted. In order to examine importance of this effect we evaluate the fraction of stars that reach the centre within the tidal radius $R_t = (M_\bullet/M_\star)^{1/3}R_\star$, i.e. we integrate the distribution function over appropriate loss-cone in the space (a, C_1, e, ω). We consider a distribution function in the form $D_f(a, C_1, e, \omega) = K a^{1/4} e \eta^{-1}$, where $\eta \equiv (1-e^2)^{1/2}$, which corresponds to spatially isotropic Bahcall & Wolf (1976) distribution in energy and linear distribution of eccentricities.

In a purely Newtonian regime the loss-cone would be defined as $a(1-e) < R_t$ and the appropriate fraction of stars with pericentre below R_t is then

$$\mathcal{F}(R_t) \simeq 10\, R_t\, a_{\max}^{-1}\,, \qquad (3.1)$$

where $a_{\max}$ is an upper limit on the semi-major axis. An upper estimate for the case when the Kozai mechanism drives the secular orbital evolution is determined by the loss-cone

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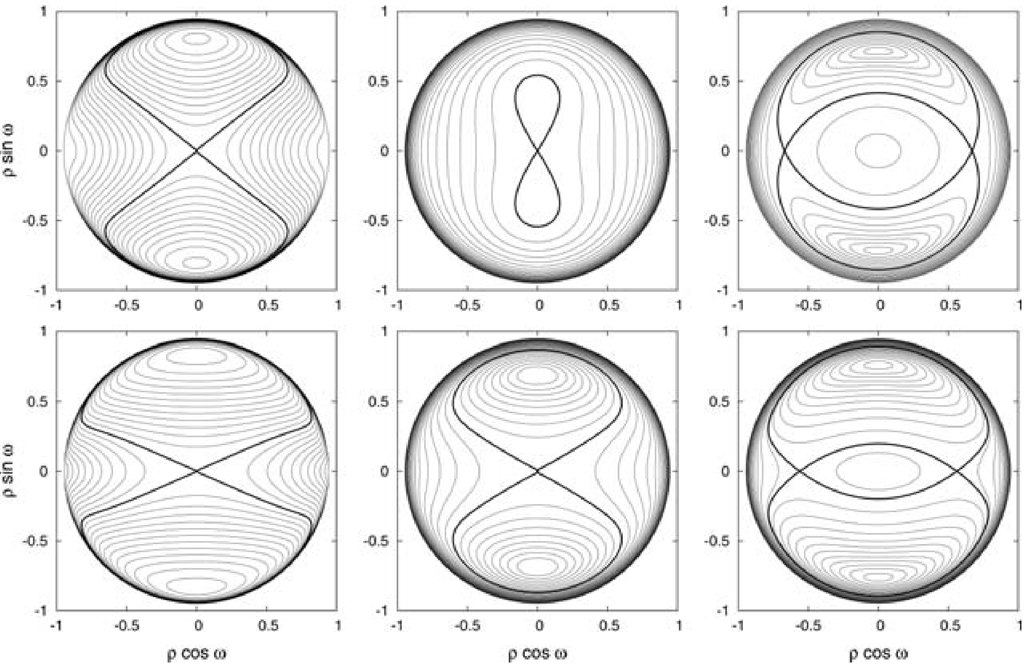

Figure 3. Evolutionary diagrams for different forms of the perturbing potential. Axisymmetric perturbation $V_{\rm d}$ due to the ring (top) or the disc (bottom) is considered in all panels. In the middle panels, the perturbing potential consists of $V_{\rm d} + V_{\rm PW}$ and in the right panels all terms $V_{\rm d} + V_{\rm PW} + V_{\rm c}$ are included. Common parameters are: $C_1 = 0.1,\ a = 1.2 \times 10^4 R_{\rm g},\ M_\bullet = 3.5 \times 10^6 M_\odot,\ M_{\rm d} = M_{\rm c} = 0.01 M_\bullet$ and $R_{\rm d} = R_{\rm h} = 5 \times 10^4 R_{\rm g}$.

defined as: $a(1 - (1 - C_1)^{1/2}) < R_t$ which gives:

$$\mathcal{F}(R_{\rm t}) \simeq \tfrac{10}{3}\sqrt{2R_{\rm t}/a_{\rm max}}\,. \tag{3.2}$$

The real values should lie somewhere between the two estimates, depending on the perturbing terms considered. Example of numerically determined fraction $\mathcal{F}(R_{\rm t})$ is shown in Figure 4. We employed a single-parameter model with $M_{\rm c} = M_\bullet$, $M_{\rm d} = 0.01 M_\bullet$ and $R_{\rm d} = R_{\rm h} = 2.25 \times 10^{10} (M_\bullet/M_\odot)^{-1/2} R_{\rm g}$. In the left panel, $\mathcal{F}(R_{\rm t}, a)$ represents a partial fraction of stars with given value of semi-major axis that reach the central BH within $R_{\rm t}$. Again, the way how the two additional spherical perturbations damp the Kozai oscillations can be easily distinguished. The relativistic effect sharply drops $\mathcal{F}(R_{\rm t}, a)$ to the lower estimate (3.1) at $a \simeq a_{\rm t}$ which can be estimated (Karas & Šubr 2007) as:

$$a_{\rm t}^7 \simeq \tfrac{32}{9}\, R_{\rm d}^6\, R_{\rm g}^2\, R_{\rm min}^{-1}\, \mu^{-2}\,. \tag{3.3}$$

In contrary, the gravity of the extended cluster damps amplitude of the Kozai oscillations over the whole range of semi-major axis, without any clear threshold. Fraction $\mathcal{F}(R_{\rm t})$ integrated over the semi-major axis as a function of $M_\bullet$ is plotted in the right panel of Figure 4. We see that the relativistic damping is becoming important for $M_\bullet \gtrsim 10^6 M_\odot$, while the damping due to the cluster potential influences $\mathcal{F}(R_{\rm t})$ in the whole mass spectrum. In general, we may conclude that the tidal disruption loss-cone increases with decreasing mass of the system.

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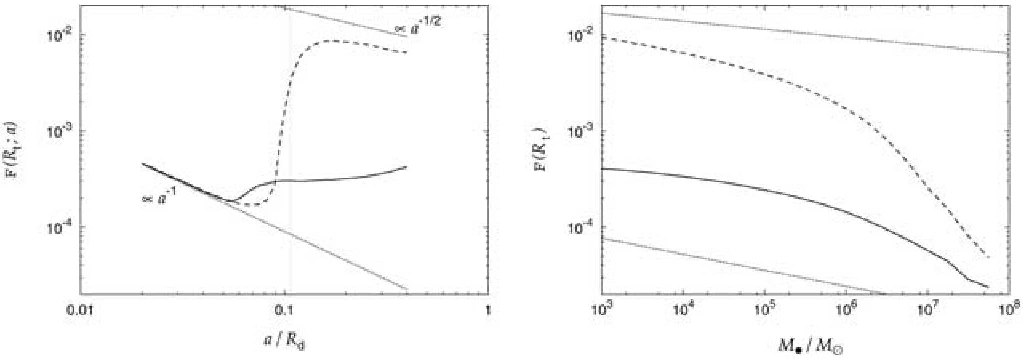

Figure 4. Fraction of stars from the cluster that reach R_t due to Kozai oscillations. Left: $\mathcal{F}$ as a function of semi-major axis for $M_\bullet = 10^4 M_\odot$; right: integrated fraction as a function of $M_\bullet$. Dashed and solid curves correspond to models omitting or including gravity of the stellar cluster, respectively. Dotted lines represent the lower and upper estimates (3.1) and (3.2). Vertical thin dotted line indicates the terminal value of semi-major axis determined by eq. (3.3).

4. Application to the Sgr A*

Recent observations of the Galactic Centre have revealed two structures that could be sources of the perturbation of the central potential required by our model: (i) At relatively large radii it is the circumnucelar disc of molecular gas ($M_\mathrm{d} \approx 0.1 M_\bullet$) which extends to $r \gtrsim 1.5$pc (e.g. Christopher *et al.* 2005). Its influence on the stellar orbits of the bound cluster can be in the first approximation simulated with a ring of radius R_h. This configuration leads to $\mathcal{F}(R_\mathrm{t}) \approx 3\times10^{-4}$, which can be translated to few$\times$100 tidally disrupted stars. (ii) Another structure is a coherently rotating disc of young stars, which can be modelled by a disc of surface density $\Sigma \propto R^{-2}$ (e.g. Paumard *et al.* 2006) with $0.04\text{pc} \leqslant R \leqslant 0.4\text{pc}$ and $M_\mathrm{d} \approx 0.01 M_\bullet$. This latter source competes more successfully with the damping effects; therefore, the loss-cone gets enlarged, giving $\mathcal{F}(R_\mathrm{t}) \approx 2\times10^{-2}$. Nevertheless, the part of the cluster that is affected by this source is small. Hence, the number of stars tidally disrupted due to this perturbation is again few$\times$100. These stars can be pushed to the tidal radius within one Kozai period (i.e. order of 1 Myr). Such events may lead to episodes of enhanced activity of the Galactic SMBH.

Acknowledgements

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