CONSTRAINTS ON THE DARK MATTER FROM OPTICAL ROTATION CURVES

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ABSTRACT. From the observed rotation curves of Sa, Sb, and Sc spiral galaxies, it is possible to deduce a dozen constraints on the nonluminous matter in spirals. Within the optical image, the dark matter is less concentrated than the luminous, and contributes about 1/2 of the mass, for spirals of all Hubble types and luminosities.

1. INTRODUCTION

The determination of rotation curves for spiral galaxies has been a fruitful industry for the past decade. High dispersion spectrographs on large telescopes, coupled with electronic enhancement of incoming photons, has made it possible to derive accurate emission line rotation velocities for the optical disks of Sc's, Sb's and even for some Sa's. From a systematic study of the dynamical properties of about 60 relatively nearby Sa, Sb, and Sc field spirals, Rubin and colleagues (1985 and references therein) have discussed dynamical properties as a function of various galaxy parameters. Virtually all of this work has been done photographically.

Within the past few years, a dramatic change in observing techniques has been introduced. Replacing the photographic plate with a CCD detector permits accurate digital subtraction of weak nightsky emission features and hence velocity measurements of weaker galaxy emission; it also presents simultaneously an accurate record of the variation of emission line strengths across the galaxy disk. Rotation curves can be obtained for galaxies with redshifts as great as $z=0.05$ and probably even 0.1. (I hope that some day I will be amused at the conservative nature of this prediction). Dynamical properties of entire new classes of galaxies can be studied.

I show in Figure 1 an image of a portion of the compact group Hickson 88, and a spectum of Hi88a. Note the entanglement of the [NII] 6583A line with the nightsky line. Such confusion is easily eliminated, now that I measure CCD spectra at a computer terminal with a new facility developed by Kent Ford, based in part on software supplied by Schechter, Tonry, and Boroson. Velocities measured from the galaxy plus

J. Kormendy and G. R. Knapp (eds.), Dark Matter in the Universe, 51–65.

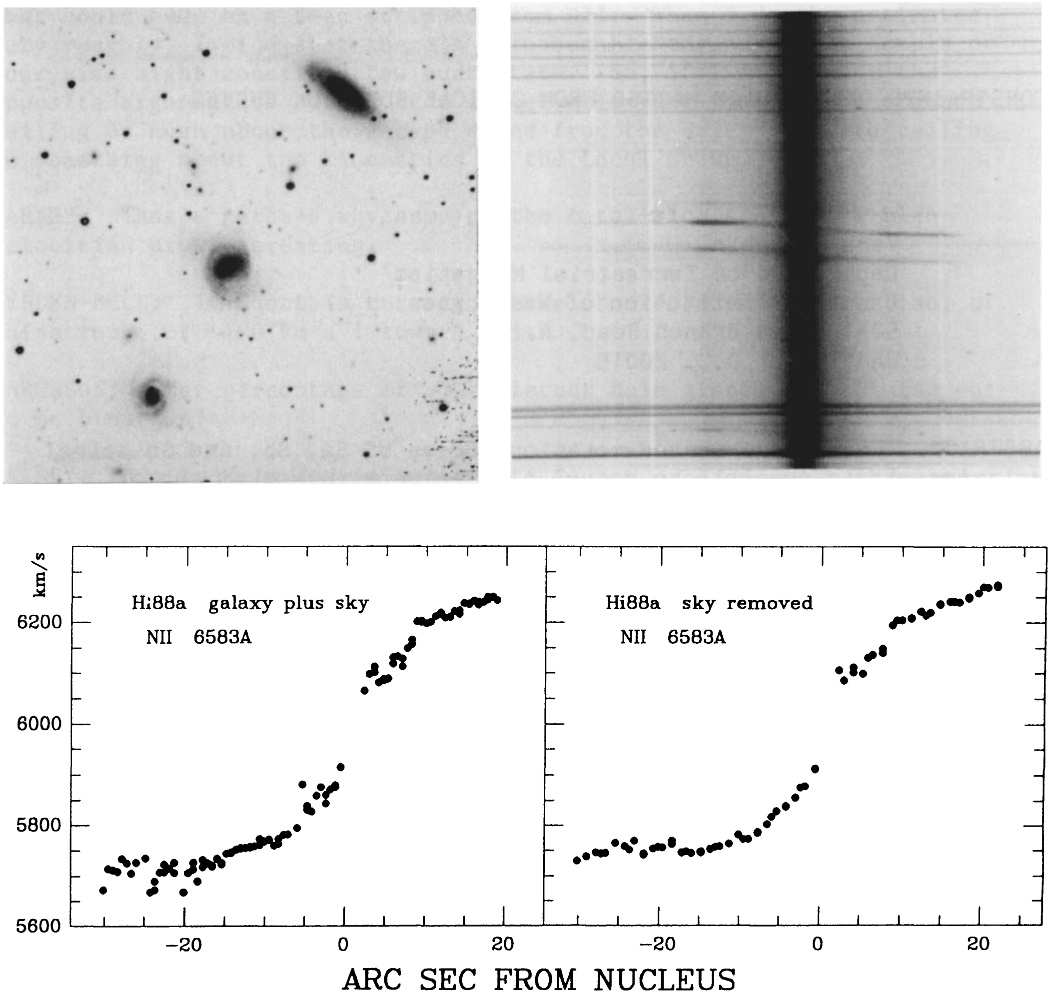


Figure 1. Upper Left: The inner region of the compact group Hickson 88, from a KPNO 36-inch CCD + R (Mould filter) frame. Hi88a is the NE spiral. Upper Right: Hα and [NII] lines in the spectrum of Hi88a, from a Palomar 200-inch spectrum recorded on a CCD; wavelength increases down. Original dispersion and scale are 0.8A/pix and 0.59"/pix; exposure 6000 sec. The nightsky line between Hα and [NII] (right) merges with the low velocity [NII] (left). Lower: Measures of [NII] velocities, with and without sky subtraction, found by computer fits to the continuum levels and emission line shapes. Generally, 2 or 3 columns are summed in making the fits. Note the reduced scatter and the increased extent after sky subtraction. Additional accuracy, especially in the central region, can be gained by removing the galaxy continuum.

sky frame and for the galaxy minus sky (Figure 1) attest to the decrease in scatter when the sky spectrum is removed. Increased velocity accuracy offers us the capability of asking and answering questions, for example, about subtle alterations in dynamics arising from environmental causes.

While these techniques offer great promise for the immediate future, in this paper I want to probe what we have learned in the past. Rather than discuss rotation curves as a tool for studying galaxy dynamics, I want to describe what rotation curves tell us about the nonluminous matter in spirals. Specifically, my aim is to compile a list of the properties of the dark matter, as they are inferred from the observations, generally optical, of the dynamics of galaxies, generally spirals.

It seems to me remarkable that rotation curves for galaxies of a variety of Hubble types and a wide range of luminosities exhibit such a simplicity of form. This in turn implies a simplicity of mass distribution. It seems equally remarkable that a comparison of the rotation curves, or mass forms, with optical properties makes it possible to place a dozen or so contraints on the dark matter, something we have so far detected only by its gravitational effect. The observations from which these properties are inferred are already known to many of you, but here the emphasis will be slightly different. Due to space limitations, the less controversial points will be mentioned only briefly; literature references are incomplete, but can be traced from papers cited.

2. WHERE IS THE DARK MATTER?

2.1. It is clumped around spiral galaxies

Since the early work of Zwicky (1933) and Smith (1936), evidence has existed for nonluminous matter in clusters of galaxies. Only in the past decade has the study of field spirals demonstrated that nonluminous matter is a property of isolated spirals as well (Bosma 1978). For a galaxy with a flat rotation curve, the mass interior to R grows linearly with R, $M(R) = kV^2R$, and the density ρ falls as R^{-2}; for many spirals where the outer rotation curve is slowly rising rather than flat, the density falls more slowly, $\rho \sim R^{-1.7}$. At the isophotal radii of massive spirals, the density has fallen only to about 10^{-25} or 10^{-26} gm/cc which is 4 orders of magnitude higher than the closure density of the universe. Thus field galaxies with their attendant halos represent density peaks in the distribution of matter in the universe.

If density continues to fall as R^{-2}, it requires halos of a few Mpc extent, which is about one-half the average distance between galaxies, to have sufficient mass to close the universe. There is presently no evidence that space is filled with these large halos.

2.2. It is less concentrated than the luminous matter

In a disk galaxy, the surface brightness of luminous mass falls exponentially, while mass density falls more slowly, nominally as R^{-2}. Thus the ratio of M(total)/M(lum) increases significantly with increasing radial distance across a galaxy disk. Resolution of spirals into component parts by Carignan and Freeman (1985; see other papers this volume, especially Bosma, Athanassoula, and van der Hulst) shows that M/L(local) increases by a factor of about 100 across the 10 kpc disks of late-type spirals.

2.3. It is more extended than the luminous matter

It is a rare circumstance that permits the measurement of optical velocities beyond the optical disk; SO galaxies with polar rings offer such an opportunity. For a few of these galaxies (Schweizer et al. 1983; Whitmore et al. this volume) velocities have been measured over a ring whose spacial extent is several times that of the disk it girdles. In all cases, the constant ring velocity matches that in the disk. Hence mass continues to rise linearly with radius to a distance several times that of the disk isophotal radius. For polar ring galaxies, the dark matter extends beyond the galaxy disk.

For some spirals, HI gas is significantly more extended than optical luminosity. Almost without exception, measured HI velocities show flat rotation curves to the Holmberg radius (Bosma 1981, Sancisi 1983), beyond which warps become prominent and rotation velocities uncertain. For NGC 3198 (van Albada et al. 1985), accurate 21 cm velocities exist to radii almost twice the Holmberg radius. Cumulative M/L values increase from unity at R(25) to 2 at R(Holmberg) to 4 at the limits of HI observations. In all of these special cases with extensive measurements, the dark halo extends well beyond the optical galaxy.

3. WHAT IS THE FORM OF THE DARK MATTER DISTRIBUTION?

3.1. Gravitational potential is more nearly spherical than flat

SO galaxies with polar rings offer the very unique possiblity to sample the 3-dimensional form of the gravitational potential of a disk galaxy. The velocity of particles in the ring at polar distance R can be compared with the velocities of particles in the disk at radial distance R. For four polar ring galaxies (Schweizer et al. 1983; Whitmore et al. this volume) the ratio V(ring)/V(disk) is unity to within 10%, implying a nearly spherical form for the potential. Moreover, the constant velocities in the ring which match and extend those in the disk offer evidence that the density distribution of the dark matter is not discontinuous at the limits of the optical disk, at least for these objects.

3.2. Forms of rotation curves, and hence forms of mass distributions, are unrelated to galaxy morphology

I show in Figure 2 two sets of rotation curves, plotted in log V, log R, coordinates to emphasize their forms. The lower three curves come from an Sa, Sb, and Sc galaxy whose bulge-to-disk ratios range over a factor of 40. NGC 2639, an almost diskless Sa, and NGC 801, an almost bulgeless

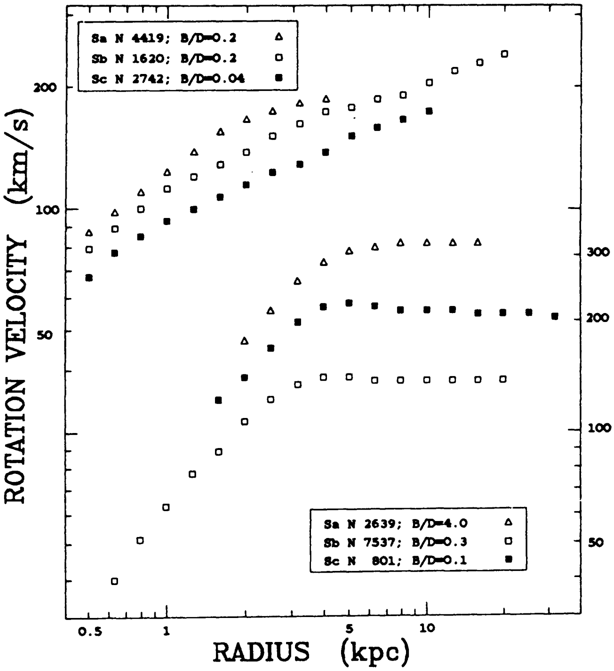


Figure 2. Lower: Rotation curves of similar form for an Sa, Sb, and Sc galaxy; bulge-to-disk ratios differ by a factor of forty among these three. Upper: Rotation curves of similar form for 3 other galaxies, whose form differs from that below. The form of the rotation curve is not closely correlated with the Hubble type or luminosity.

Sc, both exhibit a similar form for their rotation curve. The three upper rotation curves exhibit a similar form among themselves, but a very different one from the lower set. Thus, galaxies of similar morphology can have very different rotation curves, while galaxies of the same morphology can have very different rotation curves. Galaxy morphology is not a useful predictor of the distribution of mass within a galaxy. This is a surprising result, but a fairly well-founded one.

3.3. Rotation curves, and hence forms of mass distribution, come only in a limited variety

It is a remarkable fact that the forms of the rotation curves of galaxies, in distinction to their amplitudes, show a notable sameness, when scaled to appropriate units in radius and velocity. Rotation curves cover only a small fraction of the V,R plane; velocities at moderate and large R range only from slightly rising to slightly falling. Burstein and Rubin (1985) show that this simplicity transforms to an equivalent simplicity in integral mass distributions; these are useful in discussing overall galaxy properties. When we assume that the relation between the rotation curve and the projected mass distribution is the same for all galaxies, then $M(R) = kV^2R$ is the mass interior to R. We can classify the forms of mass variations by examining a plot of M(R) vs R.

In Figure 3 we superpose plots of the mass interior to R vs R for 20 galaxies (11 Sc, 5 Sb, 4 Sa), each identified by a letter. Radii and masses are scaled to give minimum scatter. All of these galaxies exhibit a similar form of rotation curve and hence mass distribution, even though they differ markedly in morphology and in luminosity. From the restricted

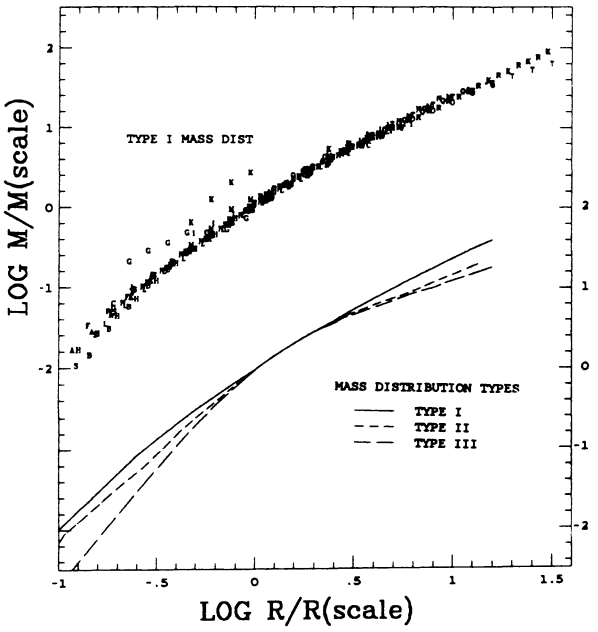


Figure 3. Upper: Mass interior to R plotted vs R for 20 galaxies; each coordinate is scaled to produce minimum scatter. The resulting curve we call mass type I. Lower: Mean lines for each of the three integral mass distributions, plotted together to emphasize the difference in curvature between the mass forms, and the continuity of forms among the types.

continuum of mass forms, we have identified 3 principal ones, shown schematically in the lower part of the Figure. Mass type II contains 2 Sc, 4 Sb, and 5 Sa spirals; type III contains 1 Sc, 5 Sb,and 4 Sa's. Twelve spirals have intermediate forms, and 7 have complex (uncertain) forms. Although a larger number of Sc galaxies are of mass type I, there is a selection of all Hubble types among all mass types.

Mass forms are differentiated by the amount of curvature on a [log M(R), log R] plot; they illustrate the progression in the form of the rise of the rotation curve relative to the radius at which the rotation curve becomes nearly flat. For most spirals, maximum curvature in the rotation curve occurs within one or two radial scale lengths, generally of order 10" or 20" for the galaxies we observe. In the limit that the rise is unresolved, the mass form is a straight line. Thus it is significant that many virtually bulgeless Sc galaxies have a faster rise (i.e., type I) than the large-bulged galaxies of mass type III.

3.4. Dark matter contributes to the galaxy potential on all scales

There is little controversy that dark matter dominates the galaxy potential on large scales. Here we question the smallest scales on which the dark matter is significant. The already-classic procedure for discussing the distribution of mass in spiral galaxies (Kaljnas 1983, van Albada et al. 1985, Bahcall and Casertano 1985, Carignan and Freeman 1985) is to decompose the galaxy into a nucleus, bulge, disk, and halo and deduce the relative mass for each, subject to the condition that each component separately satisfy the observed luminosity properties, and that all components sum to predict a gravitational potential which produces the observed rotation curve. In practice, the rise of the rotation curve is attributed solely to the luminous disk mass, producing what van Albada et al. have called the "maximum disk." For example, in NGC 3198, the halo mass contributes negligible mass interior to one scalelength (2.7 kpc = 60"). I would now like to argue, on admittedly tenuous grounds, that although the contribution of the dark matter is not dominant at small radii, it is present here too.

If gas in a spiral disk has noncircular motions anywhere, these are likely to exist near the nucleus. However, optical observations give only minimal evidence for such. In many galaxy spectra, emission lines can be traced smoothly as close as one or two arc sec to the nucleus, typically 100 or 200 pc along the major axis (and of order 500 pc along the minor axis) for our sample. The width and character of the lines do not change as they approach the nucleus; minor axis measures give little evidence for radial motions. (In practice, a slight slope along the adopted minor axis is taken as evidence of a small error in the adopted line of nodes, which can have an uncertainty of several degrees due to complex galaxy morphology).

If we accept the conclusion that the disk gas is in circular orbit within a few hundred pc of the nucleus, then how are we to interpret the similar observed mass form at small radii (Figures 2 and 3) for galaxies of very different nuclear morphology, or the steep velocity rise for Sc galaxies of minimal bulge? One simple explanation is that the contribution of the dark matter is already signficant by 0.5 or one kpc. More detailed optical observations might help settle this question; spectra of nuclear regions of the nearest galaxies from the space telescope will be crucial. To date, most radio observations are unable to resolve velocities at small R. Thus while the evidence is not overwhelming, I think we may learn that dark matter is a nontrivial component of nuclear regions of spiral galaxies.

4. HOW MUCH DARK MATTER MAKES A SPIRAL GALAXY?

4.1. The amounts of dark and luminous matter are related

Within the sphere defined by the radius of its optical disk, a spiral galaxy has equal parts luminous matter and dark matter. This proportion holds for galaxies whose optical luminosities differ by as much as a factor of 100. These conclusions come both from statistical studies, and from mass decomposition of a few individual galaxies. This incredible concordance of luminous and nonluminous matter must offer one of the primary clues as to the nature of the dark mass.

I show in Table I the dynamical mass-to-luminosity ratios which are derived from the rotation curves of the field spirals (Rubin et al. 1985). Within each Hubble type, there is a good correlation of dynamical mass with optical luminosity over the entire observed luminosity range. A comparison of the derived M/L values with M/L values predicted (Larson and Tinsley 1978) from luminous mass in stars plus gas shows that for each spiral type, the dynamical (i.e., luminous plus dark) mass exceeds the luminous mass by a factor of about two. Hence the mass in dark matter equals the mass in luminous matter interior to R(25).

For those galaxies whose mass distributions have been deconvolved

Table 1. Ratio of Dark-to-Luminous Matter in Spiral Galaxies

Hubble Type	M(R25)/L Dynamical	n	(M/L)stars Tinsley Larson	(M/L)stars plus gas	(M/L)dynamical / (M/L)stars,gas
Sa	6.1±0.7	11	3.1	3.1	2.0
Sb	4.5±0.4	22	2.0	2.1	2.1
Sc	2.6±0.2	20	1.0	1.2	2.2

into component parts (NGC 247, 300, and 3109, Carignan and Freeman 1985; NGC 891, Bahcall 1983; NGC 3198, van Albada et al. 1985; NGC 4564, Casertano 1983a; NGC 5907, Casertano 1983b; see other contributions this volume), the same 1:1 ratio of dark-to-luminous matter is derived. This conclusion is discussed more fully by Bahcall and Casertano (1985). Finally, Bahcall (1984; see also Oort 1960) has shown that the mass required in the galactic disk in the solar neighborhood to account for the observed z-motions of stars exceeds by a factor of two that mass which is observed. However, there is as yet no evidence that this dark disk material is related to the more extended dark halo matter.

4.2. M(dark+lum) has an upper limit of order 10^{12}M⊙

It is exceptional for a spiral mass interior to R(25) to exceed 10^{12}M⊙. I have obtained rotation curves for two spirals for which Giovanelli et al. (1982; 1985) had discovered exceedingly high rotational velocities,

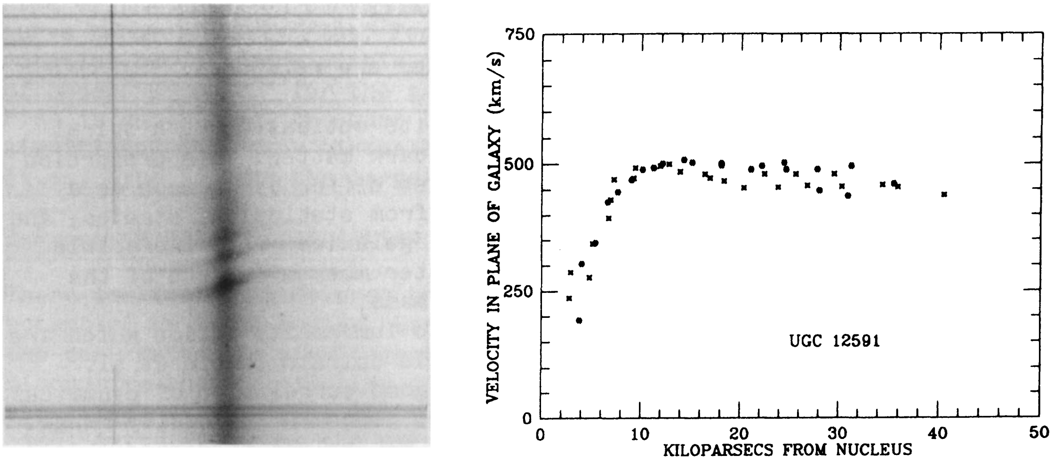
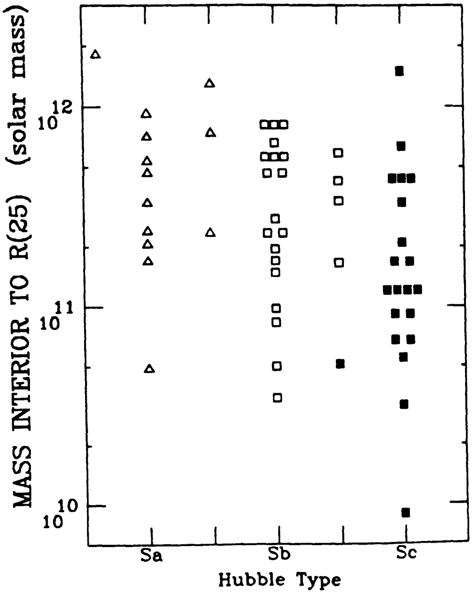


Figure 4. Left: Spectrum of UGC 12591 from a Palomar 200-inch CCD frame. Dispersion and scale 0.8A/pix and 0.59"/pix; integration 5400 sec. Hα emission is here bracketed by [NII]. Right: Rotation velocities for UGC 12591, the most rapidly rotating spiral disk known, from Hα and [NII]; different symbols refer to opposite sides of the galaxy.

Figure 5. Distribution of mass in spiral galaxies for which we have determined rotation curves. The normal spiral galaxies of highest mass are UGC 12591 (S0/a, Vmax = 500 km/s), NGC 669 (Sab, Vmax = 363 km/s), and UGC 2885 (Sc, Vmax = 304 km/s, but R(25) is over 100 kpc).



NGC 669 [Sab, V(max)=363 km/s] and UGC 12591, [S0/a, V(max)=500 km/s]. The rotation curve for UGC 12591 is shown in Figure 4. Even though its rotation velocity exceeds by 25% that of the next highest spiral, [IC 724, V(max)=374 km/s], the mass of UGC 12591 is not significantly in excess of 10^{12} M⊙ (Figure 5). A lower limit for spiral masses is probably not well established from our sample.

5. WHAT ROLE DOES THE ENVIRONMENT PLAY IN DETERMINING SPIRAL DYNAMICS?

5.1. Statistics of forms of rotation curves, and hence mass forms, differ for field and cluster spirals

It seems well established that field galaxies are embedded in massive, extended halos composed of nonluminous matter. What happens to halos of galaxies in clusters? Are they modified, or indeed, never given the opportunity to form, in the denser cluster environment? In an attempt to see if the dynamics of spirals in clusters differs from those of field spirals, Burstein, Whitmore, and Rubin are analyzing rotation curves of about 20 spirals in clusters: Peg I, Cancer, Hercules, and about 10 from the southern Dressler (1980) cluster DC1842-63 (Figure 6).

Individually, the rotation curves for cluster galaxies show the characteristic turnover and nominally flat outer portions that are observed for field spirals. Statistically, however, there is a difference in the distribution of galaxies among the three mass types, which we believe is environmentally produced. Of the 18 cluster galaxies with well defined mass types, 7 are Sc, 4 Sb, 3 Sa or Sab, 2 S..., and 2 distorted; a distribution not notably different from the field spirals. But whereas 50% of the field spirals show mass type I, the type with the

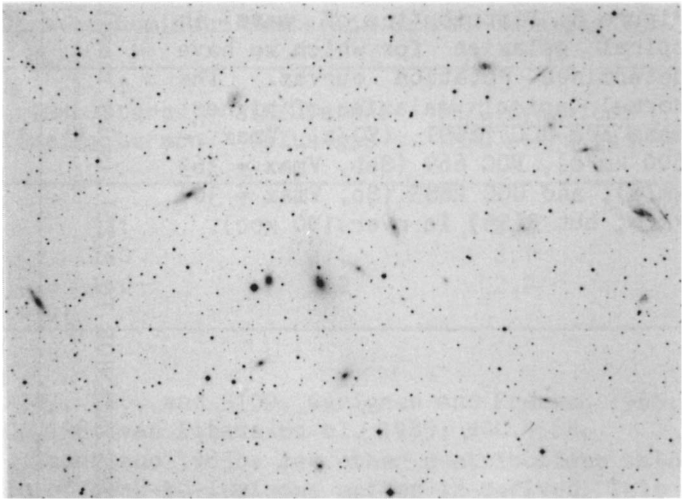


Figure 6. The central region of the southern Dressler cluster DC1842-63, from a Las Campanas 100-inch plate. Baked 103aO+GG385 filter, exposure 2 hours.

most rapid mass increase in the outer parts, none of the cluster spirals do. Conversely, while only 20% of field spirals are mass type III, the type with slowest mass increase, 50% of the cluster spirals are type III. Thus, cluster galaxies do not exhibit the slowly rising outer portions which are so characteristic of field spirals, but have flat or slowly decreasing velocities, evidence of a more rapid decrease in mass density with radius. A likely explanation is that the denser cluster environment has modified the distribution of dark matter surrounding the spirals which still exist there. In an effort to examine halo properties of spirals in a yet denser environment, we are presently obtaining rotation curves for galaxies in Hickson (1982) compact groups, where the galaxy surface density is as high as it is in the centers of the large clusters. Ultimately, mass forms may be an important clue to the evolutionary and environmental history of spiral galaxies.

5.2. What role does "dim matter" play in spiral galaxies?

In concluding this discussion of the properties of matter that we cannot see, I would like to call attention to the matter that we can almost not see. I refer to the low surface brightness "dim matter" which is not visible on the Palomar Sky Survey prints, the images from which many of us formed our ideas of what a galaxy is. On the Sky Survey prints, UGC 10205 appears as a bulge crossed by an indistinct absorption lane, sufficiently isolated and sufficiently similar to the Sombrero galaxy that we observed it spectroscopically in our sample of normal Sa galaxies. New CCD images (Figure 7) illustrate that there is more to UGC 10205 than appears on the Survey prints. At moderate light levels, UGC 10205 resembles a classic Sa galaxy seen edge-on; at lower light levels it reveals faint features like the shells and ripples which Malin and Carter (1980) and Schweizer and Ford (1984) have pointed out in Ellipticals and S0's. And at the faintest levels, barely 3% above sky, UGC 10205 shows streamers and features characteristic of Arp galaxies.

Until we understand the role of this dim matter, which we can see, in the formation and evolution of spirals, the properties of the dark matter will be difficult to illuminate.

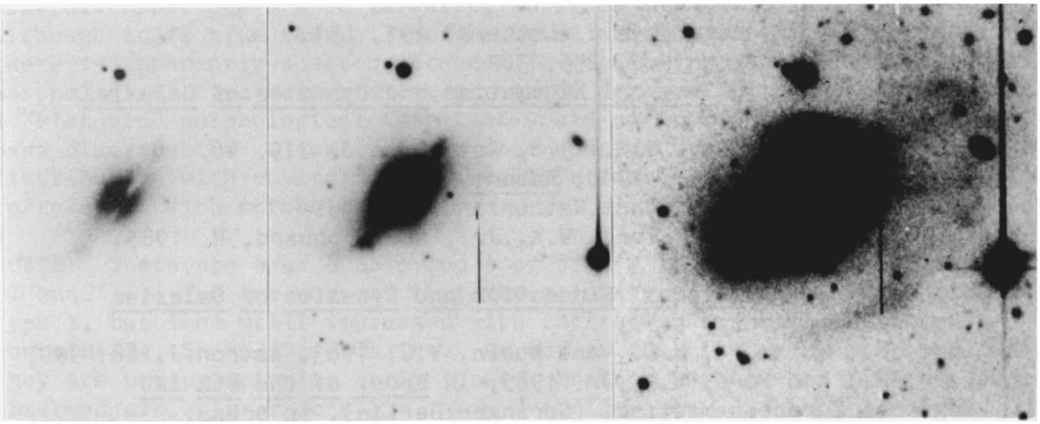


Figure 7. UGC 10205 from a KPNO 36-inch CCD + R (Mould filter) frame, showing successively lower surface brightness features. While at the highest light levels the galaxy resembles a fairly normal Sa, at low light levels it appears embedded in an extensive envelope, from which streamers and other faint features emerge.

ACKNOWLEDGEMENTS AND OTHER

I acknowledge with thanks the valued support of my observing colleagues, Kent Ford, Deidre Hunter, Paul Schechter, and Brad Whitmore and thank them especially for putting up with me on cloudy nights. Much of the work discussed here was done in collaboration with them and with Dave Burstein and Norbert Thonnard. Thanks also to the Directors of Cerro Tololo, Kitt Peak, Las Campanas and Palomar Observatories for observing time. Finally I wish to announce a Vmax Prize consisting of a $100 bill to the first astronomer who observes a spiral galaxy with a rotation velocity of 600 km/s (after correction for 1+z) or greater.

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DISCUSSION

DRESSLER: Your point 3.2 that mass distributions are not correlated with galaxy morphology seems overstated. A plurality of your Type I mass distributions are Sc galaxies, while Sa's dominate the Type III's. Although it is true that all morphologies are present for each type, there is apparently a strong trend. To me this suggests that the morphological type criteria are not quite correct, that there might be a "Platonic" morphological type that would correlate much better with mass distribution. I am further concerned that the correlation of mass distribution with environment is really a manifestation of this correlation with morphological type.

RUBIN: There are 4 or 5 Sb's and 4 or 5 Sa's in each of mass types I, II and III. You are correct that the majority of Sc galaxies have mass type I, but I am still impressed with the fact that each mass type contains Sa, Sb and Sc galaxies. As far as the cluster galaxies go, they are not principally early types. Most of the cluster galaxies we observed are Sb's or Sc's.

BURSTEIN: If one accepts our result on environmental differences in mass types, one has the following unclosed circle: a) morphology of spirals is correlated with environment; b) mass type is correlated with environment; but c) morphology is uncorrelated with mass type! How does the environment distinguish between luminous mass and total mass in the formation process of a spiral galaxy?

MILGROM: I wonder if it wouldn't be more convenient to use the density distribution. After all, the mass distribution is an integral of the density distribution, and so has little freedom to vary. It must start out at zero and go linear more or less asymptotically. So it is no surprise that all of the mass types look similar. If I understood correctly, the distinction between the types comes down to whether the rotation curve is flat or slightly rising.

RUBIN: There is also a difference in the inner parts. A rotation curve which is slightly rising in the outer parts is always rapidly rising in the inner parts. A rotation curve which has a shallow rise in the inner parts has a slight fall in the outer parts. You don't see any rotation curves which are crossovers between these two cases. So some mass types which are possible are not observed.

KORMENDY: For me, the most uncomfortable thing you said was your point 3.4 that dark matter is important at all radii. I think that this corollary would follow: The dark matter could probably not even be close to isothermal. If it were, and it had a density high enough to be important near the center, then (given typical bulge and disk scale lengths) the mass of the disk would be negligible. This would be uncomfortable, because then you couldn't make density waves, which we know are there. So I'd like to ask the people who make models: Is it possible, if you start with an isothermal halo and then embed within it a much smaller, high-density concentration, to pull the dark matter enough so that it becomes non-isothermal but important at all radii?

FABER: I don't think we fully know the answer. The best information we have comes from the adiabatic assumption, because that allows us to look at many different baryonic mass distributions. The adiabatic calculations suggest that at just about the degree of infall required to give a typical specific angular momentum, the final dark matter density equals the baryonic density within the optical radius. The amount of dark matter present before the infall was much smaller. That is, because of the squeezing effects, you have four or five times as much dark matter within the optical radius after infall as you did before. Further down toward the core, the dark matter density is smaller than the baryonic density. By the time you get to the core of an elliptical galaxy, the dark matter is really not important.

LYNDEN-BELL: I do not agree with Kormendy's point, for the following reason. It seems to me that what matters is the core radius. Just saying that something is isothermal doesn't tell you its core radius. Galaxies have roughly V = constant a long way in, and in principle you can make isothermal models with density $\propto r^{-2}$ all the way in to the middle. Then the dark matter would be important even at the center. So I think your statement is based on the idea that dark matter must be isothermal but with a large core radius. You didn't say that.

KORMENDY: That's true. I'm not used to thinking about singular isothermals because elliptical galaxies looked at carefully and dark matter distributions measured by decomposing rotation curves have all had cores. Even if dark matter can exist as singular isothermals, you can't afford to make the dark matter completely dominant at any radius where there is a disk. Otherwise, if the disk is massless, you can't make density waves.

PEEBLES: One could have the dominant mass in a spheroid and still have instabilities in the disk.

KORMENDY: Not if the disk is totally non-self-gravitating.

LYNDEN-BELL: Even if the disk is totally non-self-gravitating.

PEEBLES: Then some mechanism other than gravity would have to be making spiral arms.

LYNDEN-BELL: That's right (laughter).

FELTEN: Some of your observational points might be used to argue for or against alternative Milgrom-type theories of dynamics. For example, your result that mass types show strong environmental effects in clusters might be surprising in a Milgrom theory, in which there is no dark matter and all the matter is concentrated and tightly bound. Perhaps Milgrom will comment on this, now or later. Have you given any thought to whether some of these points can be used for or against unconventional dynamics?

RUBIN: No.

YAHIL: I want to go back to a question which Sandy posed this morning. What happens at small radii to the spheroidal component? If you take a reasonable mass-to-light ratio, e.g., $M/L \geqslant 1$, then you predict a higher velocity than is observed. Now, your calculations of the integral mass are made by assuming that $M \propto V^2R$. So it may be possible to resolve Sandy's paradox if you took the flatness of the disks into account. Also, what if there is a hole in the center of the disk?

RUBIN: It is hard to believe that disks have holes, because you observe emission lines right across their centers with no discontinuity. Observations of the Na D lines also coincide exactly with observations of the emission lines. I, too, have worried about rotation curves in the inner parts of galaxies - how accurately we know them, whether they reflect circular motions, etc. This last spring, Gallagher and Hunter took a large-scale, high-dispersion echelle plate of the inner regions of NGC 3198. Within the errors (~ 10 km s^{-1}) this very detailed rotation curve falls exactly in the HI rotation curve. This doesn't tell us that the motions are circular. But this very accurate rotation curve fits on the family curve. So if there are non-circular motions, they are fooling us, because they are similar from galaxy to galaxy and because the minor axis spectra reveal no peculiar motions.

STEIGMAN: Comparing the luminous mass to the total mass is comparing something derived from observations to something derived from theory. Am I supposed to believe the factor of two difference, or am I supposed to say, "Great agreement!"?

RUBIN: You're supposed to believe the factor of two difference. That is, if you build a galaxy out of stars, you get $M/L = 3$. If you measure its dynamics, you get $M/L = 6$.