GENERATION OF TORSIONAL OSCILLATIONS IN THE SUN

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ABSTRACT. The hypothesis is considered that the torsional wave observed on the Sun is an eigen-mode oscillation excited in the presence of a weak poloidal magnetic field. We derive asymptotic linear equations for a perturbation with a large number of nodes along the radius, assuming the rotation to be slow and the characteristic perturbation period to be much longer than the rotational period. The results of a preliminary numerical study of the stability of the torsional mode indicate that the superadiabaticity of the solar convection may contribute to the excitation of this mode. In the present work the approximation of harmonic radial dependence of the perturbation has been used.

1. Introduction

Recently Howard and LaBonte (1980) made a detailed analysis of data on the solar horizontal velocity field, and concluded that a solar torsional wave exists, which manifests itself as a modulation of the average rotational velocity. At a fixed latitude the velocity is changed with approximately an 11-yr period having an amplitude of close to 10 m s^{-1} (Howard and LaBonte, 1980; LaBonte and Howard, 1982). This wave is a travelling one showing strong symmetry with respect to the equator. The whole picture seems to be repeated with a 22-yr period.

Many authors consider the torsional wave as a dynamo wave (Yoshimura, 1981; Schüssler, 1981; Kleeorin and Ruzmaikin, 1984). However, the corresponding theory has not been worked out in detail since there have appeared difficulties, which solar dynamo theory itself has not yet overcome.

Wilson (1987), Snodgrass (1987a, 1987b), and Snodgrass and Wilson (1987) have a quite different point of view. They suggest that the torsional wave observed is not in fact an oscillation, but represents a modulation of the mean differential rotation caused by a system of giant azimuthal convective rolls with opposite direction of rotation in any two adjacent rolls of the same hemisphere. They also cite observational evidence that giant-cell convection in the Sun takes the form of equatorward migrating azimuthal rolls. However, it remains unclear whether a rotational velocity distribution exists which is self-maintained, and which satisfies the constraints of the observational data.

A suggestion that the solar torsional wave is excited due to the instability of an appropriate eigen-mode has been considered by Vandakurov (1988). In this case we need to assume that the Sun has a weak, steady, poloidal magnetic field not detectable by current observ-

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ing techniques. For the oscillations in question, a toroidal magnetic field is generated, the magnetic energy being transformed into kinetic energy and vice versa.

Torsional oscillations have been studied long ago by Walén (1948) and Layzer et al. (1955) in connection with a hypothesis that sunspot fields might represent loops of a toroidal magnetic field gererated by such oscillations and then being pulled up to the surface. However, the characteristic period of the torsional oscillations (for the fundamental mode with a magnetic field strength of around 2 G) turned out to be 25–100 times longer than the solar activity cycle (Layzer et al., 1955).

Nevertheless, the difficulty with the long oscillation period can be eliminated if the torsional mode has numerous nodes along the radius (Vandakurov, 1988). In this case the steady magnetic field is rather weak. An additional restriction of its value follows from the circumstance that in the presence of a steady poloidal magnetic field, an asymmetry should develop between the even- and odd-numbered solar cycles (Boyer and Levy, 1984; Pudovkin and Benevolenskaja, 1984). According to the latter two authors, a maximum value of 0.5 G for the dipole type field gives results consistent with the observations. Such a field can apparently be in accordance with the value 11 yr for the period of the torsional mode (Vandakurov, 1988).

The main question is whether the mode mentioned can be self-excited. This question is considered in the present paper. Asymptotic linear perturbed equations are derived, supposing that the perturbation has a large number of nodes along the radius, and that the torsional oscillation period is much longer than the stellar rotation period. We take into account different types of dissipation. Some results of this study have been discussed briefly in Vandakurov (1988).

2. Asymptotic perturbed equations

Let us assume invariance with respect to φ, the azimuth angle, and consider movements of a viscous, gravitating, compressible medium with finite conductivity in the presence of a magnetic field $\mathbf{B}$. We assume the pressure p and thermal flux $\mathbf{F}$ to be proportional to $\rho T/\mu$ and ∇T, respectively, where ρ is the density, T the temperature, and μ the molecular weight. A different expression for $\mathbf{F}$ will be considered later. In the following r, ϑ, φ are spherical coordinates, and $\mathbf{e}_r$, $\mathbf{e}_\vartheta$, and $\mathbf{e}_\varphi$ are unit vectors.

Let us now write down the φ-component of the equation of motion, as well as the div and $\mathbf{e}_\varphi\cdot$ curl of the same equation:

$$\mathbf{e}_\varphi \cdot (\mathrm{d}\mathbf{v}/\mathrm{d}t + \mathbf{B} \times \mathrm{curl}\ \mathbf{B}/4\pi\rho - \nu\nabla^2\mathbf{v}) = 0, \tag{1}$$

$$\mathrm{div}(\mathrm{d}\mathbf{v}/\mathrm{d}t + \nabla p/\rho + \mathbf{B} \times \mathrm{curl}\ \mathbf{B}/4\pi\rho - \nu\nabla^2\mathbf{v}) + 4\pi G_0\rho = 0, \tag{2}$$

$$\mathbf{e}_\varphi \cdot \mathrm{curl}(\mathrm{d}\mathbf{v}/\mathrm{d}t + \mathbf{B} \times \mathrm{curl}\ \mathbf{B}/4\pi\rho - \nu\nabla^2\mathbf{v}) - \mathbf{e}_\varphi \cdot (\nabla\rho \times \nabla p)/\rho^2 = 0. \tag{3}$$

The other basic equations are

$$\partial\rho/\partial t + \mathrm{div}(\rho\mathbf{v}) = 0, \tag{4}$$

$$\partial\mathbf{B}\partial t - \mathrm{curl}(\mathbf{v} \times \mathbf{B}) - \nu_B\nabla^2\ \mathbf{B} = 0, \tag{5}$$

$$\mathrm{d}p/\mathrm{d}t - (\gamma p/\rho)\mathrm{d}\rho/\mathrm{d}t - (\gamma - 1)(\rho\epsilon - \mathrm{div}\ \mathbf{F}) = 0, \tag{6}$$

$$\mathrm{d}\mu/\mathrm{d}t = 0. \tag{7}$$

Here $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \mathbf{v}\cdot\nabla$, ν and ν_B are kinematic and magnetic viscosities, and $\mathbf{v}$, G_0, γ, and ϵ denote, respectively, the velocity, gravitational constant, ratio of specific heats, and energy production.

We assume that in equilibrium the toroidal magnetic field is absent, and

$$\mathbf{v} = \mathbf{e}_\varphi r\Omega\sin\vartheta, \qquad B = B_0\mathbf{b}, \tag{8}$$

where Ω and B_0 are constant, and $\mathbf{b}$ is the dimensionless meridional vector. The equilibrium conditions follow from Eqs. (1)–(7) if we insert expressions (8). We do not write them down here.

To obtain the perturbed equations we insert, instead of ρ, $\mathbf{v}$, etc., $\rho+\rho^*$, $\mathbf{e}_\varphi r\Omega\sin\vartheta+\mathbf{v}^*$, etc., where $\rho^*(r,\vartheta,t)$ and $\mathbf{v}^*(r,\vartheta,t)$ are Eulerian components of the perturbation. In the linear approximation it follows from Eqs. (3) and (5) that

$$\frac{\partial v_\varphi^*}{\partial t} + 2\Omega(v_r^*\sin\vartheta + v_\vartheta^*\cos\vartheta) - \frac{B_0\mathbf{b}\cdot\nabla(B_\varphi^* r\sin\vartheta)}{4\pi\rho r\sin\vartheta} - \nu\; L(v_\varphi^*) = 0, \tag{9a}$$

$$\frac{\partial B_\varphi^*}{\partial t} - B_0 r\sin\vartheta\mathbf{b}\cdot\nabla(\frac{v_\varphi^*}{r\sin\vartheta}) - \nu_B\; L\;(B_\varphi^*) = 0, \tag{9b}$$

where

$$L = \frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{r^2\sin\vartheta}(\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} - \frac{1}{\sin\vartheta}).$$

To find the velocities v_r^* and v_ϑ^* in these equations, we need to use the equations of system (1)–(7). Let us write them in the approximations of slow rotation, very slow movements, and small h_*, the radial scale of the perturbation, i.e.,

$$r\Omega^2/g \ll 1, \qquad |\omega^2|/\Omega^2 \ll 1, \qquad h_*/h_p \ll 1, \tag{10}$$

where $g = -\mathrm{d}p/\rho\mathrm{d}r$ is the gravitational acceleration, and $h_p = -\mathrm{d}r/\mathrm{d}\ln p$ is the radial scale of the equilibrium pressure. If $\partial/\partial t \sim i\omega$, then the frequency ω will be of order $\Omega_B r/h_*$ as follows from Eq. (9), with $\Omega_B^2 = B_0^2/4\pi\rho r^2$.

One can see that the main terms in Eq. (3) are the last one and the one that contains the angular velocity Ω. A similar approximation for slow motion (but for the case of large Lorenz force) has been used by Taylor (1963). Note furthermore that from Eq. (2) we obtain the estimate

$$p^*/p \sim (h_*/h_p)(\rho^*/\rho). \tag{11}$$

Thus, neglecting small terms (but retaining those which are important at small r), we get

$$2\Omega\frac{\partial}{\partial\vartheta}(v_\varphi^*\sin\vartheta) - 2\Omega\cos\vartheta\frac{\partial}{\partial r}(rv_\varphi^*) = \frac{g}{\rho}\frac{\partial\rho^*}{\partial\vartheta}. \tag{12}$$

In addition, the equations of continuity and energy give

$$\frac{1}{r}\frac{\partial}{\partial r}r^2 v_r^* + \frac{1}{r\sin\vartheta}\frac{\partial}{\partial\vartheta}v_\vartheta^*\sin\vartheta = 0, \tag{13}$$

$$-\frac{\partial^2}{\partial t^2}\frac{\rho^*}{\rho} + \frac{a}{r}\frac{\partial v_r^*}{\partial t} - \Omega_F\left(\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} + \frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta}\right)\left(-\frac{\partial}{\partial t}\frac{\rho^*}{\rho} + \frac{sv_r^*}{r}\right) = 0, \tag{14}$$

where

$$a = \frac{1}{\gamma}\frac{\mathrm{d}\ln p}{\mathrm{d}\ln r} - \frac{\mathrm{d}\ln \rho}{\mathrm{d}\ln r}, \quad s = -\frac{\mathrm{d}\ln \mu}{\mathrm{d}\ln r}, \quad \Omega_F = -\frac{(\gamma-1)F_r\, T}{\gamma p r^2 (\mathrm{d}T/\mathrm{d}r)}.$$

Here we have taken into account that $\partial\mu^*/\partial t = s\mu v_r^*/r$, $T^*/T = -\rho^*/\rho + \mu^*/\mu$. Actually, all terms having an extra factor of order ω^2/Ω^2 or h_*/h_p have been omitted in Eqs. (12)–(14). However, we retain terms with a factor of order $r^2 h_p \Omega^2/g h_*^2$.

Eqs. (9), (12)–(14) constitute a system for the perturbation components $\mathbf{v}$, B_φ, and ρ^*. The boundary conditions are the following. Near the centre (under the condition that $b_r \neq 0$), the term with Ω in Eq. (9a) is small for the perturbation in question. Near the surface where ρ is small, this term is also small. Thus the boundary conditions are the same as those in the absence of rotation, i.e., at the boundaries $\partial(v_\varphi^*/r\sin\vartheta)/\partial r = 0$.

When deriving Eq. (14) we assumed that the heat flux is proportional to ∇T. In the convection zone where $a < 0$, this flux depends mainly on the entropy gradient. Let, for instance, F_r be proportional to $(-a)^\lambda$, where $a < 0$, and λ is a positive constant. Then, with our approximation,

$$F_r^* = (-\lambda r F_r/a)\nabla(\rho^*/\rho - p^*/\gamma p),$$

where the term $p^*/\gamma p$ is negligibly small. We see that Eq. (14) holds true if we put $s = 0$, and replace Ω_F by Ω_{FC}, where

$$\Omega_{FC} = -(\gamma-1)\lambda F_r/\gamma p r a. \tag{15}$$

3. Model with a radial steady magnetic field

A simple solution of the above equations may be found for the case of an idealized magnetic field distribution: $\mathbf{b} = b_r \mathbf{e}_r$, where $r^2 b_r = \text{const}$, and b_r is positive (negative) if $\vartheta < \pi/2$ ($\vartheta > \pi/2$). In this case, the field direction abruptly reverses when the equator is crossed. Besides, we retain in the equations only terms with the highest radial derivative of the perturbed quantities, assuming the latitudinal derivative not to be large. For example, we replace the operator L by $L_0 = \partial^2/\partial r^2$. In this approximation the term with $\partial v_\varphi^*/\partial\vartheta$ in Eq.(12) may be omitted. The solution of Eqs. (9), (12), and (13) may be expressed in the following form:

$$v_\varphi^* = [r^2(1-y^2)^{1/2}/y]\;\Lambda\;(\partial^2 E/\partial r\partial y), \tag{16}$$

$$v_\vartheta^* = [r^2(1-y^2)^{1/2}/2y^2\Omega]\;K\;(\partial^2 E/\partial r\partial y), \tag{17}$$

$$v_r^* = (r/2\Omega)(\partial/\partial y)\left\{[(1-y^2)/y^2]\;K\;(\partial E/\partial y)\right\}, \tag{18}$$

$$B_\varphi^* = [r^2 b_r B_0(1-y^2)^{1/2}/y]\;L_0\;(\partial E/\partial y), \tag{19}$$

$$\rho^*/\rho = (2r^3\Omega/g)\;\Lambda L_0\;(E), \tag{20}$$

where $K = r^2 b_r^2 \Omega_B^2 L_0 - \Lambda(\partial/\partial t - \nu L_0)$, $\Lambda = \partial/\partial t - \nu_B L_0$, $y = \cos\vartheta$, and E is a dimensionless function of r, y, and t. Substitution of these expressions in the energy equation (14) yields

$$\left(a\frac{\partial}{\partial t} - s r^2\Omega_F L_0\right)\frac{\partial}{\partial y}\left[\frac{1-y^2}{y^2}K\left(\frac{\partial E}{\partial y}\right)\right] - q r^2\left(\frac{\partial}{\partial t} - r^2\Omega_F L_0\right)\Lambda L_0\left(\frac{\partial E}{\partial t}\right) = 0. \tag{21}$$

Here $q = 4r\Omega^2/g$.

One can see that the solution of this equation for one mode is

$$E(r, y, t) = Y(y)\exp(ikr + i\omega t). \tag{22}$$

Then

$$\frac{\mathrm{d}}{\mathrm{d}y}\left[\frac{1-y^2}{y^2}\frac{\mathrm{d}Y}{\mathrm{d}y}\right] = -j^2 Y, \tag{23}$$

where j = const., while the frequency ω satisfies a cubic equation studied in Vandakurov (1988). In this paper Eq. (23) has also been investigated.

If ν and ν_B are small, the stability condition is (Vandakurov, 1988)

$$\Omega_B^2 \Omega q(a - s) \geq 0, \tag{24}$$

i.e., thermal dissipation in zones with a superadiabatic temperature gradient serves to self-excite torsional modes with numerous nodes along the radius. In contrast, both ordinary and magnetic viscosity tend to dampen these modes (Vandakurov, 1988).

4. Approximation of harmonic dependence of the perturbation on radius

The equilibrium magnetic field studied in the preceding section had a steplike change near the equator. For a more realistic field distribution, one needs to solve the complicated system of equations in partial derivatives. Since we study perturbations having many radial nodes, the dependence on the boundary conditions becomes of small significance. Then, to form a general concept of the stabilizing or destabilizing contribution of some layer, it seems sufficient to use the approximation of harmonic radial dependence of the perturbation. Thus we assume the perturbation to be proportional to $\exp(ikr + i\omega t)$, where k and ω are constant. Besides, we do not use the approximation that the latitudinal derivative of the perturbation is much smaller than the radial one. Furthermore, we assume the equilibrium magnetic field to be

$$b_r = 2\cos\vartheta, \qquad b_\vartheta = -\beta \sin\vartheta. \tag{25}$$

Here $\beta = \partial \ln(r^2 b_r)/\partial \ln r$, and $|\beta/kr| \ll 1$. Eliminating B_φ^* from Eq. (9), we find

$$\begin{aligned}\left(1 + \frac{r^2\beta^2\Omega_B^2}{i\omega\nu}\sin^2\vartheta\right)\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial W}{\partial\vartheta} &= \frac{2r^2\Omega}{\nu}(w_r^* + w_\vartheta^*\cot\vartheta) - \\ -2\cot\vartheta\frac{\partial W}{\partial\vartheta} + k^2r^2W + \frac{2kr^3\Omega_B^2}{\omega\nu}&\left[\beta\sin 2\vartheta\frac{\partial W}{\partial\vartheta} - (2ikr\cos^2\vartheta + \beta)W\right],\end{aligned} \tag{26}$$

$$\frac{\Omega_F}{i\omega\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial\Theta}{\partial\vartheta} = \left(1 + \frac{k^2r^2\Omega_F}{i\omega}\right)\Theta + (a - s)w_r^*, \tag{27}$$

$$s\frac{\partial w_r^*}{\partial\vartheta} = \frac{\partial\Theta}{\partial\vartheta} + \frac{2i\omega r\Omega\sin\vartheta}{g}\left(\sin\vartheta\frac{\partial W}{\partial\vartheta} - ikr\cos\vartheta\, W\right), \tag{28}$$

$$\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\; w_v^* = -ikrw_r^*, \tag{29}$$

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where $\mathbf{w}^* = \mathbf{v}^*/i\omega r$, $W = w_\varphi^*/\sin\vartheta$, $\Theta = T^*/T$. Here we put $\nu_B \approx 0$.

These equations are equivalent to six first-order differential equations for six variables: W, $\partial W/\partial\vartheta$, w_r^*, w_ϑ^*, Θ, and $\partial\Theta/\partial\vartheta$. In the vicinity of the polar axis ($\vartheta = 0$, or $\vartheta = \pi$), the following expansions are valid:

$$W = W_0 + W_2 \sin^2\vartheta + \ldots, \quad w_r^* = V_0 + V_2 \sin^2\vartheta + \ldots, \quad \Theta = \Theta_0 + \Theta_2 \sin^2\vartheta + \ldots, \tag{30}$$

where

$$W_2 = \frac{1}{8}\left[k^2r^2\left(1 - \frac{4ir^2\Omega_B^2}{\omega\nu}\right) W_0 - \frac{r^2\Omega}{\nu}(ikr - 2)\, V_0\right], \tag{31}$$

$$\Theta_2 = \frac{i\omega}{4\Omega_F}\left[(a - s)V_0 + \left(1 + \frac{k^2r^2\Omega_F}{i\omega}\right) \Theta_0\right], \tag{32}$$

$$V_2 = \frac{1}{s}\left(\Theta_2 + \frac{kr^2\omega\Omega}{g}\, W_0\right). \tag{33}$$

Thus the constants W_0, V_0, and Θ_0 remain undetermined. This fact permits us to construct three independent solutions. The whole solution calculated with the initial point at $\vartheta = 0$ coincides with that found with the initial point at $\vartheta = \pi$ if at the equator ($\vartheta = \pi/2$) the quantities $\partial W/\partial\vartheta$, $\partial\Theta/\partial\vartheta$, and w_ϑ^* are zero. These conditions give three linear algebraic equations for W_0, V_0, and Θ_0. The condition that these three equations are solvable,

$$D(\omega) = 0, \tag{34}$$

provides an equation for the complex eigenvalue ω.

5. The case of a chemically homogeneous medium

If $s = 0$, Eqs. (26)–(29) need some modification. Differentiating Eqs. (26) and (27) with respect to ϑ, and excluding (using also Eq. (28)) the derivatives $\partial\Theta/\partial\vartheta$ and $\partial w_r^*/\partial\vartheta$, we find

$$\begin{aligned}
&\left[\frac{\nu}{r^2} + \left(\frac{\beta^2\Omega_b^2}{i\omega} + \frac{4r\Omega^2\Omega_F}{ag}\right)\sin^2\vartheta\right] \frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial W}{\partial\vartheta} = \\
&= 4kr\left(\frac{\beta\Omega_B^2}{\omega} + \frac{ir\Omega^2\Omega_F}{ag}\right)\sin^2\vartheta\cos\vartheta\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial W}{\partial\vartheta} + \\
&+ \left\{\nu k^2\sin^2\vartheta - \frac{2kr\Omega_B^2}{\omega}(2ikr\cos^2\vartheta + 3\beta)\sin^2\vartheta + \right. \\
&\left. + \frac{4r\Omega^2}{ag}[(i\omega + k^2r^2\Omega_F)\sin^2\vartheta + 2ikr\Omega_F]\sin^2\vartheta\right\}\frac{\partial W}{\partial\vartheta} + \\
&+ 4ikr\left[\frac{2kr}{\omega}\Omega_B^2 - \frac{r\Omega^2}{ag}(i\omega + k^2r^2\Omega_F)\right]\sin^3\vartheta\cos\vartheta\, W - \\
&- \Omega\left[ikrw_r^*\sin 2\vartheta - 2w_\vartheta^*(\sin^2\vartheta - 2)\right].
\end{aligned} \tag{35}$$

Now Eqs. (35) and (29) reduce to a system of four first-order differential equations for W, $\sin\vartheta\ \partial W/\partial\vartheta$, $\sin\vartheta\ (\partial/\partial\vartheta)(\sin\vartheta\ \partial W/\partial\vartheta)$, and w_ϑ^*. The velocity $i\omega r w_r^*$ in these equations is determined by Eq. (26). The expansions in the vicinity of the polar axis turn out to be given by the first two expressions in Eq. (30), in which the constant W_2 is determined by Eq. (31). One can see that the constants V_0 and W_0 are arbitrary, so by setting the variables $\partial W/\partial\vartheta$ and w^*_ϑ at the equator ($\vartheta = \pi/2$) equal to zero, we find two equations for V_0 and W_0. The determinant $D_*(\omega)$ of these two equations should be equal to zero, i.e., the equation for ω is

$$D_*(\omega) = 0 \tag{36}$$

6. Numerical study of the unstable modes

The excitation of the torsional wave observed on the Sun can occur due to the superadiabaticity of the solar convection zone. In general the approximation that the perturbation has a large number of nodes along the radial direction is not well-founded in the case of the convection zone (Vandakurov, 1988). Nevertheless, some preliminary estimates can be made using Eqs. (35), (29), and (26).

We have carried out a numerical solution of these equations considering the convection zone as a chemically homogeneous medium with a superadiabatic temperature gradient ($a < 0$), with a turbulent viscosity, and, of course, with a convective thermal conductivity. We choose the following values for the parameters: $kr = 10$, $a = -10^{-5}$, $\nu = 1.3 \times 10^{12}\text{cm}^2\ \text{s}^{-1}$, $\Omega_{FC} = 5\nu$, $r = 6 \times 10^{10}\text{cm}$, $r\Omega = 2 \times 10^5\text{cm s}^{-1}$, $r\Omega^2/g = 2 \times 10^{-5}$, $\beta = 0.5$, $\Omega_B^2 = 9.72 \times 10^{-19}\alpha_B$, where α_B is either equal to 1 or to 0.1. If $\rho = 0.001\ g\ \text{cm}^{-3}$, these values of α_B correspond to 6.6 G (if $\alpha_B = 1$) and 2.1 G ($\alpha_B = 0.1$). In the case of smaller values of α_B, the computation becomes more time-consuming.

Complex solutions of Eq. (36) were found by the Newton method generalized to cover the case of two-dimensional variables. We searched only for a solution with a positive real part of the quantity $i\omega$. Such solutions imply instability. Note that attempts to find similar solutions for the case of some models with positive values of a did not succeed.

It turns out that the dependence of $D_*(\omega)$ on some trial values of ω is extremely complicated, so the procedure mentioned is convergent only if the trial ω-value is sufficiently close to an eigen-solution. Under the conditions $\vartheta_{st} = 0.006$ and $\alpha_B = 1$, we found the solution $i\omega = (0.5285 - i\ 0.0116) \times 10^{-8}\ \text{s}^{-1}$, where ϑ_{st} is the initial value of the angle ϑ. For other small values of ϑ_{st}, the quantity $i\omega$ may differ from the above value by several percent, and fixing $i\omega$ exactly appears to be rather troublesome. However, the latitudinal dependence of the perturbation undergoes only minor changes during the procedure of making the frequency ω more accurate. The dependence of w^*_ϑ on latitude is shown in Figure 1. Here we assume that at the pole ($\vartheta = 0$) W is unity. In the region of $\vartheta \gtrsim 20°$ the perturbation amplitude is very small (if $\vartheta = 90°$, then $w^*_\varphi = 9 \times 10^{-9}(1 + i)$, and the real (imaginary) part of w^*_φ goes through zero at $\vartheta = 74°(80°)$).

The radial velocity v^*_r has many nodes in the vicinity and to the left of the point $\vartheta = \vartheta_*$, where $\vartheta_* = 9°.1$. This is because the coefficient in brackets on the left-hand side of Eq. (35) is small. If $\vartheta < \vartheta_*$, then the closer ϑ is to ϑ_*, the larger is w^*_r, with a maximum value as large as $-5220 + i\ 2590$. We do not know whether these large values of w^*_r are consistent with our approximations or not.

In the case that $\alpha_B = 0.1$ we found a solution $i\omega = (0.08145 - i\ 0.00538) \times 10^{-8}\ \text{s}^{-1}$ which apparently belongs to the same mode as that considered above. These solutions correspond to nearly exponentially growing modes with a characteristic growth time of the order of several years. Overstable modes are possible if there are zones in which the perturbation is propagating. Thus the study of models having not only convective but also radiative zones is needed.

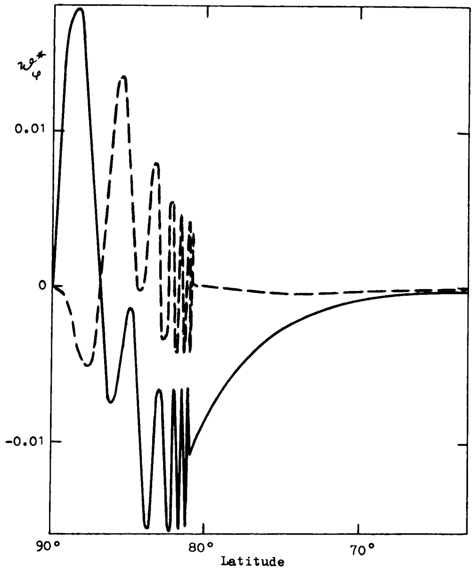

Figure 1. Real (solid curve) and imaginary (dashed curve) parts of w^*_φ, the dimensionless azimuthal displacement, as functions of $\pi/2 - \vartheta$, the latitude.

Note in conlusion that the aforementioned nearly exponentially growing unstable mode can coexist together with the torsional wave found by Howard and LaBonte (1980). We suggest that this mode having large radial velocities in some zones near the poles is the cause of the solar activity observed at high latitudes (Makarov and Sivaraman, 1989). We may relate the existence of such a mode to the weak polar poloidal magnetic fields whose direction reverses periodically. Then the growth of the instability is supposed to begin after the new polar fields have formed. The growth time can be smaller than the cycle duration if the parameter Ω_B is larger than in the previous examples.

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