

Precision versus Accuracy: some astrophysical and operational limitations

R. E. M. Griffin¹

Oxford, UK

Abstract. The measurement of accurate radial velocities for A- or B-type dwarfs poses a separate category of problems: the small number of suitable lines, the wide wavelength range over which such lines are distributed, thermal broadening, rotational broadening, line distortions caused by rotating spots, spectrum variations due to obvious duplicity, and low-level velocity variations due to undetected companion spectra. Moreover, the fact that A- and B-type spectra fall into very distinct groups, each with its own sub-set of the above problems, means that it may be both difficult and unsatisfactory to specify velocity standards, since intrinsic uncertainties caused by differences in spectral type may exceed the measuring errors. The investigation summarized here into the nature and magnitude of some of these intrinsic errors employs wide spans of high-resolution, high-dispersion spectra of Sirius and Vega as ‘natural’ templates. Attention is also drawn to systematic errors which may arise (a) when modelling a cross-correlation ‘dip’ and (b) whenever a spectrum or a cross-correlation dip is measured in the unavoidable presence of another such spectrum or dip.

1. General

The terms *precision* and *accuracy* are frequently interchanged. They will be used here in the strict statistical sense, namely:

- accuracy (A) is the nearness of your result is to the true answer;
- precision (P) describes the size of the scatter of your attempts.

Precision is therefore a measure of the quality of the measurements, but accuracy involves a precise identity of what is being measured. Precision is roughly analogous to random errors, and may be improved by concentrating on techniques; accuracy largely concerns systematic errors, and may invoke astrophysics. If $P > A$, experiments to reduce P are under control and meaningful. But if $A > P$, experiments to reduce P are not well defined.

Describing radial velocities as ‘precise’ assumes that the quantity being measured is open to unambiguous interpretation, yet Dainis Dravins has already

¹Visitor, Department of Physics, University of Oxford, UK

mentioned conditions under which that assumption breaks down. It is therefore important to sift assiduously all the information which a measured Doppler shift may contain.

This paper touches on three different but often related aspects of radial-velocity determinations: (a) instrumentation issues, where the formal precision can be swamped by external causes that have been overlooked, (b) factors influencing cross-correlation results, and (c) binary spectra. It does not so much offer solutions as identify problems.

2. Effects of instrumentation

2.1. Modelling or measuring a cross-correlation 'dip'

Measurements of Coravel-type dips are frequently carried out using the assumption that the shape of the dip can be adequately described by a gaussian, implying that the dip itself is symmetrical and that the continuum outside it is level. However, that may not necessarily be true. The characteristics of the mask may cause the continuum level to swerve permanently (Fig. 1a), or to change in slope occasionally; changes in adjustment of the instrument can also give rise to a slope in the continuum. Unless such possibilities are investigated rigorously and checked frequently, and adequate representation of the instrumental profile is included in the reduction procedures, systematic errors are likely, possibly with a spectral-type dependence. Even if only the far wings of a dip profile deviate significantly from a mathematical expression, it is still advisable to employ an empirical version of the actual dip profile when determining radial velocities. Velocities derived from double- or multiple-lined dips (e.g. as in Fig. 1b) will be particularly noticeably affected, but since those errors will depend on the nature of the dips and their mutual separation the effects will appear random rather than systematic. The centroid of a dip that is substantially broadened by rotation will be displaced systematically if the continuum is not flat; limitations to the *accuracy* of such a measurement are obvious if the dip is broad enough to extend to one end of the scan (Fig. 1c).

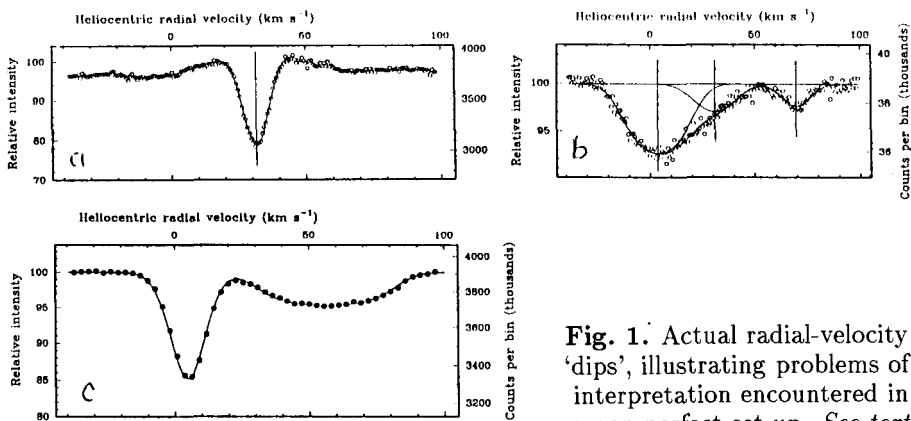


Fig. 1. Actual radial-velocity 'dips', illustrating problems of interpretation encountered in a non-perfect set-up. *See text*

2.2. Modelling the spectrograph

In the context of this meeting, 'precise' is defined as 100 m/s or better, so I may refer to a paper in 1973 by Griffin & Griffin which was one of the first to discuss the possibility of determining stellar radial velocities with a precision of 10 m/s. That paper applied two unusual concepts: the use of telluric lines as the wavelength reference frame (a principle incorporated into the HF-cell and iodine-cell techniques), and the use of the grating equation, instead of an arbitrary polynomial, to calculate a wavelength scale. Although our spectra were not specially optimized for the task, they nevertheless yielded radial velocities for Arcturus and Procyon with internal precisions (r.m.s.) of a few tens of m/s.

The grating equation works with 3 unknowns: the incident angle (A) on the grating, the focal length (F.L.) of the camera, and a constant related to the zero of the wavelength scale. It takes as input the linear positions of lines in the raw spectrum, and their wavelengths (which should be homogeneous – measured solar values or high-quality laboratory data). It adjusts the angle of incidence by minimizing the wavelength residuals, and produces a wavelength scale that fits all points optimally; it also yields the focal length of the camera.

The grating equation works well for classical grating systems. I have also applied it successfully to separate orders of échelle spectra, though the wavelength range there is insufficient to derive the full benefits of the method. Mostly I have used a detector 20 inches long and divided physically into two halves, with a wavelength range between about 200 and 800 Å. The grating equation describes precisely the optical path, and its application relies only on knowledge of the physical quantities mentioned, so one would intuitively expect the two separate halves of the recorded spectra to give identical results for the focal length, within the limits of random errors. But they don't; see Fig. 2.

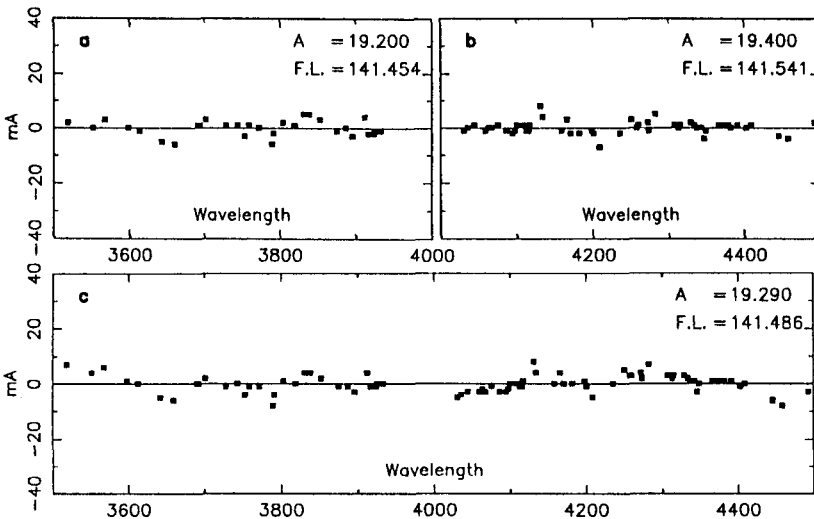


Fig. 2. Wavelength residuals from the grating equation, applied to two halves of an exposure (**a** and **b**) and to the whole exposure (**c**). The separate values of A and F.L. differ sufficiently that the combined solution shows curvature.

If I force them together, the separate halves appear to be bowed away from linearity by up to about $5 \text{ m}\text{\AA}$ (400 m/s in the blue), and always in the same sense. The precision of the measured wavelengths is therefore destroyed by a lack of conformity in some physical or optical property; there may be departures from the spherical surface which the grating equation assumes, or imperfections in the spectrograph or detector holder. I have found the same problem in 3 out of 3 different coude systems examined in this way.

3. Factors influencing the accuracy of the CCF

Apart from roAp stars, early spectral types have so far been conspicuous in this meeting by their absence. There are important astrophysical applications that call for accurate velocities of early types, e.g. the dynamics of stars in young clusters, the velocity of Galactic spiral arms, the mass ratios of the components of binary stars. However, because of the nature of early-type spectra there may be sufficient correspondence for reliable cross-correlation between some spectra that are one class apart in type, yet major differences between CP types even within a sub-class; cross-correlating with selected digital templates is therefore likely to be more 'accurate' than with the physical mask that is common in instruments for measuring late-type stars.

In order to investigate the scale of the problems encountered with early-type stars, and thus to design a workable solution for achieving optimum conditions for cross-correlations, I am making a series of experiments using real stellar spectra as the templates. The objectives are to distinguish between random and systematic errors, and to assess the importance of mismatch between spectral type, luminosity, rotational broadening and composition differences. Real spectra are free from uncertainties in tabulated wavelengths, and have line shapes and intensities that are actually encountered. The spectral *appearance*, rather than the exact temperature of a model which may simulate parts of it, is regarded as the better guide. Indeed, the Coravel mask is based on a real spectrum; the original Cambridge RV spectrometer used an imprint of the spectrum of Arcturus, but with line intensities truncated at 50% of the continuum height and the contrast made infinite, i.e. a sequence of slots. The power in the CCF comes from the deepest parts of lines, and its contrast from the slopes of lines used. I therefore tried truncating my early-type spectra at different percentages of the continuum. I also tried isolating line types: Balmer lines, metallic lines, Ca II K line or Mg II $\lambda 4481 \text{ \AA}$. All the spectra used were sampled in steps of $10 \text{ m}\text{\AA}$. The wavelength scale was in the rest frame of the star, so an accurate CCF should have its peak precisely at zero.

Conclusions from a pilot study are summarized below. They consider intrinsic precision (which is presumably controlled by noise), effects of mis-match of spectral type, of resolution, and the influence of a faint secondary. A full study, to include other homogeneous sets of early-type spectra, is planned and should refine the results quantitatively.

3.1. Minimum errors

The intrinsic precision (basic random error) was estimated by cross-correlating individual very high-resolution, high S/N spectra of Sirius against one another;

a typical error was 1–2 mÅ (about 120 m/s). An amalgamation of those spectra constituted the template for the investigations of program stars. However, cross-correlations of the Sirius template with spectra of Vega of similar quality produced discrepancies of 2–3 times the random error just quoted, and appeared to be systematic. The K line in Sirius, for example, showed a consistent blue-shift by 450–550 m/s, and examination of the line profile revealed a distinct asymmetry attributable to extra absorption on the blue edge of the core. Weak metallic lines, on the other hand, gave only small CCF displacements, mostly well below 100 m/s.

3.2. Spectral-type mismatch

A random sample of high-dispersion, sharp-lined spectra ranging from B2 V to A1m was selected from the DAO plate archive and digitized. Spectra were extracted in steps of 0.01 Å in the stellar rest-frame, and cross-correlated with the Sirius template. The results indicated that individual differences between spectra, especially idiosyncracies of CP types, appear to be at least as influential in affecting the peak of the CCF as do gross differences in spectral type. Somewhat unfortunately, three of the program stars proved to be SB systems, and their CCF peaks gave deviations from zero that showed strong systematic trends up to about 800 m/s with increasingly low cut-off levels, implying that it was the cores of the Balmer lines that were most affected in those particular examples. All cases gave bad discrepancies for the K line, and not always of the same sign. More examples must be tested before a general statement can be made about the behaviour of this line at different spectral types; however, its strength varies so considerably with temperature that it should probably be avoided altogether when high-precision radial velocities are required.

3.3. Resolution

Cross-correlations between lower-dispersion spectra of Sirius and smoothed versions of the program spectra showed marked deviations from zero for the strong lines but not for the weak lines. Moreover, when I blurred the Sirius spectrum such that its rounded lines were as wide as the lines in Vega, which tend to have somewhat square profiles, the CCFs from all types of lines were then displaced by nearly 100 m/s.

3.4. Influence of an undetected secondary

To investigate the extent to which an unsuspected companion could affect the position of the CCF peak, a grid of composite spectra was constructed in which small fractions of high-quality spectra of Vega were added to another set of Sirius spectra, constituting a series of companions with $\Delta m = 2.5$ to 5.0. For each value of Δm the secondary was displaced in velocity in steps from 80 m/s to 400 m/s. Cross-correlations with the standard Sirius template indicated that the companion must be 5 mag. fainter than the primary before its presence has negligible effects upon the CCF. However, such a model has little counterpart in reality, so another grid of composite spectra was constructed in which the secondary was a faint F dwarf (spectra from the *Procyon Atlas* were used). Cross-correlations with the standard Sirius template showed that an early A-

type primary can have an F-type dwarf secondary no brighter than $\Delta m = 3$ without affecting the velocity measured for the primary.

4. Binary spectra: CCF measurements on composite spectra

The masses of single stars are extremely difficult to determine with any acceptable precision, but the mass *ratio* of the components of spectroscopic binaries can be determined directly from their radial-velocity ratio. To determine the masses of representative giants, therefore, we select composite binaries containing a cool giant primary and a hot dwarf secondary, and measure their relative displacement in velocity. The only assumption to be made is the mass of the secondary, which has to be inferred from its spectral type. Unfortunately, any measurement of the velocity of the secondary in the presence of the primary is extremely unreliable, so I have developed a technique for isolating the secondary's spectrum by subtracting away that of the dominating primary. However, unless one has an exact match to the primary spectrum (and only the rare ζ Aur binaries offer an opportunity to obtain the spectrum of the primary, during total eclipse of the secondary) then the uncovered secondary spectrum is likely to contain weak shadows of parts of the primary spectrum.

Tests carried out warn unambiguously that, unless the secondary spectrum is cleanly separated, its measured velocity can be affected substantially by a systematic error due to the presence of such shadows. The nature of the error will probably (but not necessarily) be such as to pull the result towards zero. The error will of course be completely passed over to the mass ratio. The broad features of many of these early-type secondary spectra also expose the measurements of their CCFs to the specific difficulties mentioned in section 2.1 above.

5. Additional comments

Astrophysical factors that can influence the instantaneous position of a line centroid have already been discussed by other speakers. Granulation, pulsations and oscillations operate on a different scale according to luminosity, magnetic activity, composition or atmospheric stratification. Absorption lines are formed in *dynamic* equilibrium within a maze of local surface velocity fields and macroscopic forces, and it will be necessary to model the nett effects of such influences realistically in individual cases before the bodily displacement of line centroids can be accurately interpreted as due to a velocity of the star as a solid body. We are still some distance from that situation, i.e. $A > P$, and this must represent an important constraint on the achievement of *precise* radial velocities.

Acknowledgments. I am grateful for financial support from the University of Antwerp, where these studies were initiated, and to Marc David, Werner Verscheuren and Herman Hensberge individually for their time and patience.