STABILITY OF A MASSIVE CURRENT SHEET SUPPORTED BY A TWO-DIMENSIONAL POTENTIAL MAGNETIC FIELD.

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ABSTRACT. Necessary and sufficient criteria of linear ideal magnetohydrodynamic stability are derived for a massive current sheet - idealizing a solar prominence - supported against gravity by a two-dimensional potential magnetic field.

1. INTRODUCTION

A solar prominence is often idealized by a vertical infinitely thin massive current sheet extending in the "coronal half-space" $\{z > 0\}$ parallel to the x-axis and in equilibrium under the opposite actions of a uniform vertical gravitational field and of an x-invariant potential magnetic field (see e.g. the review by Anzer, 1989). In this communication, we analyze the linear ideal magnetohydrodynamic stability of this simple model, taking into account the stabilizing influence (neglected in the earlier work by Anzer, 1969), of the low-beta coronal plasma.

2. MODEL

We consider a simple model of solar prominence in which : i) the corona is represented by the half-space $D = \{z > 0\}$ (in a cartesian frame (Oxyz)) ; D is filled up with a pressureless perfectly conducting plasma ; ii) the prominence is represented by the strip : $\Sigma = \{-\infty < x < +\infty \; ; \; y = 0 \; ; \; 0 < a \leq z \leq b\}$; Σ is made of perfectly conducting matter having a mass density $\sigma(z)$ (satisfying $\sigma(a) = \sigma(b) = 0$) ; iii) Σ is in equilibrium under the opposite actions of a uniform vertical gravitational field $\underset{\sim}{g} = - g \hat{z}$ $(g > 0)$ and of an x-invariant magnetic field $\underset{\sim}{B}(y,z) = \nabla A(y,z) \times \hat{x}$ which is y-symmetric $(A(-y,z) = A(y,z))$, has finite energy per unit of x-length, and is potential outside Σ ($\Delta A = 0$ in D/Σ) ; thus $2\pi \sigma g = B_y B_z^+$ on Σ $(B_z^+ = B_z(0^+,z))$.

E. R. Priest and V. Krishan (eds.), Basic Plasma Processes on the Sun, 341–342.

3. IDEAL LINEAR STABILITY OF THE MODEL

Let us apply to the configuration we have just described a displacement ξ (x,y,z) vanishing on $\partial D = \{z = 0\}$ (line-tying assumption) and periodic in x. By computing the resulting average variation of the energy of the system per unit of x-length and using the classical "energy principle", it may be shown that :

i) the equilibrium is stable with respect to any perturbation which does not depend on x (this result is still true if D/Σ is a vacuum) ;

ii) the equilibrium is stable with respect to any arbitrary perturbation <u>if and only if</u> one has at all the points of Σ

$$K \, \partial \, |B_y| \, / \, \partial z \leq 2\pi \, \sigma \, g \tag{1}$$

$$-K \, \partial \, |B_z^+| \, / \, \partial z \leq B_y^2 \, , \tag{2}$$

where $K(z) = \int_{\mathcal{C}(z)} B \, ds$ ($\mathcal{C}(z)$) is a field line of B connecting a point of Σ at height z to the plane ∂D ; it may be shown that all the lines of B necessarily cut ∂D). The positive RHS of Eq. (1) and (2) are related to the stabilizing effect resulting from the lines of B being frozen in the plasma in D (thus any displacement of Σ in the x-direction necessarily leads to a bending of the lines). Would D/Σ be a vacuum, both terms would be replaced by zero (Anzer, 1969).

4. CONCLUSION

For a massive current sheet supported by a 2D potential magnetic field, we have derived stability criteria which are less severe than Anzer's. When Anzer's criteria predicted that all the models with a > 0 considered here are unstable, ours indicated indeed that stability holds at least in some cases, e.g. when σ is not too large and the lines of the "unperturbed" field B_0 (potential in the whole D and satisfying $B_{0z}(y,0) = B_z(y,0)$) have a "dip" at the points where they cut Σ (such a situation may be encountered above a quadrupolar region).

5. REFERENCES

Anzer, U. (1969). "Stability analysis of the Kippenhahn - Schlüter model of solar filaments", Solar Phys. 108, 26.

Anzer, U. (1989). "Structure and equilibrium of prominences", in E.R. Priest (ed.), Dynamics and Structure of Quiescent Solar Prominences, Kluwer Academic Publishers, Dordrecht, p. 143.