NUMERICAL SIMULATIONS OF ENCOUNTERS OF HARD BINARIES

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ABSTRACT. Results from numerical integrations of random binary-binary encounters have been used to obtain various cross-sections and outcome distributions for the four-body scattering. The initial orbital elements were chosen randomly except the Kepler-energies for which various selected values were used. Rough estimates for mass effects were obtained by simulating encounters of binaries with unequal component masses.

We developed a semi-analytical theory for obtaining the types and energies of the outcome configurations. The theory contains some adjustable parameters, the values of which we deduced by comparing the theory and experiments.

The energy transfer rate by collisions (=outcome is not two binaries) dominates over that due to fly-by's by an order of magnitude, provided that the binaries are hard. The formation of a hierarchical three-body system is fairly common. In a collision of energetically similar very hard binaries the probability is about 20 percent, while it is greater than 50 percent if the binding energies differ by a factor of more than four.

1. INTRODUCTION

Many-body systems with more than two bodies are usually unstable against disruption. Therefore binaries are the most important subsystems in a star cluster. However, binaries may make close encounters with single stars and with other binaries. The problem of three-body interactions has been widely studied by many authors (e.g. Saslaw, Valtonen and Aarseth 1974, Hills 1975a,b,1980, Hut & Bahcall 1983, Heggie 1975, and others). However, the binary-binary scattering has been studied rather little. A few papers in this field to be mentioned are Saslaw et al (1974) and Hoffer (1983). This paper is a short summary of the work done by the author (Mikkola 1983,1984a,b) and includes also some new preliminary results concerning the effect of unequal masses.

J. Goodman and P. Hut (eds.), Dynamics of Star Clusters, 335–338.

https://doi.org/10.1017/S0074180900147564 Published online by Cambridge University Press

2. NUMERICAL EXPERIMENTS

The Keplerian elements ω, Ω, M_0, cos(i), and e^2 of the binaries and the semiparameter p of the relative orbit were drawn from uniform distributions. Selected values were used for the Kepler energy T_∞ of the relative motion (which we call the 'impact energy') and for the ratio of the initial binding energies. Many experiments which were made with equal masses have been reported elsewhere (Mikkola 1983,1984a,b). Here we include also some results obtained from 9000 experiments with unequal masses. Both fixed mass families and certain random mass families were studied.

3. CLASSIFICATION OF THE FINAL CONFIGURATIONS

When the binaries are hard i.e. their binding energy exeeds the impact energy, only the following outcomes are possible:

(1) two binaries in a hyperbolic relative orbit
(2) one escaper and a hierarchical three-body system
(3) one binary and two escapers

A type (1) encounter is called a 'fly-by', while types (2) and (3) are classified as 'collisions'. The fraction of type (2) outcomes from all the collisions increases when the ratio of the initial semi-major axes increases. In the equal mass case we may approximate this fraction by

$$f(2) = 20\% + 15\%*\log_2(a_>/a_<) \qquad (1)$$

where $a_>$ and $a_<$ are the larger and smaller semi-major axes. This is valid for very hard binaries and has been obtained from samples in which $\log_2(a_>/a_<) < 2.5$. We conclude that this outcome dominates for $(a_>/a_<) > 4$.

4. A MODEL FOR THE COLLISIONS

4.1. Cross-section

We have attempted to obtain formulae for the collision cross-section. It was found (Mikkola 1984a) that a reasonable approximation may be constructed as follows: We assume that a collision occurs if the perturbation of the Kepler-energy of the relative motion is such that this energy is negative after the close approach. We suppose further that the cross-section satisfies the relation

$$\sigma \propto s_x^{\,2} \qquad (2)$$

where s_x is the maximum possible value of impact parameter for a collision. The dependence of s_x on the parameters of the system was obtained by writing the perturbation y proportional to T_∞ plus a cut-off energy ε. We have thus an equation for s_x:

https://doi.org/10.1017/S0074180900147564 Published online by Cambridge University Press

$$y(s_x) \propto T_\infty + \varepsilon \tag{3}$$

When deriving the expression for y the corresponding three-body result by Heggie (1975) was used. Finally the coefficients of proportionality in (2) and (3) was determined by fitting the experimental data. We noted that the data can be fitted within the expected statistical uncertainty.

4.2. Outcome energies

A simple statistical model for the disruption of the four-body system formed during a collision has been discussed in Mikkola (1984a). Here we give only a short description of this theory: At first the system ejects one of its stars. Depending on the configuration at the moment of ejection the remnant three-body system may disrupt or form a stable configuration. We consider in more detail only the type (3) outcomes. Let the first ejected star (number 4 say) have a Kepler energy E_4 with respect to the centre of mass of the other stars. Later the remnant three-body system disrupts with disruption energy E_3. These disruptions are independent (except that the energy of the first disruption affects the energy of the second) and thus the angle between the ejection directions is random (cosine of the angle has uniform distribution). The various energies may be written in the form

$$E_4 = \tilde{E}_4(\xi_1^{-2/7}-1) \; ; \; \tilde{E}_4 = 1.8((E_{12}+T_\infty)^{-1}+(E_{34}+T_\infty)^{-1})^{-1} \tag{4}$$
$$E_3 = \tilde{E}_3(\xi_2^{-2/7}-1) \; ; \; \tilde{E}_3 = 0.82(E_4+E_{12}+E_{34}-T_\infty)$$

where E_{12}, E_{34} and T_∞ are the initial binding energies and the impact energy. The ξ's are two random numbers uniform in the interval (0,1).

A treatment of the fly-by's in the case of equal masses may be found in Mikkola (1984b). We state here the main results: The fly-by transfer of energy between the binding energy of the binaries and the kinetic energy of their relative motion is typically small. For hard binaries the collisional rate dominates by an order of magnitude. In most situations the fly-by's are thus negligible as compared with collisions.

5. ON MASS EFFECTS

In our formula for the collision cross-section the mass effects are included (approximately at least). Consideration of the energy distributions from our various unequal mass samples do not show significant differencies against the equal mass case. A possible exception is a sample in which we have very large mass ratios (typically 10). In this case the mean values of the energy transfer are about 1.5 times greater than in other cases. We thus conclude that the energy distributions are not sensitive to the masses.

An additional interesting question is: which masses are likely to be ejected and which finally form the remaining binary? More precicely

https://doi.org/10.1017/S0074180900147564 Published online by Cambridge University Press

we may ask for the probability of a given mass to be ejected, first from the four-body system and then from the remaining three-body system. When we consider the outcome type (3) we in fact restrict the consideration to a sample in which it may be reasonable to take the probability to be a function of the masses only. For a restricted mass range a simple power law is a plausible approximation. We assume that for three- and four-body systems the escape probabilities are, respectively, proportional to

$$P_j \propto m_j^{-n} \; ; \; P_j \propto m_j^{-k} \qquad (5)$$

We wrote down the expressions for the probability that a given pair forms the remaining binary and then we used the maximum likelihood method to determine the most likely values for the exponents. The result was $n = 2.5$, $k = 1.9$. The goodness of this law was also studied by calculating quantities like the mean mass ratio and mass sum of the remaining binary. A good agreement with experiments was found.

6. CONCLUSION

A rough statistical understanding of the binary-binary scattering has been reached. We have found that the mass effects enter the collisional interaction mainly through the cross-section and the escape probability while the energy distributions do not depend sensitively on the masses. However, especially for the case of very large mass ratios the situation is yet unclear. We must also emphasize that the bad statistics due to small number of experiments makes these results only preliminary. We hope to publish elsewhere the details for the unequal mass case.

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