

THEORY AND STATISTICS OF LONG-RANGE DEPENDENT RANDOM PROCESSES

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Functional data indexed by points of multidimensional space and/or time is usually modelled using random processes and fields. The integral functionals of random fields play an important role in statistical problems related to such data. For example, by choosing functionals with integrands being polynomials, one can study the moment properties of random fields. Various geometrical properties of random fields, which are used in areas such as medical imaging, meteorology and astrophysics, are classically studied using excursion sets. Choosing the integrands to be the indicator functions of random fields, one obtains the first Minkowski functional (also called sojourn measures), the characteristic that measures volumes of excursion sets. One can also obtain the moments of excursion sets, among other characteristics, by choosing integrands to be polynomials of the indicator functions. Another important application where the integral functionals of random fields appear naturally is the research on fractional diffusion equations with random spatial data, for example the heat equation, the diffusion wave equation and the Burgers equation. Rescaled solutions of these equations can be presented as integral functionals of random fields. Linear regression models with errors being random fields observed on the surface of a sphere can be used to show that the least-squares estimator of the regression slope can be presented as a weighted integral functional of the random field.

In recent years, the models involving non-Gaussian data have grown considerably in popularity. Any Gaussian random field is completely defined by its first two moments, a property that non-Gaussian fields generally do not possess. This property plays a crucial role in the classical approaches in the theory of random fields. Therefore,

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non-Gaussian random fields are considerably harder to study. The common practical way of dealing with non-Gaussian data is to consider it as a nonlinear function of a Gaussian random process or field. This approach offers good data approximations in many cases.

The most important case is when the covariance functions of the random fields are decaying hyperbolically and hence decaying ‘slowly’. Such random fields are said to be long-range dependent and are observed in areas such as hydrology, geophysics, cosmology and finance. In contrast to the classical Donsker–Prokhorov results, the limits in the presence of long-range dependence can be non-Gaussian Hermite-type random variables and the normalising coefficients can be different from the ones found in central limit theorems.

The purpose of the thesis is to consider the asymptotic behaviour of functionals of Gaussian random fields under various scenarios. As mentioned above, these functionals appear in many statistical problems and their asymptotics have been of great interest in recent decades. However, in most cases, knowing the limit distribution is not enough for practical applications and more quantitative results, such as rates of convergence, are required. Despite the interest in the asymptotic behaviour, the rate of convergence in noncentral limit theorems almost has not been studied. The results on the rate of convergence in noncentral limit theorems in the thesis are new and derived under rather general conditions. Also, statistical applications of the results are demonstrated and various simulation studies are conducted.

First, the rate of convergence of nonlinear functionals of arbitrary Hermite rank polynomials of homogeneous isotropic Gaussian random fields defined on convex solid bodies is considered. In the existing literature, only the rates of convergence to Hermite-type random variables of orders 1 and 2 have been studied. This research relaxes the assumptions used to study the case of Rosenblatt limit distributions (order 2) in Anh *et al.* [1]. Under these weaker assumptions, the rates of convergence of functionals of random fields to Hermite-type random variables with arbitrary ranks are obtained. Also, some fine properties of Hermite-type distributions are studied. Specifically, Lévy concentration functions for such distributions are estimated.

Next, nonlinear integral functionals of homogeneous isotropic Gaussian random fields defined on hypersurfaces are studied. Various applications, for example in earth science or cosmology, require studying random fields defined on hypersurfaces. However, most known results on asymptotic behaviour are concerned only with random fields defined on solid figures. We obtain limit theorems analogous to the classical results. The rates of convergence in these theorems are derived. Some applications of the results to the studies of geometric properties of random fields are shown and several limit situations are demonstrated.

Finally, we demonstrate potential applications of the methodology developed in the thesis to statistical problems. The asymptotic behaviour of least-squares estimators of the unknown parameters in regression models on spheres with errors being long-range dependent random fields is studied. The limit distributions and the rates of convergence are derived. The theoretical findings are supported by simulations.

Numerical studies suggest that similar results should also hold in the case of other surfaces. The detailed methodology for practical simulations and studying the weighted functionals of random fields observed on surfaces and its implementation in the statistical programming language R is provided.

The results of the thesis were published in the papers [2–5].

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