PART II. THE ROTATION OF THE EARTH

THE IRREGULAR ROTATION OF THE EARTH

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Abstract. A mechanical explanation of the observed irregular rotation of the surface of the earth is proposed. From this probabilistic model the correlations between the apparent fluctuations of the motion of the moon in successive years are derived and a modified least-square method is developed for finding the secular accelerations of sun and moon.

Let ΔL be the difference between the observed and the tabular longitude of the moon, and $\Delta L'$ the same difference for the sun. Newcomb and De Sitter represented ΔL by a formula

$$\Delta L = x + yT + zT^2 + B,$$

where x, y, z are constants, T being the time in centuries since 1900, and B an irregular residue. Spencer Jones succeeded in representing $\Delta L'$ by a similar formula

$$\Delta L' = x' + y'T + z'T^2 + B/13.4,$$

in which B is the same as before. This means that both irregularities may be reduced to zero by one and the same clock time correction F. Thus it appears that the irregularities are not due to the moon, but to the daily rotation of the earth.

To explain the irregularities in the earth's rotation it is not necessary to assume (as some have done) considerable changes in the moment of inertia of the earth. In a paper to be published in the *Astronomical Journal* the following assumptions were made:

1. The retarding couple acting on the earth due to the friction of the tides is constant $= K$.
2. The earth's moment of inertia is constant $= C$.
3. The mantle of the earth is a rigid body. Its angular momentum is $m = c\omega$, where ω is the angular velocity and c the moment of inertia.
4. The earth as a whole is not a rigid body. If M is the angular momentum of the whole earth, we may define its angular velocity Ω by $M = C\Omega$.
5. The frictional couple between the mantle and the core is proportional to $\Omega - \omega$; $K_1 = f(\Omega - \omega)$. The coefficient f may be a random variable, varying between a positive lower bound f_0 and ∞.
6. In addition to this, there is another couple acting on the mantle due to irregular motions within the core: $K_2 = \Psi(T)$.
7. This $\Psi(T)$ is, for any given time T, a random variable with mean value zero; $E(\Psi) = 0$.
8. Values $\Psi(T_1)$ and $\Psi(T_2)$ at times T_1 and T_2 far apart are practically uncorrelated.

The differential equations for ω and Ω are

$$C\dot{\Omega} = - K \tag{1}$$

$$c\dot{\omega} = f(\Omega - \omega) + \Psi(T) - K. \tag{2}$$

The solution is

$$\Omega = \alpha - \beta T \tag{3}$$

$$\omega = \omega_0 - \beta T - \theta(T), \tag{4}$$

where $\theta(T)$ is again a random variable with expectation zero, and such that values of θ for times T_1 and T_2 far apart are practically uncorrelated.

"Far apart" may mean several years or several centuries apart; we do not know that beforehand and have to find it out from the observations. We are not concerned with seasonal variations in C, c and ω due to melting snow or ice, but only with changes in the mean value of ω from year to year.

Integrating Eq. (4) from 0 to T, we obtain the displacement of any place on the surface of the earth (say Greenwich, where most observations have been made) since 1900:

$$\int_0^T \omega dt = \omega_0 T - \tfrac{1}{2}\beta T^2 - \int_0^T \theta dt. \tag{5}$$

The first two terms give the displacement of a fictitious "mean Greenwich," which rotates with a uniform retardation due to the friction of the tides. The last term is the deviation of actual Greenwich from mean Greenwich. This deviation, divided by ω, gives the clock time correction F^*. Multiplying F^* by the mean motion of the moon in longitude, we obtain the apparent fluctuation of the moon

$$B^* = k \int_0^T \theta dt \tag{6}$$

and the correction to the tabular longitude

$$\Delta L = x + yT + zT^2 + B^*. \tag{7}$$

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The theoretical B^* is marked by an asterisk to distinguish it from the observed B, which is defined by the conventional formula

$$\Delta L = x_0 + y_0 T + z_0 T^2 + B, \qquad (8)$$

in which x_0, y_0 and z_0 are fixed values derived from the papers of De Sitter and adopted by Spencer Jones, Clemence and Brouwer.

From Eqs. (6), (7) and (8) we obtain, eliminating ΔL,

$$B^* = B - (a + bT + cT^2) = k \int_0^T \theta dT. \qquad (9)$$

The problem is to find out whether the constants a, b and c can be determined in such a way that

$$B^* = B - (a + bT + cT^2)$$

behaves like an integral $k \int_0^T \theta dt$, as it should do according to the theory, to estimate the constants

$$a = x - x_0, \quad b = y - y_0, \quad c = z - z_0$$

and to estimate the standard errors of the estimates of b and c.

Putting $t = 0$ in Eq. (9), we obtain $a = B(0)$. This means that in 1900 mean Greenwich coincided with actual Greenwich. This is an arbitrary convention, which may be changed at will.

Subtracting (9) for $T + \frac{1}{2}\delta T$ and $T - \frac{1}{2}\delta T$, and dividing by δT, we obtain

$$k\bar{\theta} = \frac{\delta B}{\delta T} - (b + 2cT). \qquad (10)$$

Here $\bar{\theta}$ is the integral mean of θ over the period δT, of which T is the middle; δB is the observed change of B during this period. The expectation of $\bar{\theta}$ is zero, hence:

b + 2cT *is the expectation of the observed difference quotient* δB/δT *over any period* δT.

A statistical analysis of the observed δB's by Van Woerkom shows that differences δB, taken for two one-year periods less than 10 or 12 years apart, have a high positive correlation, but if the periods are taken 20 or more years apart, the correlation practically vanishes. This is just what was to be expected from theory. The analysis was based upon 130 differences δB of B-values observed in successive years.

An estimation of b and c from ancient, medieval and modern observations by a modified method of least squares based upon the statistical properties of the δB's, resulted in $z = z_0 + c = 6.1$. This estimate has a standard error of approximately 0.8. The deviations of the observed B's from the parabola $B = a + bT + cT^2$ were not larger than might be expected from the variance of the δB's and the errors of the ancient observations.

FLUCTUATIONS AND SECULAR CHANGES IN THE EARTH'S ROTATION

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Abstract. Geophysical considerations favor seeking the cause of the random changes in the earth's rate of rotation in the turbulent motion in the core of the earth. Professor van der Waerden's analysis applies to this interpretation. The observational evidence indicates that the character of the fluctuation curve may be intermediate between that expected on the basis of a theory with frictional couple and one without. If confirmed, this would indicate that two different causes contribute to the changes in the earth's rate of rotation.

Seven years ago (Brouwer 1952a,b) I attempted a solution of the problem of the irregular changes in the earth's rate of rotation by assuming that the second differences of the annual values of the fluctuation curve in the moon's mean longitude are of a random character, uncorrelated from one year to the next. These second differences were assumed to have a normal distribution with a fixed standard deviation to be determined from the observational data. Their mean value would differ from zero by a small quantity corresponding to a correction to the secular acceleration term in the moon's tabular mean longitude.

The analysis was tested by Van Woerkom (1953) in two ways: by examining the properties of artificially constructed fluctuation curves and by a statistical discussion of the annual values of the observed fluctuation curve since 1820. He concluded that, on the whole, the observational data support the hypothesis, but that there may be an unexplained contribution to the variance of the first differences. Also, the amplitude of the observed fluctuations over the twenty centuries for which scattered data are available appears to be rather smaller than might be expected from the character of the fluctuations since 1820. Comparison with arti-

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