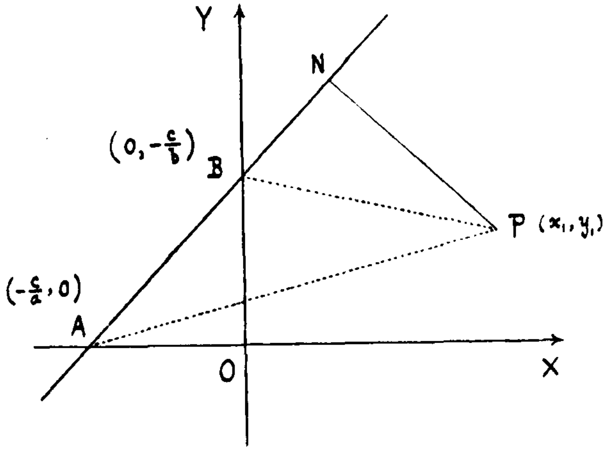
Length of the Perpendicular from a given Point to a given Straight Line.

To obtain the length of the perpendicular from the point $P(x_1, y_1)$ to the line $ax+by+c=0$ the following simple method, which requires only a knowledge of the formula for the area of a triangle in terms of the coordinates of the vertices, may be employed.



(i) The given straight line cuts the x-axis in the point $A\left(-\frac{c}{a}, 0\right)$, and the y-axis in the point $B\left(0, -\frac{c}{b}\right)$. The area, Δ, of triangle ABP is therefore given in sign and magnitude by

$$2\Delta = \begin{vmatrix} -\frac{c}{a}, & 0, & 1 \\ 0, & -\frac{c}{b}, & 1 \\ x_1, & y_1, & 1 \end{vmatrix}$$

$$= \frac{c}{ab}(ax_1 + by_1 + c).$$

Also $AB^2 = \frac{c^2}{a^2} + \frac{c^2}{b^2} = \frac{c^2}{a^2 b^2}(a^2+b^2)$, and if PN is the perpen-

https://doi.org/10.1017/S0013091500035835 Published online by Cambridge University Press

dicular from P to the line

$$2\,\triangle = AB \cdot PN$$

$$= \pm \frac{c}{ab}\ \sqrt{a^2+b^2}\,.\,PN.$$

Equating these two expressions for $2\,\triangle$ we have at once

$$PN = \pm \frac{ax_1 + by_1 + c}{\sqrt{(a^2+b^2)}}.$$

(ii) The special case in which the straight line passes through the origin (when A and B coincide with O, and $\triangle = 0$) may obviously be regarded as the limiting case of (i) for $c \rightarrow 0$.

Or, if it is desired to avoid the limit-conception, we have only to note that the perpendicular from P to $ax + by = 0$ is equal to the perpendicular form O to the parallel through P, namely

$$a\,(x - x_1) + b\,(y - y_1) = 0,$$

and that, by (i), the length of that perpendicular is

$$\pm \frac{a\,(0 - x_1) + b\,(0 - y_1)}{\sqrt{(a^2+b^2)}} = \pm \frac{ax_1 + by_1}{\sqrt{(a^2+b^2)}}.$$

(iii) The reader may be interested to refer to a method of finding the length of the perpendicular by projections, given by Prof. R. J. T. Bell in *Mathematical Notes*, No. 18 (May 1915), pp. 206–207.

J. M'WHAN.

Formulae for the Construction of Right-Angled Triangles.

Use the formulae

$$[a\,(a+2b)]^2 + [2b\,(a+b)^2]^2 = [2b\,(a+b) + a^2]^2 \qquad \text{(i)}$$

or

$$[2a\,(a+b)]^2 + [b\,(2a+b)]^2 = [b\,(2a+b) + 2a^2]^2. \qquad \text{(ii)}$$

In these a and b need not be integers, and may be positive or negative.

The formulae develop into two systems of sets of triangles.

https://doi.org/10.1017/S0013091500035835 Published online by Cambridge University Press