Convection in Astrophysics
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Thermal convection in accretion disks

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Abstract. For a long time it was believed that thermal convection could serve as the driving mechanism for turbulence and angular momentum transport in accretion disks. Even it is meanwhile accepted that convection had to leave that role to the magneto rotational instability, it is still an important effect arising in a realistic treatment of accretion disks, i.e. with proper thermodynamics and radiation transport. We review the history of thermal convection in astrophysical disks and show the relevant analytic and numerical work, including energy transport by convection and the effect of "negative" Reynolds stresses. We will also place the convective instability into the context of the magnetorotational instability and planet–disk interaction.

Keywords. Hydrodynamics, instabilities, convection, turbulence, accretion disks, planets and satellites: formation

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1. Introduction

The coplanar orbits of nearly all planets in the Solar System inspired Laplace to propose that they are formed in a gaseous protoplanetary disk, the primordial Solar Nebula, centered around the proto-Sun. See figure 1 for a schematic diagram describing the process of star and planet formation.

Observation tells us that such protoplanetary disks exist around many young stars. These disks sustain an accretion flow onto their central objects. Thus they are often called protoplanetary accretion disks. The accretion is only possible if angular momentum is transported in the radially outward direction. Such a transport can be explained by an anomalous viscosity that can result from self–gravity and turbulence. It was Cameron (1978) who suggested that "...a driving force for turbulence could arise from thermally-driven convection. This would require that superadiabatic temperature gradients exist within the gas where there is a significant gas pressure gradient." Even Cameron was pessimistic, that thermal convection would be important as a source for turbulence in comparison to "meridional currents" and "infall of material" onto the disk, his paper was still influential enough that for the next 20 years thermal convection was a popular source for turbulent viscosity in protoplanetary accretion disks, inspiring extended research in the field.

2. Observations of accretion in protoplanetary systems

Interferometric observations of the CO line emission demonstrate that the circumstellar material around young stars has a flattened structure and is in Keplerian rotation (Simon *et al.* 2000). Dust grains suspended in the gas scatter the stellar light, often revealing the disk–like geometry. Direct images of these disks have been obtained by the Hubble Space Telescope and adaptive optics systems on ground–based telescopes (e.g. Weinberger *et al.* 2002). Continuum images in the millimeter range suggest that most of the mass is located at rather large distances from central objects ($r \geqslant 30 \sim 50$ AU). Using a gas-to-dust ratio of 10^2, analyses of the dust emission indicate that the total (dust + gas) masses are in

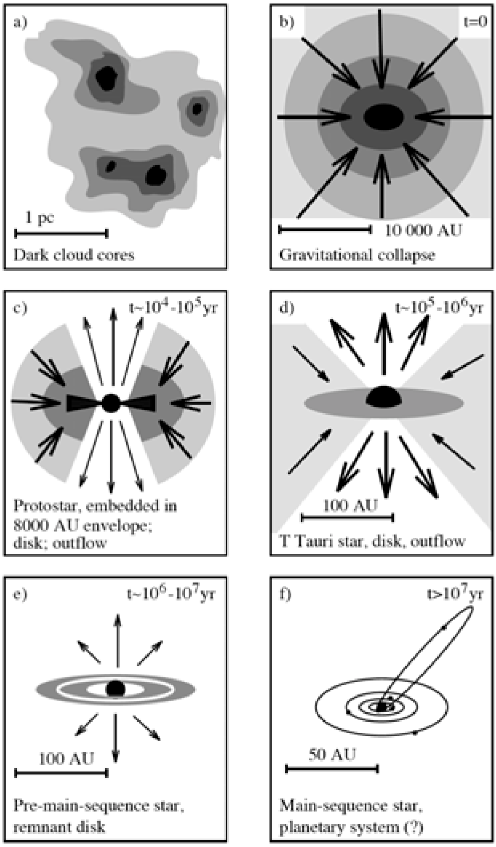

Figure 1. Schematics of Star and Planet Formation. With kind permission from Michiel Hogerheijde (after Shu *et al.* 1987).

the range 0.001 to 0.1 $M_\odot$. However estimating the disk mass is difficult and the present estimates are rather uncertain (Dutrey *et al.* 2005). The lifetimes of protoplanetary disks range from 10^6 to 10^7 yr (Haisch *et al.* 2001). The accretion rate $\dot{M}$ decreases by several orders of magnitude within the lifetime of a disk (Hartmann *et al.* 1998). The youngest, highly embedded systems accrete at a few 10^{-5} $M_\odot$/yr, and in systems which undergo FU Orionis-type outbursts $\dot{M}$ can reach up to a few 10^{-4} $M_\odot$/yr (Hartmann & Kenyon 1996). Presently, the most reliable estimates of $\dot{M}$ are based on measurements of the excess emission superimposed onto the intrinsic photospheric spectrum of the central object. It is generally accepted that this excess arises from the accretion shock formed as disk material falls onto the photosphere (either directly or along magnetic field lines). Characteristic of this type of flow are emission lines, whose equivalent widths decrease as the system ages. Best studied are low–mass objects (0.1 $M_\odot < M < 1$ $M_\odot$), which, depending on the equivalent width of the H_α line, are classified as either classical or weak-line T Tauri stars (CTTS or WTTS). Based on a large sample of CTTS with known $\dot{M}$ it was found that these objects typically accrete at a rate of $10^{-8} \sim 10^{-7}$ $M_\odot$/yr (Calvet *et al.* 2004).

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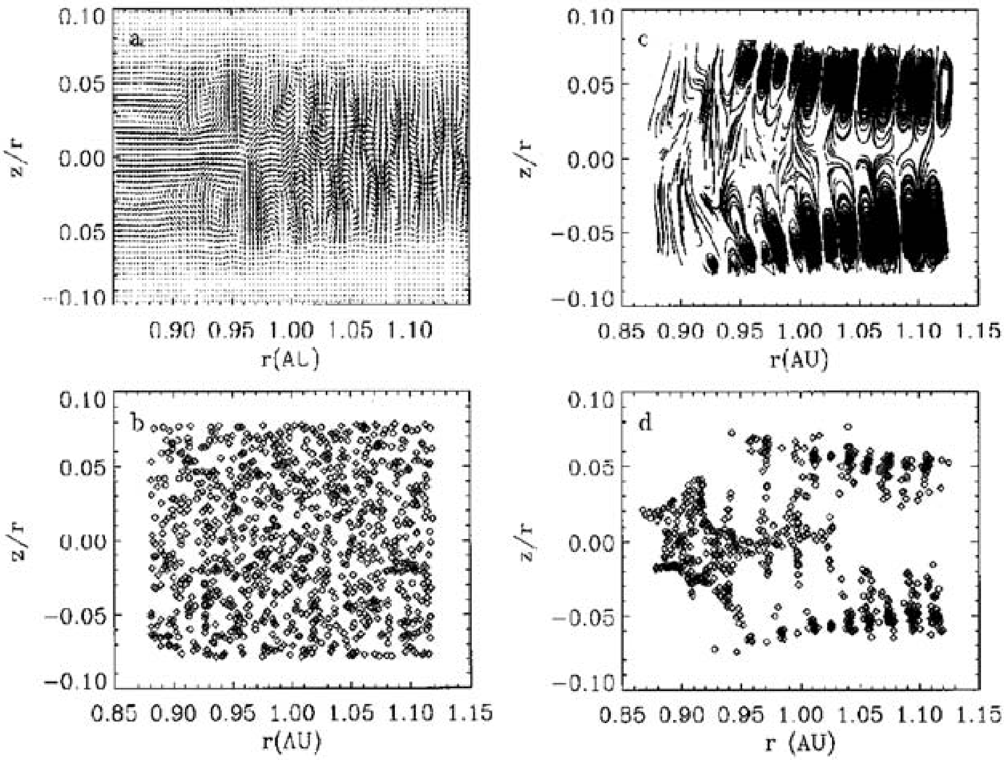

Figure 2. (a) Mass flux in the convective region of a protoplanetary accretion disk. This is the result of a 2D radiation hydrodynamical simulation. (b) The homogeneous initial distribution of 0.1-cm grains. (c) The traces of a part of the grains during 160 years. (d) The position of the grains after 160 years. (Taken from Klahr & Henning (1997)).

3. Viscous evolution

The evolution of a gravitationally stable disk is usually described in terms of viscous diffusion. A viscous disk decays on a *viscous timescale*

$$\tau_{\rm v} = R_{\rm d}^2 \nu^{-1}, \tag{3.1}$$

where ν is the effective viscosity coefficient. Given the sizes of protoplanetary disks, a viscosity

$$\nu = 2 \times 10^{17} \left(\frac{r}{500\ {\rm AU}} \right)^2 \left(\frac{\tau_{\rm d}}{10^7\ {\rm yr}} \right)^{-1}\ {\rm cm}^2 {\rm s}^{-1} \tag{3.2}$$

is needed for $\tau_{\rm v}$ to be comparable to the observed lifetimes. The source of such a large viscosity (many orders of magnitude exceeding the microscopic viscosity of the gas) remains unknown. Various candidates, among which the magnetorotational instability (MRI) (Balbus & Hawley 1991) is widely regarded as the most promising one, are discussed in the literature. Thermal convection was also among the candidates and we will discuss its history in the following sections of this chapter. First we focus on a simple approach, originally proposed by Shakura & Sunayev (1973), on which most models of accretion disks have been based. In this approach it is assumed that the viscosity originates from turbulent motions, and the viscosity coefficient is defined by

$$\nu = \alpha c_{\rm s,m} H, \tag{3.3}$$

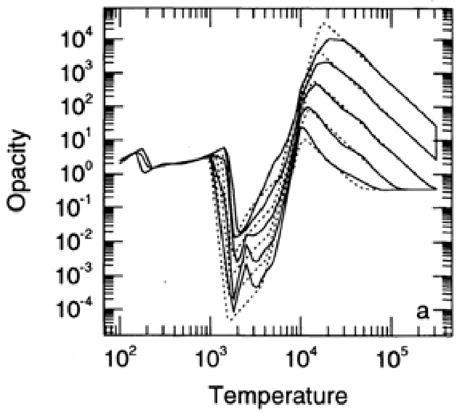

Figure 3. Opacities in $\mathrm{gm/cm^3}$ in dependence of temperature in K. Only for the very steep parts of the opacity curve (roughly $T < 200$K and 2000 K $< T <$ 20000K) a convective instability can occure. With kind permission from Robbins Bell (after Bell and Lin (1994)).

Frequency-Averaged Opacity Law over Eight Regions in Order of Ascending Temperature

Region	κ_i	a	b
Ice grains	2×10^{-4}	0	2
Evaporation of ice grains	2×10^{16}	0	-7
Metal grains	0.1	0	1/2
Evaporation of metal grains	2×10^{81}	1	-24
Molecules	10^{-8}	2/3	3
H-scattering	10^{-36}	1/3	10
Bound-free and free-free	1.5×10^{20}	1	$-5/2$
Electron scattering	0.348	0	0

Figure 4. Opacities in dependence of temperature. Only when the exponent b is steeper than 1 convection will occur. With kind permission by the authors (after Bell & Lin (1994)).

where α is a dimensionless parameter and H (the disk scale height) is a natural upper limit for the size of turbulent eddies. Adopting (3.3), one effectively assumes that the characteristic velocity associated with the largest eddies is αc_{s}. As the turbulent motions are most probably subsonic (otherwise they would quickly dissipate due to shocks), α must be smaller than 1.

Based on the assumption that the shear stress is proportional to pressure, Equation 3.3) is often cast into a more general form (Bell *et al.* 1997), e.g.

$$\nu = \alpha c_{\mathrm{s}}^2 \Omega^{-1} = \frac{\alpha P}{\Omega \rho}. \tag{3.4}$$

In the absence of external torques the viscous disk conserves its angular momentum. While most of its mass *loses* angular momentum and is accreted onto the central body, a small amount of mass *gains* angular momentum and moves away from the central body. The orbital energy of the accreted matter is transformed into heat. A thin stationary

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disk is heated at a rate

$$Q_{\mathrm{v}} = \frac{9}{4}\left(\frac{GM}{r^3}\right)^{0.5} \nu\Sigma \tag{3.5}$$

per unit area (for the derivation see e.g. Spruit (2001)), and cooled at the same rate by the radiative flux emerging from its surface.

Modeling and observations suggest that in protoplanetary disks $\alpha \approx 10^{-3} \sim 10^{-2}$, whereas cataclysmic variables (CV) have $\alpha \approx 10^{-1}$ and in accretion discs around black holes $\alpha \approx 10^{-2} \sim 10^{-1}$. The variations in α can lead to the idea that there might be more than one unique mechanism to create turbulence in a disk or shows how little we have understood so far about MRI turbulence.

4. Planet Formation

Planet formation starts with inelastic collisions between dust grains in the solar nebula. Each sticking leads to growth. Starting from kilometres in size the planetesimals can attract material by gravity. Once a planetary embryo is several times the mass of the earth it starts to accrete gas and becomes a gas giant. But if the disk is laminar there are several bottle necks in this growth scenario. For instance when the dust is roughly meters in size it is raining out of the nebula, because the pressure supported gas moves on a sub-Keplerian orbit. Large objects decouple from the gas and small dust grains are frozen in the gas, but for meter size boulders the radial drift time is shorter than the characteristic growth time. In Klahr & Henning (1997) the authors showed that the flow pattern of thermal convection can capture particles (see figure 2). Similar studies have recently performed by Johansen *et al.* (2006) for the case of turbulence driven by the MRI. It is nowadays clearly understood that the formation of planetesimals needs turbulence as a vital incredient (for a review see Klahr *et al.* (2006)).

5. Turbulence

Keplerian disks are hydrodynamically stable configurations according to the Rayleigh criterion,

$$\frac{dj^2}{dr} > 0, \tag{5.1}$$

where r is the radius and $j = r^2\Omega$ is the angular momentum per unit mass. An early idea on turbulence came from Cameron (1978) speculated that thermal convection could do the job. Thermal convection needs a superadiabatic temperature gradient. Lin & Papaloizou (1980) could show that the vertical temperature gradient is determined by the opacity in the disk: If one assumes a power law for the opacities in dependence of temperature $\kappa = \kappa_0 T^b$ one can derive the criterion

$$\frac{1}{4-b} \geqslant \frac{\gamma - 1}{\gamma}\, 0.3 \tag{5.2}$$

with the adiabatic index $\gamma = 1.5$ this leads to the simple criterion $b \geqslant 1$ for thermal convection. Figure 3 and figure 4 show typical opacities for accretion disks (Bell & Lin 1994) indicating that only part of the temperature regime of an accretion disk will lead to convection. This should then lead to a self-regulation of turbulence. If it gets too hot, convection and turbulence will shut off. Canuto *et al.* (1984) derived α values in terms of the growth rate of the unstable convective modes driving the turbulence and conclude that α should be a function of radius and temperature in the disk. Based on this idea

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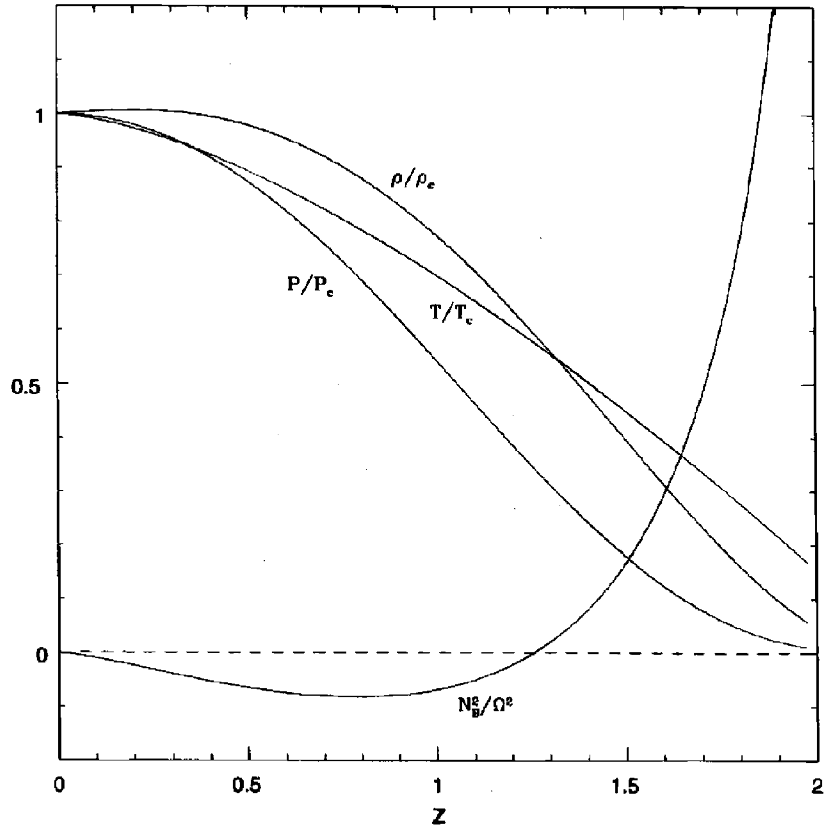

Figure 5. Vertical profiles of the density, temperature, pressure, and Brunt-Väisälä frequency in the initial state. The model is convectively unstable where $N_B^2/\Omega^2 < 0$. With kind permission by the authors (after Stone & Balbus (1996)).

Cabot *et al.* (1987a) and Cabot *et al.* (1987b) developed a model for the solar nebula, in which convection is assumed to be the sole source of turbulence that causes the nebula to evolve. They find that α is generally pretty low and sensitive to opacity and the surface density of the disk. This sensitivity produces an inverse *accretion rate – surface density* relationship, potentially breaking the disk up into rings. They conclude that convection can not be the dominant source of turbulence.

Ryu *et al.* (1992) considered linear growth of non-axisymmetric disturbances in convectively unstable disks. They used the shearing-sheet approximation in a uniform disk and found that the flux of angular momentum was inwards, which is sometimes referred to as *negative* α values. Lin *et al.* (1993) performed a linear stability analysis of non-axisymmetric convective instabilities in disks and also allowed for some disk structure in the radial direction. Here they found again outward transport of angular momentum.

Figure 5 shows the vertical profile of an accretion disk model used for a three dimensional yet local shearing sheet simulation of thermal convection (Stone & Balbus 1996). One nicely recognizes how small and weak the unstable region is as indicated by a negative Brunt-Väisälä frequency with growth rates at most $\frac{1}{3}$ of the orbital frequency. Bell *et al.* (1997) have derived vertical structure models assuming some fixed α values and radiation transport in the flux limited diffusion approximation. They compare models with and without incorporating thermal convection as a transport of heat and find only little difference in the structure (see figure 6). As already expected only for low temperatures

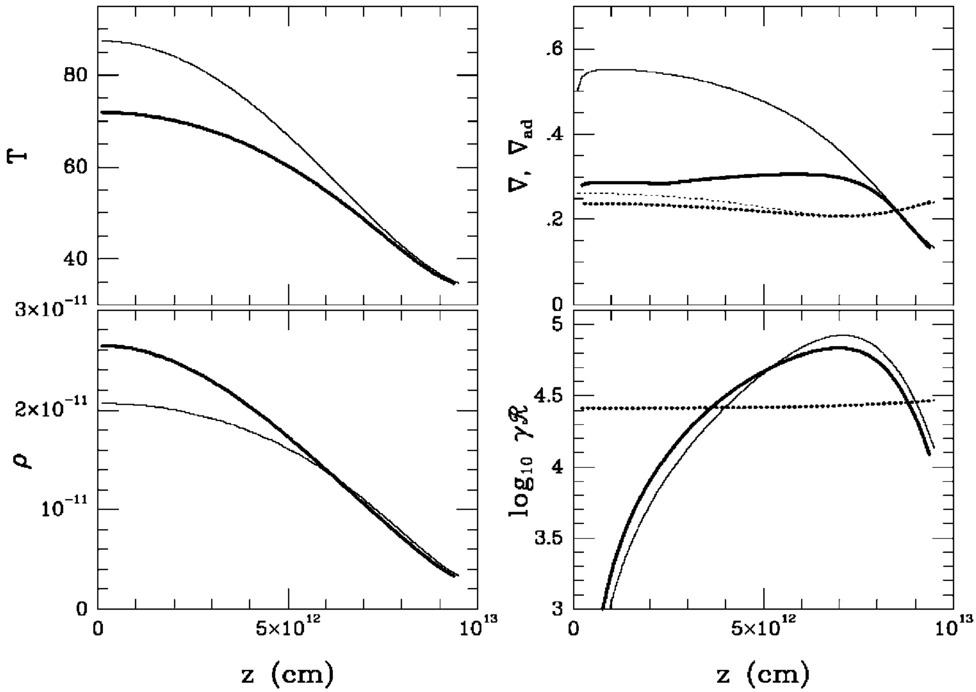

Figure 6. Annulus with $\dot{M} = 10^{-7} M_{\odot}\mathrm{yr}^{-1}$ at 7 AU showing vertical structure with convection (heavy lines) and without (light lines). In upper right panel, true gradients are solid; adiabatic gradients for the two models are dotted. Note the similarity of T and ρ for the models. In the lower right is shown the convective criterion discussed in Bell *et al.* (1997). With kind permission by the authors.

there is some heat transported by convection (see figure 7) stressing the inefficiency to turn heat into motion.

Stone & Balbus (1996) and Cabot (1996) approached the problem of the sign of α by three-dimensional hydrodynamical calculations in the shearing-sheet approximation in order to simulate a tiny box of a disk in pseudo-Cartesian coordinates. Stone & Balbus used the inviscid ZEUS3D code. The convection was driven by a superimposed heat source. They measured a net inward transport of angular momentum corresponding to a very low and negative α with a value of $\alpha = -4.2 \times 10^{-5}$. Cabot used a high and constant kinematic viscosity in his simulations. This viscosity, together with the shear, leads to a two-dimensionalization of the convection (see figure 8). All azimuthal structures were smeared out, and these two-dimensional (2D) flows tend to transport angular momentum inward (see figure 9).

In Klahr *et al.* (1999) and Klahr & Bodenheimer (2003) such simulations were extended to global 3D non-viscous simulations of thermal convection. In these simulations axissymmetry was broken by turbulence and the α values were at a reasonable value of 10^{-3}–10^{-2}. Nevertheless, convection still had to be driven by additional heating in the midplane. If the heating was switched off the convection died out and the disk cooled down to the ambient temperature. Eventually the disk also broke up in annuli much like the rings observed in Cabot *et al.* (1987b). Again the conclusion was that thermal convection cannot be the sole source of turbulence in accretion disks.

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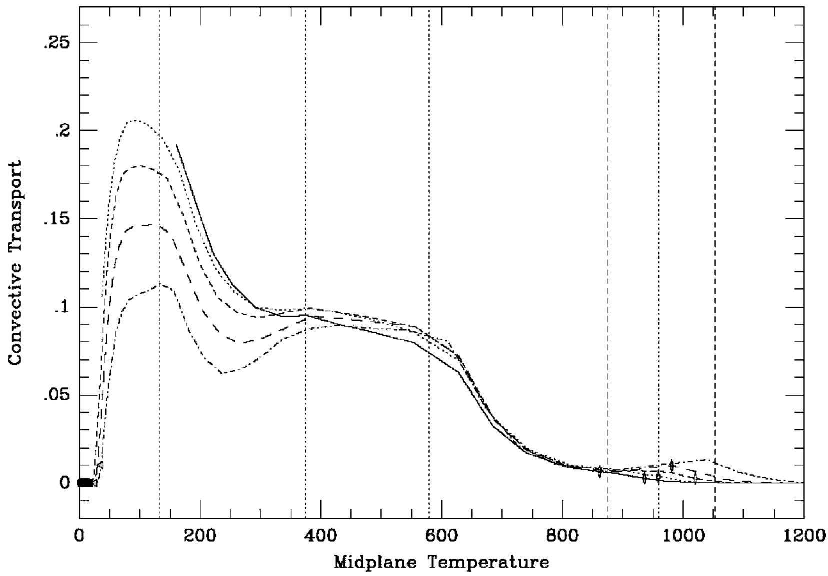

Figure 7. Radial distribution of convection. "Convective transport" is a vertically averaged measure of the fraction of flux transported by convection. Each line represents a constant mass flux model; line types solid, dotted, short dashed, long dashed, and dot dashed represent models with, respectively, $10^{-5},^{-6},^{-7},^{-8}$, and 10^{-9} $M_{\odot}yr^{-1}$. Symbols are as in Vertical dotted lines mark the dust destruction fronts ("Max. Temp." column of Figure 2 opacity regimes 1, 3, 5, and 7); the two vertical dashed lines indicate the temperature of the final dust destruction front varying the density an order of Magnitude in either direction. The peak in (narrow diamonds) coincides closely with this final dust destruction zone. The convective regions further out in H_R/r the disk (i.e., at cooler temperatures) will be shielded from illumination by the central object. Taken from Bell *et al.* (1997) with kind permission by the authors.

6. Convection and magnetic fields

Nevertheless, differentially rotating gaseous disks with sufficiently high conductivity are unstable under the influence of a weak magnetic field. The magnetorotational instability (MRI) has been known for more than three decades, but the importance for accretion disks was first pointed out by Balbus & Hawley (1991).

The two important conditions for MRI are first that there is a Keplerian rotational profile in the disk and secondly, that fluid elements are connected via magnetic field lines so they can exchange momentum. Both numerical simulations (Hawley *et al.* 1995) and analytical work (Balbus & Papaloizou 1999) show that the MHD turbulence leads to a viscosity consistent with the description in Equation (3.4), however, α may vary both in space and time.

Even the MRI provides a heating of the disk it still does not lead to thermal convection. First the MHD turbulence is way more violent than any convection in disks simulated so far and secondly the turbulent diffusion erases the vertical entropy gradient making the disk convectively stable anyway.

Nevertheless, a good coupling between the gas and the magnetic field is necessary for the MRI to operate. Since proto-planetary disks are weakly ionized, in some regions the resistivity of the disk may be so high that the magnetic field decouples from the gas and the MRI decays. The MRI-free region is usually referred to as a *dead zone*, whereas the remaining part of the disk is referred to as *active* (see figure 10). Two-dimensional axially symmetric radiation hydrodynamic models of layered disks were obtained by Wunsch *et al.* (2005) and Wunsch *et al.* (2006). The authors show that variations of the

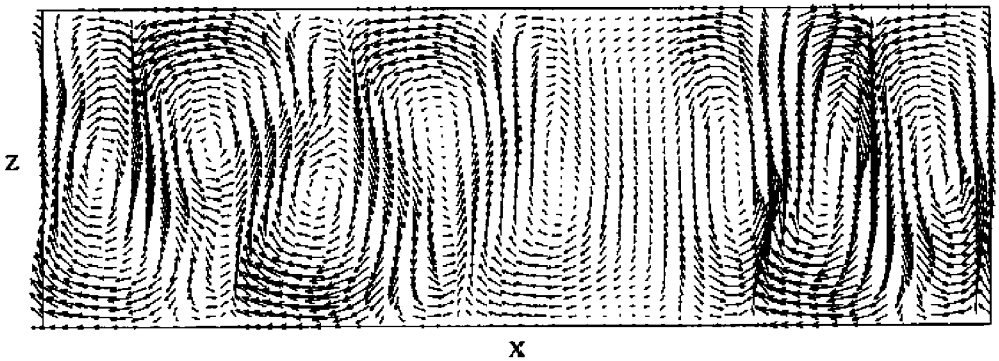

Fig. 2a

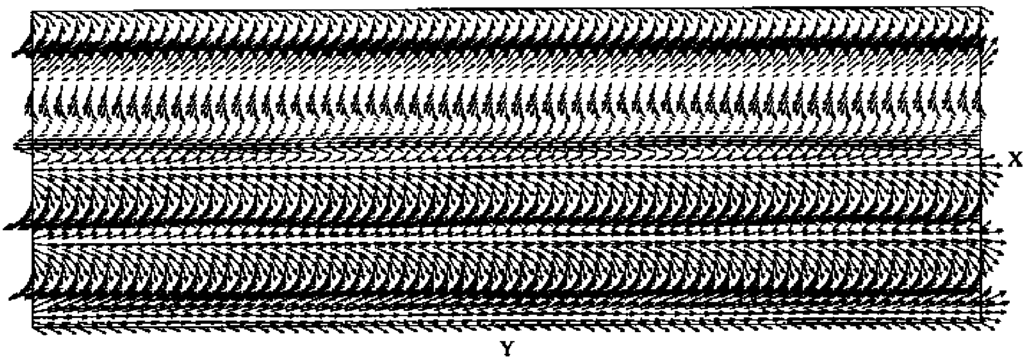

Fig. 2b

Figure 8. Instantanious velocity vectors viewed in (a) an x, z plane and (b) an x,y plane at a height $z = 0.09\ R$ near the top of the convection zone. Taken from Cabot (1996) with kind permission by the author.

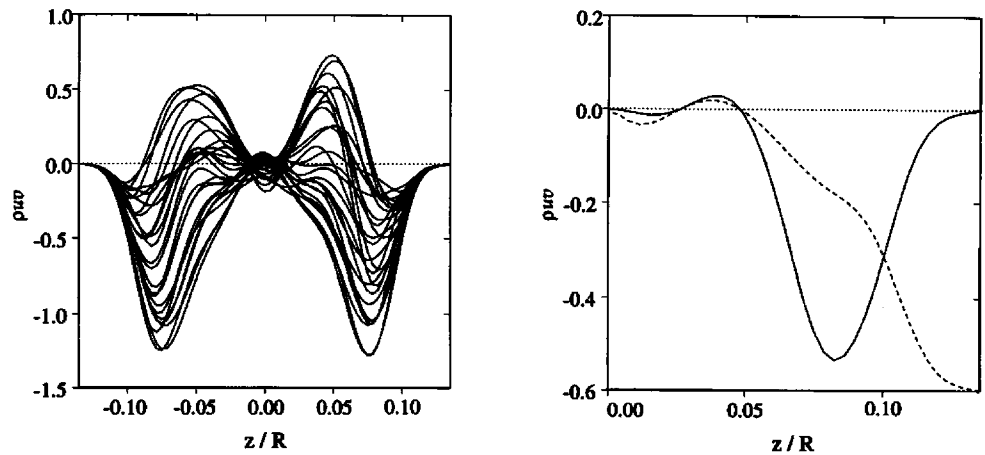

Figure 9. Angular momentum flux ρuv (scaled by the mean midplane value $10^{-3}\rho c_s^2$), where c_s is the sound speed): (a) time series of plane-average values over 2.5 shear times (a quarter of a rotation period); (b) the Reynolds stress $\rho\bar{u}v$ (*solid line*) and its correlation coefficient, $\rho\bar{u}v/(\rho\bar{u}^2\rho\bar{v}^2)^{1/2}$ (*dashed line*). Taken from Cabot (1996) with kind permission by the author.

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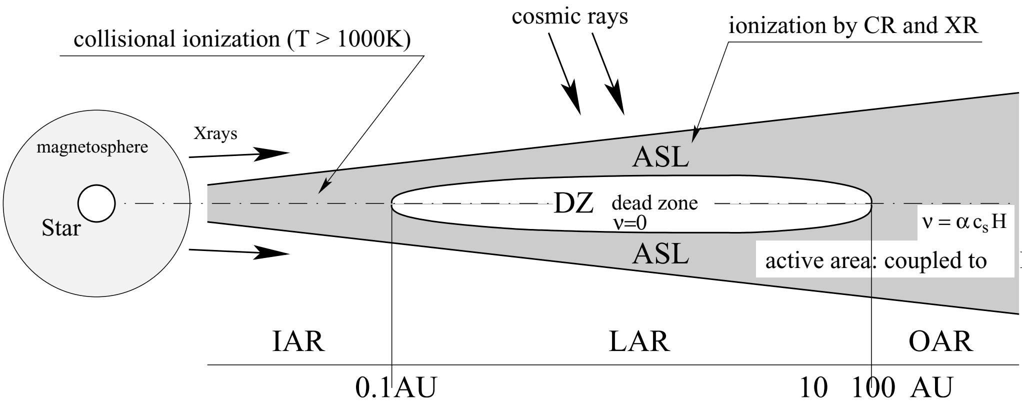

Figure 10. The schematic structure of the layered disk: the inner active region (IAR), the layered accretion region (LAR) with two active surface layers (ASL) and the dead zone (DZ), and the outer active region (OAR). By permission of the authors taken from Wunsch *et al.* (2005).

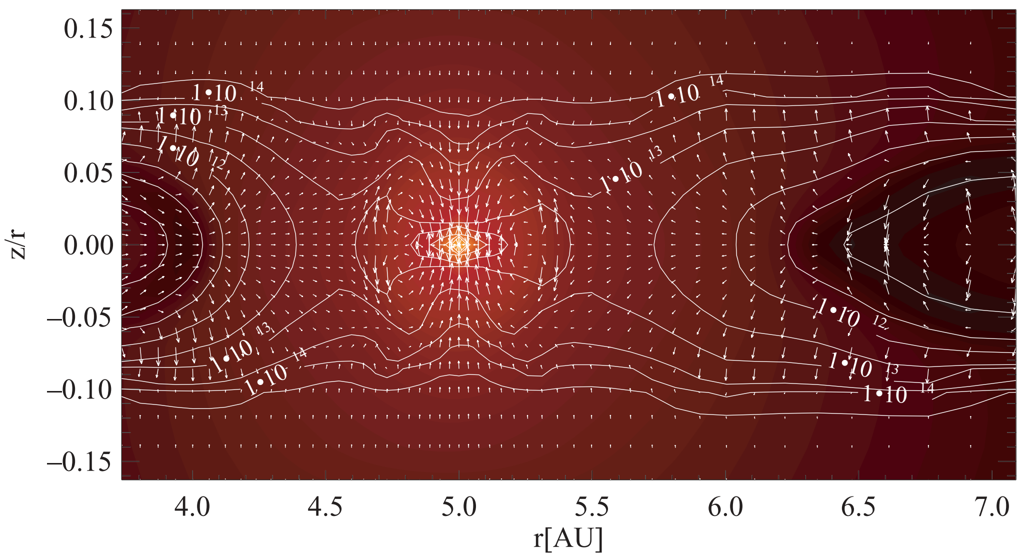

Figure 11. Planet-disk interaction drives thermal convection: Temperature in the r-θ plane of the protoplanetary disk at the azimuthal location of the planet after 141 orbits. Brightness is logarithmic temperature (lighter = warmer) between 30 K and 1500 K, contours are equi-density lines (in g/cm^3) and vectors give the logarithmic mass flux. See the online edition of Klahr & Kley (2006) for a color version of this plot.

thickness of the dead zone influence the structure of the surface active layer, leading to growing perturbations of the mass accumulation rate. As a result, the dead zone splits into rings. The model also shows that radiation from the inner disk heats the dead zone and makes it active shortly after mass accumulation has started. Furthermore thermal convection can be observed in the dead zones driven from the heat generated in the active part of the disk.

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7. Convection by planet disk interaction

During their formation process, massive, gaseous planets are believed to undergo a phase of evolution where they are still embedded in the protoplanetary disk. The gravitational influence of the planet onto the ambient disk leads to such features as spiral arms and, for planets sufficiently massive, an annular gap at the planetary radius. The back-reaction of the perturbed disk onto the planet generates torques, which induce a change of the orbital elements (semi-major axis and eccentricity) of the planet.

Klahr & Kley (2006) perform radiation hydrodynamical planet–disk calculations in three dimensions.Thus they study directly the dynamical influence of the planetary accretion luminosity and determine the three–dimensional temperature structure in the vicinity the planet.

The authors find that planets are most likely to form a circumplanetary pressure–supported envelope rather than an accretion disk around them, with strong convective vertical flows (see figure 11). The relative pressure scale height in the circumplanetary material is at least 0.5, in which case the approximations for a thin Keplerian accretion disk are no longer valid. What results is a cloud which rotates at only 50 percent of the Keplerian value.

One observes strong convection flows in the early gap opening phase. Considering the entire mass accretion phase of the young planet starting from a few earth masses, one finds that these vertical fountains e.g. convective overshooting may last for hundred-thousands of years before the planet opens its gap, making the effect clearly relevant for observations. As a result there would be locally a stronger flaring of the disk, and more radiation could interact with the small dust grains in the surface layer above the planet. Future telescopes should be able to observe this asymmetry in the scattered light from disks in which planets are forming. Thus even thermal convection is not the source for disk evolution it may still be possible in the very near future to detect the phenomenon of thermal convection in protoplanetary accretion disks by direct observation.

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Discussion

Kupka: Are the accretion disks around young hot planets considered and investigated under the prospect of formation of moons, at least the larger sized ones?

Klahr: Yes, this is considered for explaining for instance the 4 large satellites of Jupiter. Even our research indicates that the young disk around Jupiter was too hot for satellite formation, it will cool down once the accretion onto Jupiter stops. In the aftermath satellites might form from the remaining disk material

Wuchterl: How did you find out that a planet of 9 $M_{\rm Earth}$ is accreting?

Klahr: Our models do not deal with the actual accretion onto a planet itself. The accretion occurs way below our finest resolution. What one measures by the accretion rate is the amount of mass that can flow through the Roche lobe. So to speak an upper limit for the material that can eventually be accreted by the planet.

Toomre: Although convection in planetary disks is likely short-lived or episodic, have you looked at convection in accretion discs with mass transfer from a companion star?

Klahr: Even convection might occure in these systems, it is doubtful that it plays a mayor role in producing an effective turbulent viscosity with $\alpha \approx 0.1$.

Rincon: Results showing inwards or zero angular momentum transport have been obtained using at most 64^3 simulations (Stone & Balbus 1996), so using a realistic Reynolds number may lead to quite different conclusions.

Klahr: Our global 3D simulations show indeed outwards transport, while 2D results predict inwards transport. Nevertheless, convection is too inefficient in its angular momentum transport to make it a self-sustaining process. As long as more than 90% of the heat released by the accretion process get lost via radiation, there can be no sufficient feedback from the released potential energy into the convective flow. Thus it appears not to be a question of the Reynolds number whether convection can sustain itself or not.