

STAR COUNTS

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I. INTRODUCTION

The number of stars counted along a particular line of sight depends on the spatial distribution of stars, the luminosity function, and the absorption. Thus star count programs are designed to constrain or determine one or more of these functions. Early efforts to understand the structure of our Galaxy, including the fundamentals of stellar statistics, were largely based on work that involved star counts. Since then a growing appreciation has developed for the variety of forms the density function $D(r)$ and the luminosity function $\phi(M)$ can take, especially the recognition of different stellar populations, each with different density and luminosity functions. In the simplest formulation two distinct populations are considered: disk and halo. This suggests two distinct formation histories, but uncertainty in the picture remains (Eggen, Lynden-Bell and Sandage 1962; Ostriker and Thuan 1975; Saio and Yoshii 1979; Jones and Wyse 1983). To discriminate between various models, more information is needed. The problem is that star counts integrate over the details of the stellar statistics to a greater or lesser extent, depending on how much information is available to separate the populations. The challenge to observers is to obtain sufficient resolution. Population type indicators are kinematics, metal abundance, and age distribution, so that star counts combined with proper motions, or radial velocities, or colors, or line strength indices, etc. would naturally provide the greatest constraints. A basic question in this context is whether or not the two-population idealization retains its usefulness when considered in detail, and this issue could be regarded as a point of departure for further work (e.g., Strömgren 1976).

There are two ways to use star counts to develop a picture for what is going on. The first method is to assume forms for the functions $D(r)$ and $\phi(M)$ for each population, along with absolute magnitude - color relations, etc., and then model the expected observations along a line of sight in order to compare with data (e.g., Bahcall and Soneira 1980). The second method is to attempt to derive $D(r)$ and $\phi(M)$ directly from

the data via the fundamental equation of stellar statistics (McCusky 1965). To a large extent it can be said that the success of the second method depends on how confidently one can assign a photometric parallax to each star -- and this depends on knowing the population types of the stars, which is often one of the questions being investigated. An example is the problem of the subgiant/turnoff star ratio (Yoshii 1982), since these stars can have quite different luminosities at about the same color.

II. STAR COUNTS WITH SCHMIDT TELESCOPES

Reckoned in terms of information gathered per unit time, Schmidt-type telescopes are unrivaled, but some types of star count programs benefit from the unique capabilities of Schmidt telescopes more than others. The number of stars seen at magnitude m_2 within a distance shell of width Δr is proportional to $\phi(M)D(r)r^2$, where m , M , and r are related in usual way, taking into account the effects of absorption (see e.g. van Rhijn 1965). This function may have a simple form -- peaked at some distance r -- or it could be more complex. Where the function $\phi(M)D(r)r^2$ is small can be of great astrophysical importance, and this is where Schmidt telescopes can contribute, since the large field can produce usable statistics even for types of stars of low surface density on the sky.

a) r small

The stars of lowest luminosity can be observed only at close range, within which the volume is necessarily small. Even though these stars never contribute significantly to the general star counts, it is of considerable interest to know whether or not the (bolometric) luminosity function declines at very faint absolute magnitudes. In order to find the few stars of very low luminosity, a "pointer" is required, like red colors or large proper motion.

A comment can be made here concerning the relative efficiency of larger telescopes at finding stars of low luminosity. The gain in volume sampled due to the increased depth of a 4m telescope compared with a 1.2m Schmidt does not compensate for the much larger field area of the Schmidt. However, with the larger telescope the "S/N" is higher; at high latitudes, the thinning out of the stellar distribution suppresses the number of distant stars, so the relative number of low-luminosity stars at a fainter apparent magnitude is larger. Another method of suppressing background stars is to observe a field in front of a dark cloud (Haro and Luyten 1961; Herbst and Sawyer 1982). However, in this case the wide field of the Schmidt is not as useful, and the fainter limiting magnitude obtainable with the larger telescope may arguably produce a better result.

b) $\phi(M)$ small

Intrinsically rare stars are also best discovered with wide field instruments. This category includes degenerate dwarf stars, OB sub-dwarfs, blue stragglers, and various types of variable stars. Most of these stars share the property of being blue, and Schmidt telescopes completely dominate the discovery of faint blue stars. The following is an incomplete but (I trust) representative list of some of these surveys: Humason and Zwicky (1947), Luyten and collaborators (1955), Iriarte and Chavira (1957), Feige (1958), Haro and Luyten (1962), Rubin, Moore and Bertiau (1967), Richter, Richter, and Schnell (1968), Jaidee and Lynga (1969), Barbieri and Rosino (1972), Berger and Fringant (1977), Steppe (1978), Green (1980), and Noguchi, Maehara and Kondo (1980). Interest in faint blue stars accelerated in the mid-1960's after the identification of extragalactic objects in extant lists. Still, the Galactic problems were exciting enough, as documented in the Strasbourg conference of 1964 (Luyten 1965). (The proceedings of this conference include an interesting historical discussion by Zwicky on the early role of Schmidt telescopes for survey work.) Generally omitted from the list of papers cited above are surveys explicitly designed to discriminate against Galactic stars.

An important case is the event when $\phi(M)$ is relatively small and r is small -- this is true for white dwarf stars, for which the Schmidt surveys of Luyten and Green have been especially productive at generating finding lists. One key question is the shape and amplitude of the white dwarf luminosity function, especially for the cooler stars (Sion and Liebert 1977). Discovery by proper motion appears to be by far the most efficient technique (Liebert *et al.* 1979).

c) $D(r)$ small

At very large distances, the density function $D(r)$ falls to small values. Thus stars at very great distance are expected to have low surface density, and Schmidt telescopes can contribute towards mapping the outer regions of our Galaxy. An example of the discovery of a faint carbon star at high latitude is Sanduleak and Pesch (1982); indeed, Haro and Luyten (1962) had earlier remarked on the discovery of five extremely red stars, three of which they took to be N stars. (See also Weinberger and Poulakos 1977 and Poulakos 1978.) A good statistical sample of very distant stars does not yet exist (other than RR Lyraes) because of the lack of luminosity indicators easily applicable to faint stars.

III. COLOR AND PROPER MOTION DISTRIBUTIONS

Two main generalizations have been made so far: star counts need to be accompanied by some other measurement, such as proper motion or color, in order to separate different groups of stars; and Schmidt telescopes are uniquely capable of finding stars of low surface density.

In this section I review a variety of Schmidt studies which have included either color or proper motion as an observed quantity, with emphasis on complete, magnitude-limited samples. Since other speakers at this conference will develop the scientific content of at least some of these surveys, I shall restrict myself to one or two remarks in each case about special features of the data. Also, this discussion is biased in favor of high-latitude studies because the interpretation is relatively insensitive to the adopted absorption and reddening. Examples of star count surveys at low latitude are the Case series (McCusky 1965), Becker's (1979) work towards the Galactic center, and Morales-Durán's (1982) study of a region in the anticenter.

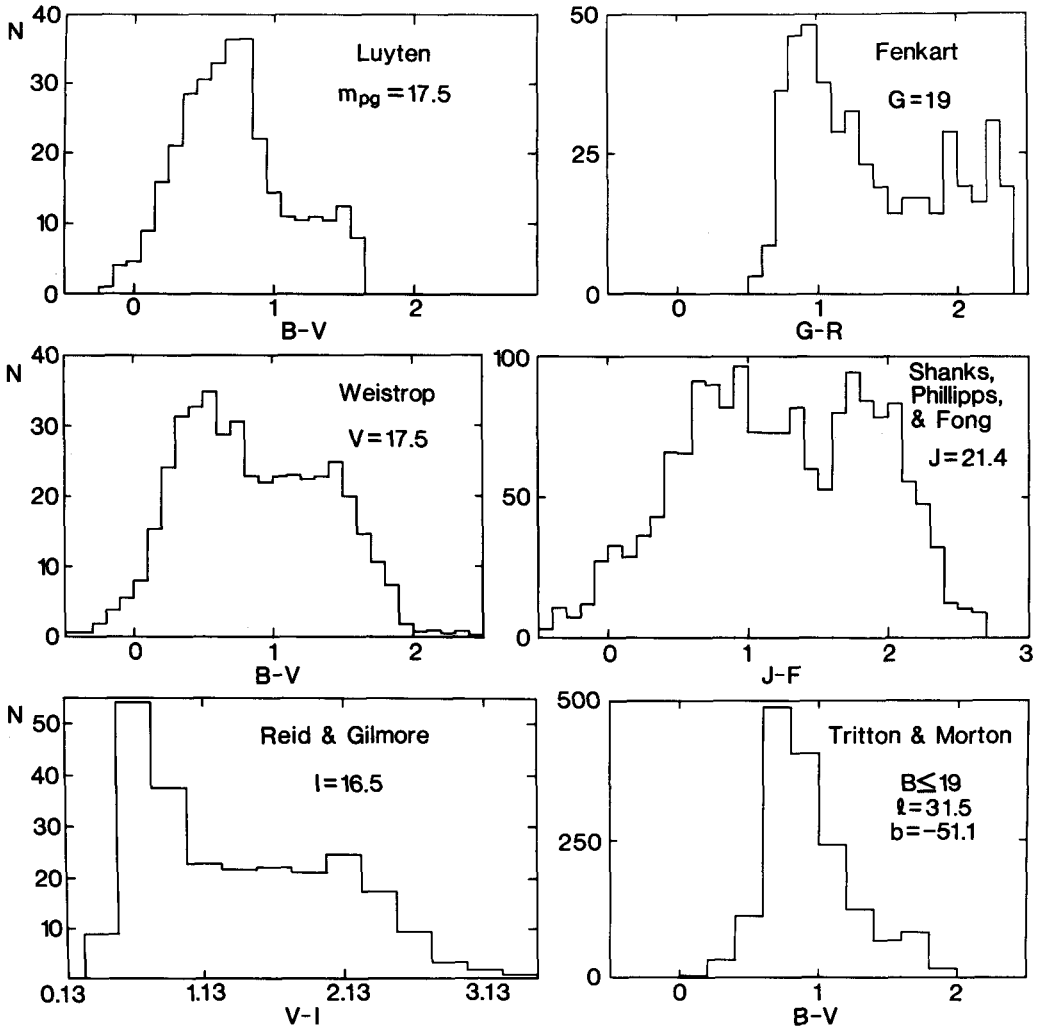
Luyten (1960) produced a color-magnitude array derived from five high-latitude fields. It indicated many more faint blue stars than predicted by a simple model. The model did not include evolved stars, which as realized by Luyten could well help to resolve the problem. Attention was given at the Strasbourg conference to the possibility of systematic color errors near the plate limit (see especially Sandage and Luyten 1967). In Figure 1 Luyten's color distribution is presented for the interval one magnitude brighter than the nominal plate limit.

[The issue of "the nature of the faint blue stars" is relatively convoluted. In part it depends on exactly what is meant by blue: blue in B-V may not be equivalent to blue in U-B, especially as far as the discovery techniques are concerned. Also, it is clear that there is no single identity to these stars, but rather several stellar types appear in the counts (Luyten and Anderson 1962, Kinman 1965), depending sensitively on apparent magnitude and, for lower latitudes, on galactic latitude.]

Uppgren (1962, 1963) made an objective-prism survey of the North Galactic Pole, yielding the numbers of stars as a function of both apparent magnitude ($m_{pg} < 13$) and spectral type (including luminosity class). This work contributed to the picture of differing scale heights for disk stars of differing characteristic ages. There have in addition been several objective-prism surveys for specific types of stars for specific purposes. Among these could be mentioned the Case surveys (e.g., Sanduleak 1976), and Thè and Staller (1974), both aimed at the problem of the faint end of the luminosity function. Proper motions are available for many of the Thè-Staller stars, and all of the Sanduleak stars have been measured for motion (Luyten 1976; Jones and Klemola 1977). The small motions have been found to be largely due to distance, rather than due to low space velocities.

The large RGU survey by the Basel group began with SA 51 (Becker 1965), and the North Galactic Pole was observed by Fenkart (1967) to a limit $G = 19.5$. Their principal analysis technique is the separation of Pop I from Pop II via the positions of stars in the U-G, G-R diagram, according to apparent magnitude. The distribution of stars in the two-color diagram changes markedly with increasing apparent magnitude, with

Figure 1. Color histograms for each of six complete high-latitude samples. All counts reduced to 1 deg^2 , 1 mag interval in the indicated waveband (except for Morton and Tritton), and 0.25 mag interval in the indicated color.



(among other effects) progressively greater ultraviolet excess. The morphology of the two-color diagram is rich in structure and thus provides excellent constraints on star count models.

Subsequently Weistrop (1972) made an accounting of the stellar population towards the North Galactic Pole using similar observational techniques (UBV instead of RGU, $V < 18$) but a rather different analysis procedure. Globular cluster CM diagrams were assumed to apply for halo stars, and for conventional halo parameters reasonable agreement with the data was obtained, at least for the bluer stars.

Shanks, Phillipps, and Fong (1980) used the UKST for star counts and colors at the South Galactic Pole. The data were obtained on fine-grained emulsions, and automatic machine measurements (COSMOS) were used instead of an iris photometer. At their limit of $B - J = 21.5$, star/galaxy separation becomes a problem because the Schmidt scale is small and the number of galaxy images is at least comparable (Hubble 1936). Godwin and Peach (1982), in a work dealing with similar plate material, were more conservative in their assessment of the reliability of star/galaxy discrimination at faint limits, even though they used a PDS machine to digitize their plates. The Shanks, Phillipps, and Fong data are at least qualitatively in agreement with other studies, showing a bimodal color distribution at faint limits (Figure 1), in agreement with Fenkart's (1967) findings.

Reid and Gilmore (1982) have made V-I measurements at the South Galactic Pole, $I < 17$. The data are especially valuable for studies of red main sequence stars, since M_I is well-correlated with V-I. A subsequent analysis (Gilmore and Reid 1983) advocates a fat-disk model to describe the distribution of the bluer stars, but it is not clear that their interpretation is unique (Bahcall and Soneira 1983).

Bahcall *et al.* (1983) have discussed Tritton and Morton's UKST color measurements for a somewhat lower-latitude field; these are included in Figure 1 for comparison with the other B-V histograms.

As far as proper motion surveys with Schmidt telescopes are concerned, Luyten's (1963) work provides an extensive data base for the analysis of stellar statistics. A graphical summary has been given by Luyten (1977), in which he presents reduced proper motion diagrams for a total of over a hundred thousand stars. These diagrams, like the two-color diagrams, contain much structure, having in addition dynamical information. Even more structure would no doubt be apparent if the reduced proper motion diagrams were constructed for different regions of the sky separately. Two other Schmidt proper motion studies of faint stellar samples are Schilbach (1982) and Noguchi, Yutani, and Maehara (1982), but both of these were restricted to blue stars.

To fix some idea of the numbers involved, Table 1 gives motions on the basis of a very simple model: the line of sight is perpendicular to

the plane, the density distribution is assumed to be exponential with a scale height $z_0 = 350$ pc, and the number of stars $\text{pc}^{-3} \text{mag}^{-1}$ in the plane is assumed to be 0.01 (approximately valid for main sequence stars in the range $10 < M_V < 15$). These stars would have $\langle z^2 \rangle^{1/2} \sim 20 \text{ km sec}^{-1}$, so that at high latitudes the transverse velocities would be about 47 km sec^{-1} rms. Indicative proper motion dispersions in arc sec per year using this value are given. Table 1 also gives similar calculations for a stellar population like that advocated by Gilmore and Reid (1983), namely $z_0 = 1500$ pc, $\langle T^2 \rangle^{1/2} = 130 \text{ km sec}^{-1}$, and with a density in the plane $1/50$ that of the thin-disk population.

TABLE 1. Star Counts Towards the Pole per Deg²

m-M	D(0) = 0.01 z ₀ = 350 pc		D(0) = 0.0002 z ₀ = 1500 pc	
	A(m)	μ _T	A(m)	μ _T
0	0.0015	0.99		
2	0.022	0.39		
4	0.31	0.16	0.0073	0.43
6	3.7	0.063	0.11	0.17
8	29	0.025	1.45	0.069
10	79	0.010	15	0.027
12	18.5	0.0039	83	0.011
14	0.019	0.0016	103	0.0043
16			4.1	0.0017

When A(m) in the table is in the neighborhood of 0.03 stars deg⁻², we expect about one star per Schmidt field. We also require proper motion greater than about 0".015 yr⁻¹ for measurement over a baseline of, say, 25 years. According to Table 1, one expects to find one M_V = +19 star at V₁ = 21 if the space density of such stars is close to 0.01 star pc⁻³ mag⁻¹; moreover, that star should have substantial proper motion. Reid and Gilmore (1981) did in fact find one such star (using red-sensitive plates) in a UKST field. These remarks can be generalized easily to white dwarfs. For instance, if at M_{pg} = +13 one star out of 25 is a white dwarf (Luyten 1960), then according to Table 1 at a distance modulus of 6 there would be expected to be about five white dwarfs with measurable motions of this absolute magnitude per Schmidt plate, if they have a scale height of 350 pc.

IV. CONCLUSIONS

Counts of common stars might as well be done with other types of telescopes. Telescopes of longer focal length enable more accurate photographic photometry -- in a sense focal Schmidt images may be too good, i.e., too small for really accurate measurement (Zwicky 1965). Also, focus variations across the plate with resulting photometric difficulties may be less of a problem at slower focal ratios. For bright stars, the great speed of Schmidt cameras is not needed. For faint stars, the statistics are good enough in small areas, and the wide field is not needed. For very faint stars, greater plate scale is required to separate with high confidence galaxies from stars.

On the other hand, the statistics of uncommon faint stars can be obtained only with Schmidt-type telescopes, as long as some kind of pointer is available to isolate the interesting stars from the mass of common stars. Examples have been given of the many very substantial contributions of Schmidt telescopes in this respect. Surveys for distant luminous stars, nearby stars of low luminosity, and nearby stars of high velocity are all of great interest. These surveys should be designed to provide more than just counts; two colors and proper motions are not difficult to generate even for very large numbers of faint stars. At brighter magnitudes, automated spectral classification is a program well worth pursuing.

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