MODE-COUPLED MHD WAVES IN THE CORONA AND SOLAR WIND

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The purpose of this paper is to outline a model of mode-coupled MHD compressional waves in the corona and solar wind. The eventual aim of this work is to be able to compute how MHD waves propagate through the corona and into the solar wind beginning with a source of Alfven or fast mode waves at the base of the corona. The necessity for consideration of mode coupling arises because of typical scalelengths in the corona. For wave sources, such as supergranulation, with wave periods of about a day, the different modes do no propagate independently, as in the WKB approximation, but are coupled because the ratio of wavelength to scalelength is of the order of one or greater.

The mathematical basis for this work is a model of small amplitude, poloidal, axisymmetric waves in an axisymmetric background flow in which solar rotation is neglected. The main effort has been to relax the WKB assumption: that is, we treat the wave equations directly. The formulation is based on the poloidal components of the ideal MHD equations: the mass conservation equation, Euler's equation (with a scalar pressure and including gravity), the ideal Ohm's law, and a polytropic pressure relation. We have written the equations in terms of the Fourier amplitudes of the fluctuations of the total pressure, the components of the flow and Alfven velocities parallel and perpendicular to the background magnetic field, and the density. In terms of these amplitudes, the equations in hyperbolic regions are

$$\partial_\pm\left\{\rho\sqrt{(V_A^2-V^2)\left[V^2(V_A^2+c_s^2)-V_A^2c_s^2\right]}\,\delta V_\perp \pm \sqrt{(V^2-c_s^2)}\,\delta P_T\right\} = \cdots \tag{1-2}$$

$$\partial_\parallel(V\delta V_{A\perp}-V_A\delta V_\perp) = \cdots \tag{3}$$

$$\partial_\parallel\left[\rho(V\delta V_\parallel - V_A\delta V_{A\parallel})-\tfrac{1}{2}V_A^2\delta\rho+\delta P_T\right] = \cdots \tag{4}$$

$$\partial_\parallel\left[\rho(V_A\delta V_\parallel - V\delta V_{A\parallel})-\tfrac{1}{2}VV_A\delta\rho\right] = \cdots \tag{5}$$

$$\partial_\parallel\left[(V_A^2+c_s^2)V\delta\rho+\rho V_A^2\delta V_\parallel - V\delta P_T\right] = \cdots \tag{6}$$

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The right hand sides of the equations, which contain terms proportional to the amplitudes and determine the coupling, are omitted for brevity. Even though these equations are complicated, they have several simple features and are transparent enough to outline a method of solution. Of the six equations, the first two are written as partial derivatives along two families of characteristics, shown as the curved lines in the hyperbolic regions in Figure 1. They are drawn for radial field lines only because they are much more complicated for nonradial geometries. The equations contain explicit singularities which occur

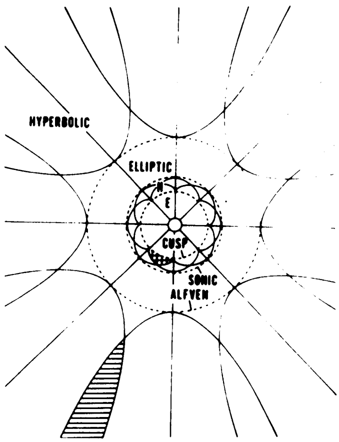

Figure 1. The characteristics and singular lines.

where the flow speed is equal to one of the MHD group velocities along the field direction: the Alfven speed, the sound speed, and the speed of the cusp at the rear of the slow mode wave front. These singularities lie along lines defined by the background flow; the singular lines are shown as dotted lines in Figure 1. The remaining four equations are written as partial derivatives along the field lines. In the hyperbolic regions, the equations can be integrated as ordinary differential equations along the characteristics and the field line. In elliptic regions, the characteristics are not defined; the first two equations take a different form there.

These equations describe the coupling of the MHD fast and slow mode waves and two entropy modes. This identification can be made by replacing $\underline{\nabla}$ by $i\underline{k}$, retaining terms involving $i\omega$, and setting the gradients on the RHS to zero. The result is that there are two fast and two slow wave modes which occur in forward and backward propagating pairs for a given $\underline{k}$. The physical significance of the characteristics is that they represent the envelopes of the fast and slow mode wave fronts: they are the Mach "cones" of the magnetized plasma (shown as cross-hatched areas in Figure 1). By a standard construction, shown in Figure 2, one can show that when the flow velocity lies inside the slow mode wave front or outside the fast mode wave front there are

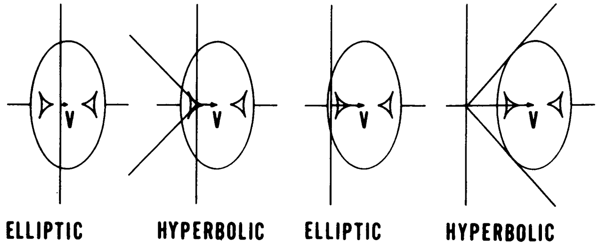

Figure 2. Relation of the wave modes to the characteristics.

real characteristics, but not otherwise. It is clear from Figure 2 that the inner hyperbolic region represents the envelope of slow mode waves propagating toward the Sun while the outer hyperbolic region represents the envelope of fast mode waves being convected away from the Sun.

The terms on the right hand side describing the coupling are quite long and tedious. However, there are only a few essentially different quantities which appear. They are: (1) the curvature of the field lines, (2) derivatives of the density, and (3) derivatives of field line constants characterizing the background flow.

In attempting to solve the equations, the singular lines are of particular importance. We have obtained regularity conditions, in the form of the vanishing at the singular lines of linear combinations of quantities related to the amplitudes, which are required for smooth flow through the lines. Physical interpretations are: the cusp line is singular for slow mode waves propagating across (i.e., $\underline{k}$ perpendicular to) the field, the sonic line is singular for slow mode waves propagating backward along the field, and the Alfven surface is singular for fast mode waves propagating backward along the field. The regularity conditions, which are similar to the Parker critical point condition, are equivalent to boundary conditions: each one reduces by one the number of free boundary conditions that may be stated at the base of the corona.

Based on this analysis, it is possible to outline a method of numerical solution. Methods in each type of region are standard. For given boundary conditions, solutions can be obtained in the elliptic regions by the method of relaxation. In the hyperbolic regions they can be obtained by integrating along the characteristics. Because the boundary conditions and the regularity conditions are not stated on the same lines, however, the solutions cannot be obtained all at once. The novel feature of this problem is that it will be necessary to iterate among solutions in the different regions in order to satisfy simultaneously the regularity conditions and the boundary conditions at the base of the corona.

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