PAC degrees then those properties will be true for PACi and PAC degrees without explicitly proving them.

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Miguel Antonio Cardona-Montoya, ***Forcing theory and combinatorics of the real line,*** Vienna University of Technology, Vienna, Austria, 2022. Supervised by Jakob Kellner and Diego A. Mejía. MSC: 03E17, 03E15, 03E35, 03E40.

Abstract

The main purpose of this dissertation is to apply and develop new forcing techniques to obtain models where several cardinal characteristics are pairwise different as well as force many (even more, continuum many) different values of cardinal characteristics that are parametrized by reals. In particular, we look at cardinal characteristics associated with strong measure zero, Yorioka ideals, and localization and anti-localization cardinals.

In this thesis we introduce the property "F-linked" of subsets of posets for a given free filter F on the natural numbers, and define the properties "μ-F-linked" and "θ-F-Knaster" for posets in a natural way. We show that θ-F-Knaster posets preserve strong types of unbounded families and of maximal almost disjoint families. These kinds of posets led to the development of a general technique to construct θ-Fr-Knaster posets (where Fr is the Frechet ideal) via matrix iterations of $<\theta$-ultrafilter-linked posets (restricted to some level of the matrix). The latter technique allows proving consistency results about Cichoń's diagram (without using large cardinals) and to prove the consistency of the fact that, for each Yorioka ideal, the four cardinal characteristics associated with it are pairwise different. Another important application is to show that three strongly compact cardinals are enough to force that Cichoń's diagram can be separated into 10 different values. Later on, it was shown by Goldstern, Kellner, Mejía, and Shelah that no large cardinals are needed for Cichoń's maximum (*J. Eur. Math. Soc.* 24 (2022), no. 11, p. 3951–3967).

On the other hand, we deal with certain types of tree forcings including Sacks forcing, and show that these increase the covering of the strong measure zero ideal $\mathcal{SN}$. As a consequence, in Sacks model, such covering number is equal to the size of the continuum, which indicates that this covering number is consistently larger than any other classical cardinal characteristics of the continuum. Even more, Sacks forcing can be used to force that $\mathrm{non}(\mathcal{SN}) < \mathrm{cov}(\mathcal{SN}) < \mathrm{cof}(\mathcal{SN})$, which is the first consistency result where more than two cardinal characteristics associated with $\mathcal{SN}$ are pairwise different. To obtain another result in this direction, we provide bounds for $\mathrm{cof}(\mathcal{SN})$, which generalizes Yorioka's characterization of $\mathcal{SN}$ (*J. Symbolic Logic* 67.4 (2002), p. 1373–1384). As a consequence, we get the consistency of $\mathrm{add}(\mathcal{SN}) = \mathrm{cov}(\mathcal{SN}) < \mathrm{non}(\mathcal{SN}) < \mathrm{cof}(\mathcal{SN})$ with ZFC (via a matrix iteration forcing construction).

We conclude this thesis by combining creature forcing approaches by Kellner and Shelah (*Arch. Math. Logic* 51.1–2 (2012), p. 49–70) and by Fischer, Goldstern, Kellner, and Shelah (*Arch. Math. Logic* 56.7–8 (2017), p. 1045–1103) to show that, under CH, there is a proper ω^ω-bounding poset with $\aleph_2$-cc that forces continuum many pairwise different cardinal

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characteristics, parametrized by reals, for each one of the following six types: uniformity and covering numbers of Yorioka ideals as well as both kinds of localization and anti-localization cardinals, respectively. This answers several open questions from Klausner and Mejía (*Arch. Math. Logic* 61 (2022), pp. 653–683).

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Francesco Paolo Gallinaro, ***Around Exponential-Algebraic Closedness***, University of Leeds, UK, 2022. Supervised by Vincenzo Mantova. MSC: Primary 03C65. Secondary 11F03, 11L99, 14K12. Keywords: abelian varieties, algebraic groups, Exponential-Algebraic Closedness, exponential function, modular *j*-function, quasiminimality.

Abstract

We present some results related to Zilber's Exponential-Algebraic Closedness Conjecture, showing that various systems of equations involving algebraic operations and certain analytic functions admit solutions in the complex numbers. These results are inspired by Zilber's theorems on raising to powers.

We show that algebraic varieties which split as a product of a linear subspace of an additive group and an algebraic subvariety of a multiplicative group intersect the graph of the exponential function, provided that they satisfy Zilber's freeness and rotundity conditions, using techniques from tropical geometry.

We then move on to prove a similar theorem, establishing that varieties which split as a product of a linear subspace and a subvariety of an abelian variety A intersect the graph of the exponential map of A (again under the analogues of the freeness and rotundity conditions). The proof uses homology and cohomology of manifolds.

Finally, we show that the graph of the modular j-function intersects varieties which satisfy freeness and broadness and split as a product of a Möbius subvariety of a power of the upper-half plane and a complex algebraic variety, using Ratner's orbit closure theorem to study the images under j of Möbius varieties.

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Rahman Mohammadpour, ***New methods in forcing iteration and applications***, Institut de Mathématiques de Jussieu-Paris Rive Gauche, Université de Paris, Paris, France, 2020. Supervised by Boban Veličković. MSC: 03E05, 03E35, 03E40, 03E55, 03E57. *Key words and phrases*. guessing model, approachability ideal, PFA, higher forcing axioms, Magidor models, side conditions.

Abstract

The Theme. Strong forcing axioms like Martin's Maximum give a reasonably satisfactory structural analysis of $H(\omega_2)$. A broad program in modern Set Theory is searching for strong forcing axioms beyond ω_1. In other words, one would like to figure out the structural properties of taller initial segments of the universe. However, the classical techniques of

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