SUPERNOVA EXPLOSIONS - THE ROLE OF A RAYLEIGH-TAYLOR INSTABILITY

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ABSTRACT

A two dimensional hydrodynamic study indicates that convectively unstable gradients which develop during core collapse and bounce give rise to large scale core overturn. It is also shown that the concomitant release of neutrini can deposit large amounts of energy and momentum in the infalling envelope and give rise to a powerful supernova explosion.

1. INTRODUCTION AND ANALYTIC ARGUMENTS

The current scenarios for type II supernova explosions involve the collapse of a dense stellar core triggered by iron dissociation and electron captures, which is halted when the equation of state stiffens at about nuclear matter density. The concomitant bounce gives rise to the generation of a shock wave. Another consequence of the collapse is the production of a large number of electron neutrinos via electron captures. Two major mechanisms are thought to be responsible for the explosion and envelope ejection: (1) Ejection by the shock wave (Van Riper 1978, Arnett 1980, Lichtenstadt et al 1979). (2) Ejection induced by the interaction of the neutrinos with the mantle, either by energy or momentum deposition. Both mechanisms and their combined actions have met with severe difficulties. Shock ejection seems to be very sensitive to details stemming either from the equation of state (such as the adiabatic index) or from neutrino damping (e.g., Van Riper 1978, Wilson 1979). Several new arguments favoring shock ejection have been recently thought up; these involve preheating of matter by the shock (Arnett 1980) and more refined calculation of the available energy (Yahil 1980). Neutrino induced ejection has been originally suggested by Colgate and White (1966). However, neutral currents cause the neutrinos to be trapped in the core at densities above 10^{12} g/cm (Arnett 1977, Mazurek 1977, Yueh and Buchler 1977). Colgate (1978) and Colgate and Petschek (1980) have

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recently suggested that a large scale core overturn could release these neutrinos and a sufficient energy transfer to blow off the envelope. Their suggestion followed by the observation made by Epstein (1979) that an unstable lepton graident develops after bounce. Some insight into this situation can be gained from the linear criterion for stability to nonradial perturbations:

$$\frac{1}{\rho}\left(\frac{d\rho}{dr} - \frac{1}{c_s^2}\frac{dp}{dr}\right) = -\frac{1}{\rho c_s^2}\left[\frac{\partial p}{\partial s}_{\rho Y_L}\frac{ds}{dr} + \left(\frac{\partial p}{\partial Y_L}\right)_{\rho s}\frac{dY_L}{dr}\right] < 0$$

An outwardly decreasing lepton number Y_L thus has a destabilizing effect. Such potentially unstable lepton graidents are obtained in collapse calculations (e.g., Wilson 1979), and they result from the fact that neutrinos are trapped inside the "neutrinosphere," while they can stream rather freely from regions above it.

A number of questions arise at this point:

1. Does a real collapse ever become Rayleigh-Taylor unstable?
2. If an unstable situation occurs, which unstable modes dominate and what are the time scales involved?
3. Is the neutrino flux released from such an overturn enough to cause an explosion?
4. Does the overturn have other dynamical consequences, which in themselves, or by helping the shock, can cause ejection?

In order to answer the second question, Livio, Buchler, and Colgate (1980) have resorted to a numerical 2-dimensional hydrodynamical modeling of the collapse. The hydrodynamics is done with a radially moving, fixed angular, Eulerian mesh. Neutrino transport is approximated with a leakage scheme; above some density $\cong 10^{12}$ g/cm, neutrinos are assumed to be trapped and in beta kinetic equilibirum, whereas below that density they are allowed to stream out with a rate parameterized to reproduce the results of one-dimensional neutrino transport calculations. The equation of state used was that of Bruenn (1975). Because of the high collapse adiabat chosen, a thermal bounce was obtained at a relatively low density (2×10^{13} g/cm). This in itself does not qualitatively affect the development of the instability, since the time scale for the overturn, like the dynamic time scale, scales like $1/\sqrt{\rho}$. In the calculation described here, the initial model was chosen as an n = 3 polytrope with central density $\rho_c = 5 \times 10^9$ g/cm, a temperature $T_c = 1.1 \times 10^{10}$ K and a composition of ^{56}Fe. The bounce occurred at $\rho_{bounce} \cong 1.2 \times 10^{13}$ g/cm and $T_{bounce} \cong 1.2 \times 10^{11}$ K. Since rotational deformation is expected to play the dominant nonradial role in the collapse, Livio et al (1980) have introduced a centrifugal acceleration of order 10^{-4} - 10^{-3} of the gravitational one, which has been switched off after about 10 ms giving rise to nonradial velocities of the order of 3% of the speed of sound. Figure 1a shows the velocity field 31 ms after the bounce and exhibits an $\ell = 2$ core overturn. The rise time for the instability is of the order of a few milliseconds. As a check, a model has been collapsed in which the unstable lepton gradient was not generated (by artificially increasing the neutrino diffusion time), in this case no overturn was obtained.

The development of smaller scale convective modes ($\ell > 2$) is of interest, and especially their interaction with the $\ell = 2$ mode. To that effect Livio, Buchler, and Colgate (1980) have introduced in addition to the $\ell = 2$ and $\ell = 8$ perturbation of the form

$$\delta v_r = \alpha[(\frac{r}{R})^{\ell-1} \Theta(R - r) + (\frac{R}{r})^{\ell+2} \Theta(r - R)] P_\ell(\cos\theta),$$

which was inspired by the exact solution to the linearized perturbation equation of an incompressible stratified sphere. α was chosen such that $\delta v_r/c_s \lesssim 0.001$. Figure 1b shows the velocity field 28 ms after the bounce. The interesting development of the perturbation is such that in the nonlinear phase one vortex is pushed outward while the others merge so that the large scale $\ell = 2$ mode takes over eventually.

In order to see the ultimate effect of such an overturn on the envelope, Bruenn, Buchler, and Livio (1979) have performed a one-dimensional simulation of the overturn using an accurate multigroup neutrino transport mode in conjunction with a Lagrangian hydrocode. They find that under the conditions described above, the net result of the overturn will be a huge release of neutrinos which then deposit a considerable amount of energy in the envelope and cause a violent explosion. The physical reason is that the hydrodynamics dredge up neutrino rich material to low densities; these neutrinos are super-

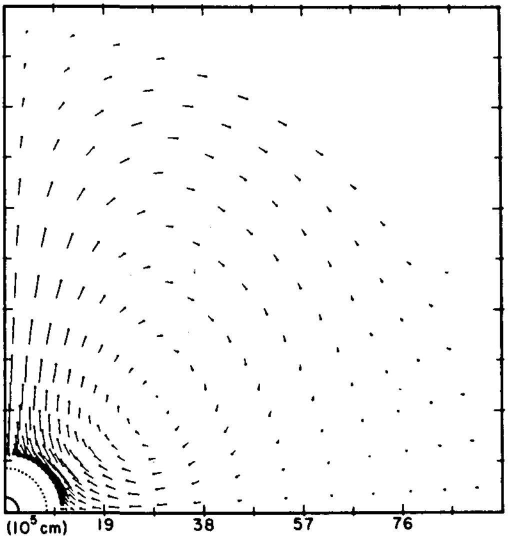

Fig. 1a. Convective flow, $\ell = 2$ perturbation, t = 31 ms.

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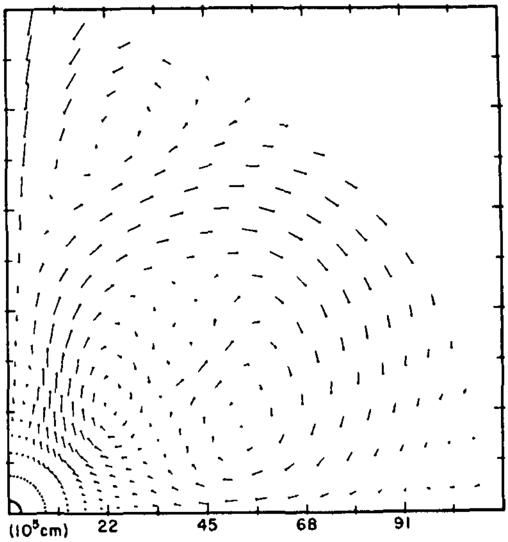

Fig. 1b. Convective flow, ℓ = 8 perturbation, at 28 ms.

thermal because at these densities they have not had time to equilibrate on the dynamic time of the overturn.

An important feature of these collapse calculations related to the above mentioned question (1) is that the entropy increase in the shocked matter, i.e., *outside* the homologously collapsed core, is insufficient to prevent a Rayleigh-Taylor instability over the whole core. This is in contrast to a subsequent calculation of Smarr et al (1980) who collapse on a lower adiabat and use a highly simplified equation of state; they obtain a shock so strong as to lead to an entropy increase of about 4 k/baryon which prevents then any convective penetration of the inner core; however, a large scale convective motion develops in the outer half of the core.

One question which arises is whether the high entropy of the shocked material can be removed by neutrinos on a sufficiently short time scale to destabilize the whole core. A rough estimate indicates that although (muon and tau) neutrino radiation from the neutrino sphere could cause a sufficiently fast cooling of the shocked region, the thermal neutrino production rates (Kolb and Mazurek 1977), mainly by electron-position pairs and de-excitation of nuclei, are too slow because of the high electron degeneracy and concomitant low pair concentration.

It has been pointed out on the basis of shell model considerations (Fuller, Fowler, and Newman 1980) that electron captures are inhibited for neutron rich nuclei. As a result the core retains a higher lepton concentration, which goes in the direction of destabilizing the core. Because of the larger lepton concentration the adiabatic index then remains also closer to 4/3 which is expected to lead to a larger homologously collapsing core (Goldreich and Weber 1980) with a shock at lower density. At the same time the pressure defect will be smaller so that one expects a gentler bounce. Both of these effects favor the occurrence of a Rayleigh-Taylor instability.

In this respect it is relevant to ask the question what entropy deleptonized matter (with say, $Y_L = 0.1$) must have so that upon adiabatic compression to some pressure it has the same density, ρ_{local} as the local matter (neutral buoyancy), whose lepton concentration is $Y_{L,local}$ i.e., $p(\rho_{local}, s, Y_{L,local}) = p(\rho_{local}, s, Y_L = 0.1)$. In other words one can plot entropy versus density curves for various values of $Y_{L,local}$, which has been done by Van Riper (1980) using the Lamb et al (1978) equation of state and is shown in Fig. 2. One can see that at a typical bounce density of 2 ρ_{nuc} and $Y_\ell = 0.35$ that the critical entropy of exterior lepton-depleted matter of $Y_\ell = 0.1$ above which overturn will be inhibited is no greater than $s/k = 2.6$. If the

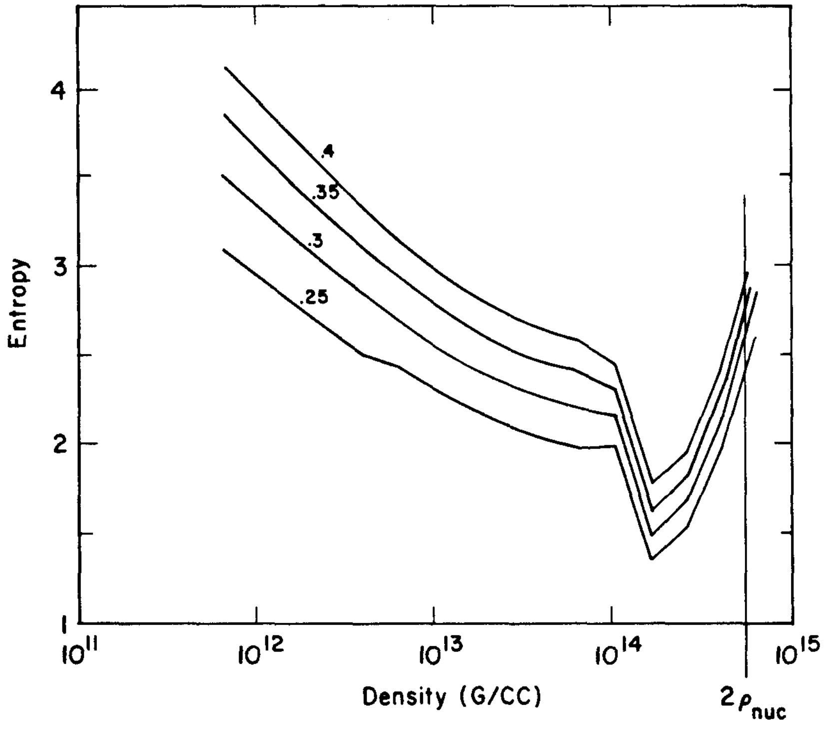

Fig. 2. The entropy that can substitute for the pressure defect of lepton depletion is shown versus density. The lepton depletion corresponds initially to the numbers on the curves and a depleted value of $Y_\ell = 0.1$. The initial entropy is $s/k = 1.2$. Curve is by Van Riper using the equation of state of Lamb et al (1978).

neutrino sphere where this matter has been depleted is at $\rho = 10^{12}$ g cm^{-3}, then an entropy of 2.6 corresponds to a temperature of only $\cong$ 2.5 MeV. This is very low compared to the shock entropy of ≥ 7 calculated by Van Riper (1980) and so would require significant time to cool. If we use the neutrino emission rate of Kolb and Lattimer (1979) assuming some nuclei are present, then $\varepsilon/\dot{\varepsilon} \cong 1$ s. This is long and indicates that the neutrino sphere will move toward greater densities. At $\rho = 10^{13}$, s/k of 2.6 results in T = 6 MeV and $\varepsilon/\dot{\varepsilon} = 10^{-5}$ s. Very few nuclei need to be present to ensure a black body emission rate. Therefore, between these densities a black body neutrino sphere should exist. The cooling is then governed by the scattering in the exterior shock heated higher entropy (s/k $\gtrsim$ 7) matter. Since no nuclei will remain, the mean free paths are long and we expect cooling characteristic of a black body at an intermediate density, say 5×10^{12} and T $\cong$ 5 MeV. At $r \cong 4 \times 10^6$, $\dot{E} \cong 10^{53}$ ergs s^{-1}. Since the total energy is of the order 5×10^{51} ergs, the cooling time to below 5 MeV will be of the order of a few 10's of ms. We therefore foresee both cooling and lepton depletion in higher density matter $\cong 10^{13}$ g cm^{-3} with the pressure of a high entropy mantle preventing further accretion. It is then possible that instability and overturn may occur.

It is clear that more work needs to be done to see what form of Rayleigh-Taylor instable overturn occurs and what effect it has on the supernova explosion.

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DISCUSSION

WHEELER: You talked about your high entropy collapse and bounce at 10^{13} g/cm^3 and a low entropy collapse and bounce at 10^{14} g/cm^3. Tohline's calculations with a not excessive amount of rotation gave a low entropy collapse which bounced at 10^{13} g/cm^3. If you don't get as much of a bounce, you won't generate that very high entropy outside, which is stopping Smarr from turning it over. Isn't that doing what you want to do, stopping it with rotation rather than with the bounce?

BUCHLER: For the overturn, you stop the collapse at the lower density. But it is extremely sensitive to lots of things. I don't know if it is the entropy or the equation of state that gives the difference.