Statistical inference on lattices

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In many scientific disciplines, there is frequently a need to describe
purely spatial interactions among objects located at the sites of a
lattice. Of particular interest are the equilibrium states of physical and
biological phenomena occurring simultaneously at sites in more than one
dimension (for example, ferromagnetism, crystal formation, patterns of
infection). Markov random fields form a wide class of intuitively
appealing models for spatial interaction. Binary ones have been studied
extensively in statistical mechanics where they are known as Ising models.

In the biological and ecological context, it is the fitting of data
and analysis of given patterns which is most relevant. Statistical
inference for Markov random fields is a relatively new field. In
particular, Ising models are notoriously difficult to analyse and the
absence of constructive characterizations has rendered Monte Carlo
simulation an ineffective tool for comparing various statistical
techniques.

This thesis provides a basis for asymptotic maximum likelihood
inference for binary Markov random fields. The general method is
illustrated by the detailed analysis of the classical Ising lattice with
nearest neighbour interactions. However, the resulting limit theorems for
this model are of considerable interest in their own right. Some rather
surprising technical problems arise in connection with the application of
the limit theorems. A solution is presented.

Also, a class of relatively tractable Ising models, which admit
constructive characterizations and direct simulation, is introduced in this
thesis. The properties of these models are explored. In particular, they

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generally have a geometric correlation structure and contain embedded
Markov chains. Moreover, they can further be used in simulation studies to
compare the relative effectiveness of various methods of spatial analysis.

Finally, although all the results are for binary models, the tools are
useful in much wider generality. They apply, for example, to gaussian
Markov random fields as well.