SELECTION EFFECTS AGAINST SMALL COMETS

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Abstract

There are three factors that inhibit detecting small comets relative to large comets: 1. They must pass closer to the Earth than large comets to be detected. 2. The resulting higher angular velocity for nearby comets means that, at comparable apparent magnitudes, there is less time available to discover a small comet relative to a large comet. 3. Small comets physically decay and vanish faster than do large comets. With these three factors, it can be shown that if the small comet input distribution $g_i(R_n)$ varies as $R_n^{-\beta_i}$, where R_n is the nuclear radius of the comet, then the observed distribution will vary as $g_i(R_n)\, R_n^{5.5} \sim R_n^{5.5-\beta_i}$.

1. Introduction

The fainter a comet is, the more difficult it is to discover. This obvious fact gives rise to a catalogued bias against faint comets that needs to be corrected before the true magnitude number distribution can be obtained. Everhart (1967) showed that the probability of detecting a comet is jointly proportional to how far (in magnitudes) the apparent magnitude of a comet remains above some threshold magnitude for detection and to how long it remains above that threshold magnitude. The optical threshold magnitude depends both upon observing conditions and upon telescopic technique but, under good conditions, is around $H_t = 12$ for most amateur comet searchers.

We adopt Everhart's formulation in order to determine how the probability of discovering a comet depends on its nuclear radius. We concentrate our analysis upon comets that are so small (not more than two or three hundred meters in diameter) that they would normally only be optically detectable near the Earth. Heliocentric variations in cometary brightness can then be neglected. We will find that there is such a strong observational bias against

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detecting small active comets that it is not surprising that small comets are not observed -- even though their true spatial density may be high.

We assume here that "small" (as defined above) active comets are detected only by reflected sunlight from their dust comas, and not from the gaseous part of the coma, nor by direct detection of the nucleus. The reason the gaseous part of the cometary coma should practically be invisible for small nuclear radii is due to the following considerations: The C_2 Swan band emissions, that are primarily responsible for the visual brightness of the gaseous coma, increase in integrated luminosity with increasing distance from the cometary nucleus up to about 3×10^4 km at one AU. Inside this "creation" distance for the C_2 bands, the coma brighness is nearly constant and depends on the square of the nuclear radius. When the nuclear radius is too small, no part of the the gaseous coma brightness will be visible against the celestial background light. The dust coma brightness, however, will still be above the visibility threshold near the nucleus of a small active comet.

2. The Dust Coma

The spatial mass density, $p(R')$, due to outflowing dust, at distance R' from the center of the comet is given by

$$p(R') = \frac{\dot{m}}{4\pi\,(R')^2\,\nu_t} = \frac{f\,\dot{R}_n}{\nu_t}\left(\frac{R_n}{R'}\right)^2 , \qquad (1)$$

where ν_t is the terminal radial velocity of dust grains due to drag by outflowing gas, $\dot{m}$ and $\dot{R}_n$ are the time rate of change of cometary mass and radius, respectively, and f is the fraction of the cometary surface that is actively releasing gas and dust. ν_t is related to cometary radius by $\nu_0 R_n^{\delta}$ where, from Probstein (1968) and Delsemme and Miller (1971), one can derive that $\delta\approx0.5$ for small comets, and ν_0 is a constant.

The brightness of the coma at an angle ϕ from the nucleus is proportional to the optical depth of dust grains along the line of sight (see fig. 1). Integrating over the coma surface brightness out to the angle at which the brightness vanishes into the background light, one obtains the integrated luminosity, L, of a comet: L is then related to comet nuclear radius, R_n, and comet-Earth distance, Δ, as

$$L \sim \frac{R_n^{\,3}}{\Delta^2} . \qquad (2)$$

This relation assumes $\delta = 0.5$. To a given limiting apparent magnitude (or integrated luminosity), the distance to the comet varies with comet nuclear radius as $\Delta \sim R_n^{3/2}$.

3. Discovery Probability Versus Cometary Radius

From Figure 7 in Everhart (1967), the probability of

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discovering a comet is given by

$$P = S_o/80 \qquad (3)$$

where S_o is defined by

$$S_o = \int_{t_1}^{t_2} (H_t - H)\, dt \quad . \qquad (4)$$

H_t is the maximum apparent magnitude, or threshold magnitude, above which the comet is unlikely to be discovered by visual comet searches, and H is the apparent magnitude of the comet at time t. t_1 and t_2 are the times, in weeks, when the comet first becomes brighter or, respectively, dimmer, than H_t. The detection geometry for a hypothetical comet is depicted in Figure 2. Depicted there are the threshold distance Δ_t at which the comet first becomes detectable, the distance Δ_{min} of closest approach to Earth, and an intermediate distance Δ.

Space limitations preclude a rigorous mathematical treatment here, but one can qualitatively deduce much of the logic from inspection of Figure 2. The probability that our hypothetical comet will pass within the threshold distance, Δ_t , for detection is proportional to Δ_t^2. In addition, it is possible to show, using equations (3) and (4) that once it passes within Δ_t of the Earth, the probability that the comet will be detected is proportional to Δ_t. One can see this result in qualitative terms, by noting that for a given chord across the circle in Figure 2, that the time of passage along that chord is proportional to Δ_t(assuming constant comet-Earth relative velocity). This is the same time that appears in the sum in equation (4).

Thus, for a comet whose discovery sphere has radius, Δ_t, its probability of discovery is proportional to Δ_t^3 . However, from equation (2) we note that $\Delta_t \sim (L_t)^{1/2} R_n^{3/2}$, where L_t, corresponding to H_t, is the threshold integrated luminosity of a comet that can be detected at distance, Δ_t. Therefore the probability, $P(R_n)$, of detecting small, active comets whose nuclear radius, R_n lies in the range dR_n, is given by the proportionality

$$P(R_n) \sim R_n^{4.5} g(R_n)\, d R_n \ , \qquad (5)$$

where $g(R_n)$ is the true (as opposed to observed) differential distribution of comets of radius R_n.

4. The Effect of Cometary Physical Decay and Conclusions

Another factor concerning small comets is that they physically decay faster than large comets. This factor also affects their probability of being detected. It is not difficult to show, and we state here without proof, that if comets are continuously input into the inner solar system with a differential input distribution,

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$g_i\ (R_n\ )$, of the type

$$g_i(R_n) \sim R_n^{-\beta_i} \quad , \tag{6}$$

then a temporal equilibrium distribution $g(R_n)$ under steady cometary decay is given by

$$g\ (R_n) \sim R_n^{-\beta_i+1} \quad . \tag{7}$$

It is assumed in this analysis, that the radii of all comets are constantly being reduced at the rate $\dot{R}_n$.

Comets that that have been input into the inner solar system with an initial size distribution, $R_n^{-\beta_i}$, will be observed with a size distribution

$$g_i(R_n)\ R_n^{5.5} \sim R_n^{5.5\ -\ \beta_i} \tag{8}$$

We see that, unless the input distribution is very steep ($\beta_i \gtrsim 5.5$), small comets would be very difficult to detect with the classic search technique. Small comets could, indeed, contribute quite significantly to the meteoritic complex without our being aware of it.

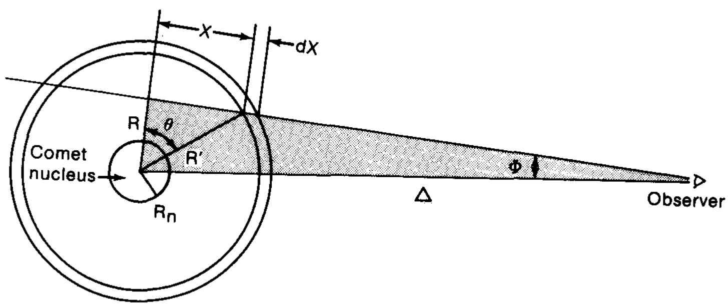

Figure 1. Geometry for observing the cometary coma brightness from distance, Δ.

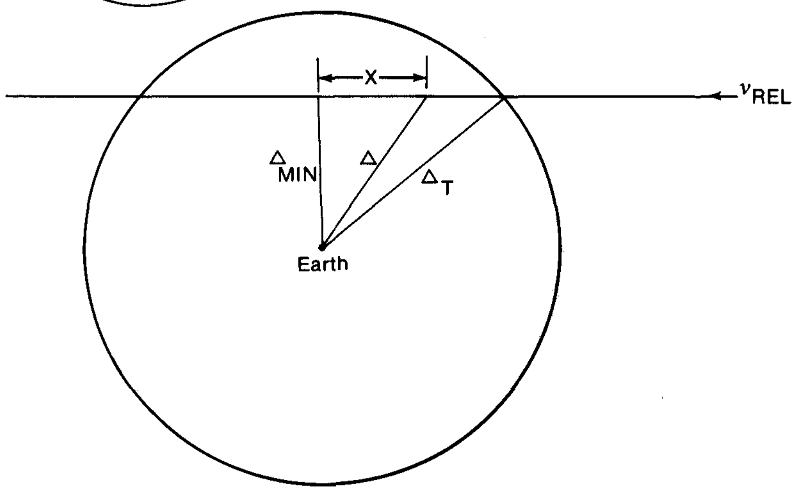

Figure 2. Geometry for a comet passing within the detection threshold distance, Δ_t, from Earth.

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