CIERVA MEMORIAL PRIZE ESSAY COMPETITION 1956

PRIZE ESSAY

Investigations of Ground Resonance in Helicopters with an Analogue Computer

By H FUCHS, B Sc , Ph D *
University of Southampton

Mr H FUCHS was born in Czechoslovakia in 1930 and emigrated to this country in 1939 He was educated at Northampton Grammar School, and obtained his Honours Degree in Physics at the Manchester University in 1952 Prior to obtaining his Ph D in Electronics, on the subject of helicopter computers in ground resonance, at Southampton University, he was employed in the Guided Weapons Laboratory at Vickers Armstrongs as an Electronic Engineer Since 1956 he has been with Blackburn and General Aircraft as Chief Electronic Engineer in charge of the Electronic Laboratory and the Electronic Design of Aircraft His work has included the design and production of Blackburn's analogue computer, and is currently engaged in designing an airborne analogue digital converter, fully transistorised, and associated electronic data handling equipment to a system of flight testing of aircraft

---

Historical

Since 1926, when Cierva produced his first autogyro, rotary wing aircraft have been known to suffer from a vibration when their undercarriage is in contact with the ground, which has become known as " Ground Resonance "

No systematic work had been done on this subject until the helicopter became a practical proposition in 1941 In that year Sikorsky built his VS-300 machine which incorporated the single rotor, swash plate control, drag and flap hinges, and the tail anti-torque propeller Soon after this the U S Army and Navy were equipped with large numbers of the R-4, a production model of the Sikorsky XR-4

The problem of ground resonance at this time became extremely

* The author wishes to express his thanks to Professor E E Zepler, of the University of Southampton, for his invaluable advice and criticism and to the Chief Scientist of the Ministry of Supply for permission to publish the work

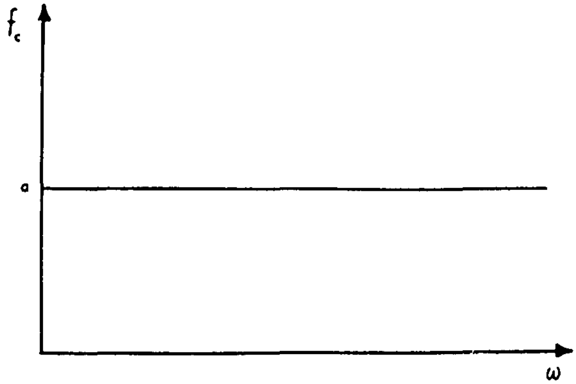


Fig 1

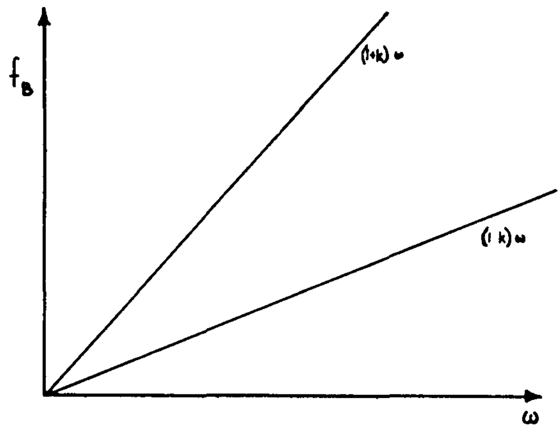


Fig 2

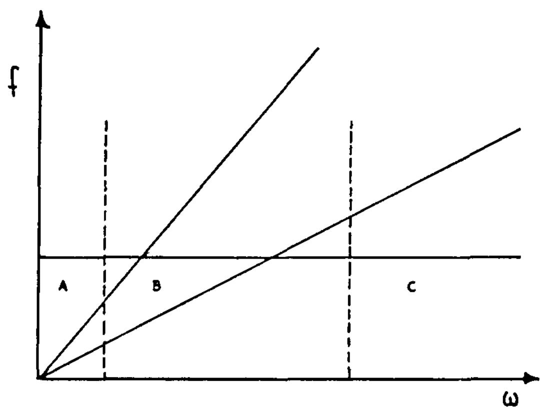


Fig 3

important, as indeed it still is, and the National Advisory Council for Aeronautics (NACA) took up the work As a result there followed the publications of Coleman (Ref 1 1 and 1 2) in 1943 and 1947, and of Deutsch (Ref 1 4) in 1946

The phenomenon under investigation can be described as follows Helicopters, either stationary or taxying, with their undercarriage in contact with the ground can, under certain circumstances, begin to vibrate The oscillation is usually a combination of lateral and transverse motion If no action is taken by the pilot, the oscillation will build up in amplitude and the machine will capsize and often become a total loss

Explanation of Coleman's Theory

A helicopter chassis on the ground has freedom to move in roll, yaw and pitch as well as in transverse, longitudinal and vertical motion Neglecting fuselage bending and twisting, this gives the chassis six degrees of freedom

If a three bladed rotor is mounted at the pylon head, each blade having a drag and a flap hinge, there are seven additional degrees of freedom, two per blade and one associated with the rotation of the pylon shaft

Hence, assuming a rigid fuselage and rigid blades, there are thirteen degrees of freedom Some of these motions will have appreciable intercoupling

In the treatment outlined by Coleman, the following assumptions are made

(*a*) All blade-chassis coupling occurs in the rotor plane, *i e*, there is no coupling to the flapping degrees of freedom
(*b*) There is no vertical motion of the helicopter
(*c*) The fuselage and blades are assumed to be rigid
(*d*) All springs are linear and all dampers are viscous
(*e*) Resonance occurs in only two degrees of chassis freedom, *viz*, longitudinal and transverse displacement

A simple helicopter may now be considered, the undercarriage of which has one degree of freedom, for example transverse motion

Let a three bladed rotor without flapping hinges have a rotational frequency ω The chassis resonant frequency f_c is measured when the drag hinges of the blades are locked This frequency will then obviously be independent of rotor frequency, *i e*, $f_c = a$ (a constant) (Fig 1)

The chassis freedom is now considered locked but the blade drag hinges unlocked The blades will swing in the field of the centrifugal force, like pendulums, and the frequency of oscillations will be proportional to the rotor speed, *i e*, $f_B = k\omega$ If all the blades oscillate in phase, then only a torque tending to turn the chassis about the pylon shaft will be transmitted, but if the phase relationship of the blades is different the centre of gravity of the blade system will be displaced from its mean position at the pylon head In rotating axes this displacement of the centre of gravity is oscillatory and takes place along a fixed line In stationary axes it is also oscillatory but has two frequency components,

$$i\,e,\quad f_{B_1} = \omega - f_B = \omega\,(1 - k)$$
$$f_{B_2} = \omega + f_B = \omega\,(1 + k)$$

A third degree of freedom corresponds, as already stated, to torsional oscillations of the pylon shaft This does not contribute to ground resonance When both curves are plotted on one graph, Fig 3 is obtained

Whereas in regions A & C the frequencies of the chassis and blades are well separated it is seen that in region B they are close together

A typical result due to mutual influence and distortion is shown in Fig 4

As is seen from Fig 4 in region A there is only one value of f for any value of ω The suppression of two frequencies indicates the presence of instability

This brief physical picture of ground resonance will now be followed by an introduction to the derivation of the equations of motion of a three bladed helicopter

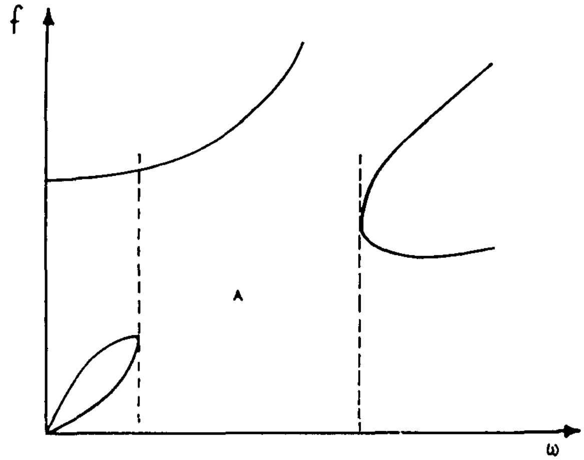


Fig 4

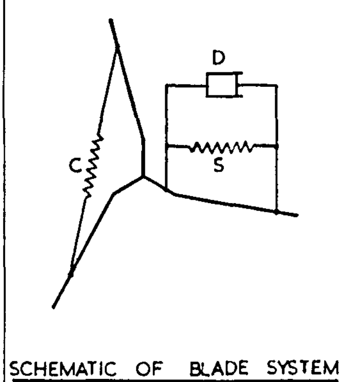


Fig 5

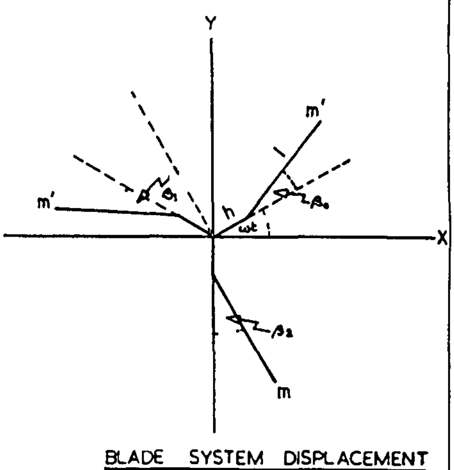


Fig 6

https://doi.org/10.1017/S2753447200003577 Published online by Cambridge University Press

The Blade Equations

Coleman and Feingold showed that if three or more blades are considered, the equations of the blade system will reduce to linear equations with constant coefficients For a two bladed rotor, however, terms in sin ωt and cos ωt remain in the coefficients of the equations of the blades unless equal chassis stiffnesses are assumed in the two directions A three bladed rotor will be considered here

The blade system is shown in plan form in Figs 5 and 6

The following symbols are used

m'	Mass of one blade
I_B	Moment of inertia of blade about C of G
S	Centring spring stiffness
D	Damper coefficient
C	Interblade spring stiffness
x_o, y_o	Acceleration of the rotor head along the two fixed axes
β_0, β_1, β_2,	Blade angular displacements

The equations for the three blades then become

1st Blade

$$(m'l^2 + I_B)\,\beta_o + D\,\beta_o + (m'h/\omega^2 + S)\,\beta_o + C\,(2\beta_o - \beta_1 - \beta_2)$$
$$- m'l\,(x_o \sin \omega t - y_o \cos \omega t) = 0$$

2nd Blade

$$(m'l^2 + I_B)\,\beta_1 + D\,\beta_1 + (m'h/\omega^2 + S)\,\beta_1 + C\,(2\beta_1 - \beta_o - \beta_2)$$
$$- m'l\left((x_o\,(- \tfrac{1}{2} \sin \omega t + \frac{\sqrt{3}}{2} \cos \omega t)\right.$$
$$\left. + y_o\,(+ \tfrac{1}{2} \cos \omega t + \frac{\sqrt{3}}{2} \sin \omega t) \right) = 0$$

3rd Blade

$$(m'l^2 + I_B)\,\beta_2 + D\,\beta_2 + (m'h/\omega^2 + S)\,\beta_2 + C\,(2\beta_2 - \beta_o - \beta_1)$$
$$- m'l\left(x_o\,(- \tfrac{1}{2} \sin \omega t - \frac{\sqrt{3}}{2} \cos \omega t)\right.$$
$$\left. - y_o\,(- \tfrac{1}{2} \cos \omega t + \frac{\sqrt{3}}{2} \sin \omega t) \right) = 0$$

The blade displacements β_o β_1 β_2 are now replaced by new variables e_o, e_1, e_2 such that

$$e_o = \frac{1}{3}\,(\beta_o + \beta_1 + \beta_2)$$

$$e_1 = \frac{1}{3}\left(- \beta_1 \sin \frac{2\pi}{3} - \beta_2 \sin \frac{4\pi}{3} \right)$$

$$e_2 = \frac{1}{3}\left(\beta_o + \beta_1 \cos\frac{2\pi}{3} + \beta_2 \cos\frac{4\pi}{3}\right)$$

e_1 and e_2 give the position of the centre of gravity of the blade system, in rotating co-ordinates, while e_o gives the summed angular displacement of the blades

The equation in e_o is —

$$(m'l^2 + I_B)\, e_o + D\, e_o + (m'h/\omega^2 + S)\, e_o = 0$$

It is seen that all the terms involving chassis motion have vanished from this equation Hence this equation represents an uncoupled mode of oscillation which does not affect the ground resonance of the helicopter

When changing the equations from rotating to stationary axes, the new variables become

$$X_1 = e_1 \cos \omega t - e_2 \sin \omega t$$

$$X_2 = e_2 \cos \omega t + e_1 \sin \omega t$$

The equations in X_1 and X_2 are then

$$(m'l^2 + I_B)\, X_1 + D\, X_1 + \left(S + 3C - (m'l^2 + I_B - m'hl)\omega^2\right) X_1$$

$$+ 2\omega\,(m'l^2 + I_B)\, X_2 + D\,\omega\, X_2 + \frac{m'l^2}{2}\, x_o = 0$$

$$(m'l^2 + I_B)\, X_2 + D\, X_2 + \left(S + 3C - (m'l^2 + I_B - m'lh)\omega^2\right) X_2$$

$$- 2\omega\,(m'l^2 + I_B)\, X_1 - D\,\omega\, X_1 + \frac{m'l^2}{2}\, y_o = 0$$

and writing

$$H = 1 - \frac{h}{l\left(1 + \frac{I_B}{m'l^2}\right)}$$

$$I = \frac{1}{2\left(1 + \frac{I_B}{m'l^2}\right)}$$

$$P = \frac{D}{m'l^2}$$

$$T = \frac{S - 3C}{m'l^2}$$

On rearranging, the equations become

$$X_1 + 2\,PI\,X_1 + (2\,TI - H\omega^2)\,X_1 \qquad \text{(basic equation)}$$

$$+ 2\,\omega\,X_2 + 2\,PI\,\omega\,X_2 \qquad \text{(blade coupling)}$$

$$+ Ix + I\,Z_H\,\theta + I\,y_H\,\mu = 0 \qquad \text{(chassis coupling)}$$

$$X_2 + 2\,PI\,X_2 + (2\,TI - H\omega^2)\,X_2$$

$$- 2\,\omega\,X_1 - 2\,PI\,\omega\,X_1$$

$$+ Iy - IZ_H\,\phi + Ix_H\,\mu = 0$$

x_H, y_H and z_H are the co-ordinates of the rotor head w r t the helicopter C of G, ϕ is the angle of roll, θ is the angle of pitch and μ is the angle of yaw

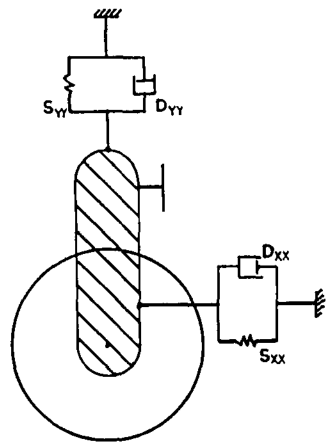


Fig 7 Helicopter with two degrees of chassis freedom

The Chassis Equations

The chassis equations like the blade equations are derived from the simple equations of motion Sufficient stiffness and damping terms are considered to cover the case of a conventional tricycle undercarriage having longitudinal symmetry (Fig 7)

The symbols used are

- S_{ij} Stiffness in freedom i due to a motion in freedom j
- D_{ij} Damping in freedom i due to a velocity in freedom j
- m Mass of helicopter
- m_H (3m′) mass of rotor

$$M = \frac{m_H}{m}$$

$$A = \frac{m_H z_H^2}{a}, \qquad B = \frac{m_H z_H^2}{b}, \qquad C = \frac{m_H z_H^2}{c}$$

a, b and c are the inertias in roll, pitch and yaw respectively

Due to the mechanical nature of the system it is completely orthogonal, *i e*, $D_{ij} = D_{ji}$ and $S_{ij} = S_{ji}$

Where $i = j$ there is the intrinsic damping and stiffness of the degree of freedom Where $i \neq j$ there is coupling between degrees of freedom

Longitudinal Motion

$$m\ddot{x} + D_{xx}\dot{x} + S_{xx}x \qquad \text{(basic equation)}$$

$$+ D_{x\theta}\dot{\theta} + S_{x\theta}\theta \qquad \text{(chassis coupling)}$$

$$+ m\,M\,\ddot{X}_1 = 0 \qquad \text{(blade coupling)}$$

Transverse Motion

$$m\ddot{y} + D_{yy}\dot{y} + S_{yy}y$$

$$+ D_{y\Phi}\dot{\phi} + S_{y\Phi}\phi + D_{y\mu}\dot{\mu} + S_{y\mu}\mu$$

$$+ m\,M\,\ddot{X}_2 = 0$$

Rolling Motion

$$a\ddot{\phi} + D_{\Phi\Phi}\dot{\phi} + S_{\Phi\Phi}\phi$$

$$- F\ddot{\theta} - E\ddot{\mu} \qquad \text{(products of inertia)}$$

$$+ D_{\Phi y}\dot{y} + S_{\Phi y}y + D_{\Phi\mu}\dot{\mu} + S_{\Phi\mu}\mu$$

$$- \frac{aA}{Z_H}\ddot{X}_2 = 0$$

Pitching Motion

$$b\ddot{\theta} + D_{\theta\theta}\dot{\theta} + S_{\theta\theta}\theta$$

$$- F\ddot{\phi} - G\ddot{\mu}$$

$$+ D_{\theta x}\dot{x} + S_{\theta x}x$$

$$+ \frac{bB}{Z_H}\ddot{X}_1 = 0$$

Yawing Motion

$$c \mu + D_{\mu\mu} \mu + S_{\mu\mu} \mu$$

$$- E \phi - G \theta$$

$$+ D_{\mu y} y + S_{\mu y} y + D_{\mu \Phi} \phi + S_{\mu \Phi} \phi$$

$$- \frac{- cC}{Z_H} \frac{Y_H}{Z_H} X_1 + \frac{cC}{Z_H} \frac{X_H}{Z_H} X_2 = 0$$

E, F and G are the products of inertia between the relevant degrees of freedom

Before deriving these equations in a slightly different manner (*i e*, by Lagrangian's methods) Coleman chooses the relevant degrees of freedom which are important in ground resonance It is necessary to consider only a maximum of four degrees of freedom in order to be able to deal with the equations analytically Although the computer could be designed to deal with a larger number, it was decided to confine it in the first instance to the four degrees of freedom which Coleman considers as playing a significant part in ground resonance These are the two degrees of freedom associated with the blade centre of gravity and the two undercarriage degrees of freedom, transverse and longitudinal motion

Coleman then proceeds to assume that a steady state oscillation exists and substitutes the value of this frequency Ω in the original equations

As an example, a simple case will be considered
Let the equation of a system be —

$$Ax + Bx + Cx = 0$$

This yields the stability equation

$$A\lambda^2 + B\lambda + C = 0$$

If one writes $\lambda = i\,\Omega$ the equation becomes

$$- A\Omega^2 + iB\Omega + C = 0$$

If a real value of Ω satisfies such an equation an undamped oscillation exists

Coleman now separates this equation into a real and imaginary part

$$E_R = C - A\Omega^2$$

$$E_I = B\Omega$$

If A, B and C vary with some parameter, say, ω, the rotor speed, he plots $E_R = 0$ and $E_I = 0$ against this parameter Where the two curves intersect both equations are obeyed simultaneously and an undamped solution exists He further argues that between the two points where the curves cross, lies the unstable range (See Fig 12)

The equations $E_R = 0$ and $E_I = 0$ in the helicopter problem are of high order in Ω but only contain terms in ω^4 and ω^2 Hence it is easier to choose Ω and calculate ω than vice versa, which is the more conventional method

The drawback to Coleman's method is

(1) It gives no information about the resonant sensitivity, *i e*, the rate of convergence or divergence of the solution This is a direct result of assuming that a steady state oscillation exists

(2) It cannot be applied to more than four degrees of freedom as the analytical approach breaks down when equations in ω^2 or Ω of higher power than the second have to be solved

(3) If a parameter such as blade mass, damping, etc, is changed, the whole graph has to be replotted, involving a complete recalculation of all the determinants

The Equations for the Computor

As already outlined it is seen that when all but the chosen four degrees of freedom are ignored, the equations which the computer is to solve become

$$X_1 + 2\,D\,I\,X_1 + (2\,S\,I - H\,\omega^2)\,X_1 + 2\,\omega\,X_2 + 2\,D\,I\,\omega\,X_2 + I\,x = 0 \qquad (1)$$

$$X_2 + 2\,D\,I\,X_2 + (2\,S\,I - H\,\omega^2)\,X_2 - 2\,\omega\,X_1 - 2\,D\,I\,\omega\,X_1 + I\,y = 0 \qquad (2)$$

$$m\,x + D_{xx}\,x + S_{xx}\,x + M\,X_1 = 0 \qquad (3)$$

$$m\,y + D_{yy}\,y + S_{yy}\,y + M\,X_2 = 0 \qquad (4)$$

It was decided to introduce into the computer all cross coupling terms so that the most general system of four simultaneous differential equations could be investigated These are not usually present in ground resonance There are a maximum of forty-eight terms in four second order linear equations, twelve in each equation

Hence the computer is able to solve equations of the form

$$A_1X_1 + B_1X_1 + C_1X_1 + D_1X_2 + E_1X_2 + F_1X_2 + G_1X_3 + H_1X_3$$

$$+ I_1X_3 + J_1X_4 + K_1X_4 + L_1X_4 = 0$$

$$A_2X_1 + B_2X_1 + C_2X_1 + D_2X_2 + E_2X_2 + F_2X_2 + G_2X_3 + H_2X_3$$

$$+ I_2X_3 + J_2X_4 + K_2X_4 + L_2X_4 = 0$$

$$A_3X_1 + B_3X_1 + C_3X_1 + D_3X_2 + E_3X_2 + F_3X_2 + G_3X_3 + H_3X_3$$

$$+ I_3X_3 + J_3X_4 + K_3X_4 + L_3X_4 = 0$$

$$A_4X_1 + B_4X_1 + C_4X_1 + D_4X_2 + E_4X_2 + F_4X_2 + G_4X_3 + H_4X_3$$

$$+ I_4X_3 + J_4X_4 + K_4X_4 + L_4X_4 = 0$$

It will now be shown how a computer solves these equations

(I) INTRODUCTION TO ANALOGUE COMPUTERS

General

Many problems in engineering and physics reduce to the solution of a set of differential equations The analytical solution is often laborious and sometimes impossible The analogue computer which has developed in the last fifteen years reduces the work involved in solving some of these equations, and makes possible the solution of many non-linear ones outside the scope of the analytical approach

An analogue computer can be defined as a device which obeys the same equations as the system under examination and hence will give the same solution In the science at present the term analogue computer is limited to an electronic or electro-mechanical device which is set up to solve the equations under examination

The reasons for using such a device can be listed as follows

(1) The equations may represent a complex system such as an aircraft which cannot easily be tested as a whole

(2) The system may become a total loss if tested as, for instance, a guided weapon

(3) It may be desired to know how the performance of the system varies with some parameter If the system is mechanical, a change of parameter may involve considerable reconstruction

(4) The analytical solution of the equations may take a considerable time, or may even be impossible An analogue computer will give the solution to the equations quickly, *i e*, in a matter of seconds The variation of a parameter merely involves the turning of the appropriate dial

The main drawback of the analogue computer is its relative lack of accuracy compared to analytical or digital methods of computation This is compensated by the ease and speed of setting up the problem on the computer and the simplicity of the presentation of the solution

The analogue computer dealing with linear differential equations, to

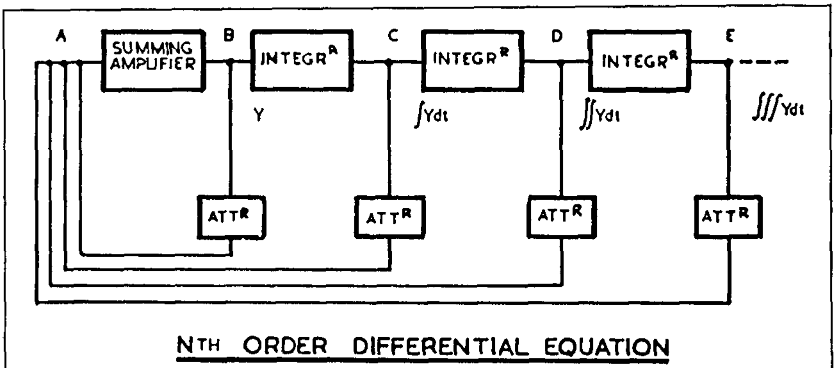


Fig 8

which this work is confined, requires three basic elements These are

(1) A device which will simulate the various rates of change of the variable with respect to time

(2) A device which will simulate the coefficients of the equation

(3) A device which will equate the sum of all the terms to zero

Referring to Fig 8 it is seen that if a variable y exists at point B, then $\int$ y dt will exist at C and $\iint$ y dt at D, etc These outputs are passed through attenuators, as shown, whose attenuation is proportional to the various constants in the equation which it is required to compute Finally, all the outputs of the attenuators are fed into the summing device, which has the property of equating them to zero The equation thus simulated is of the form

$$\sum_{0}^{n} A_r \int^{(r)} y \, dt = 0 \qquad \int^{(r)} \equiv \iiint \quad \text{up to r}$$

Differentiating n times and putting n — r = k, this becomes

$$\sum_{0}^{n} A_k \frac{d^k y}{dt_k} = 0$$

This is the general linear differential equation of the nth order having constant coefficients

Several ways are available of extending the usefulness of such a computer If a multiplier is available, powers and products of the differentials may also be included, thus solving non-linear equations Similarly the attenuators may be made to vary either with time or with the variable, in any desired way This will be dealt with later under the section on bouncing

The Operational Amplifier

The basic element in an electronic computer is the operational d c amplifier

From Fig 9 it is seen that the amplifier is connected through impedances Z_1, Z_2, etc , to various e m f 's and that its feedback impedance is Z_o The forward gain of the amplifier is —A, where A is large and the bandwidth is infinite By equating the currents at the input grid of the amplifier to zero the equation obtained is

$$\frac{V_1 - V_g}{Z_1} + \frac{V_2 - V_g}{Z_2} \qquad + \frac{V_o + V_g}{Z_o} = 0$$

provided that no current flows into the amplifier

$$\text{Also } V_g = -\frac{V_o}{A}$$

$$\text{Hence } \sum_{o}^{n} \frac{V_j}{Z_j} = -\frac{V_o}{A} \qquad \sum_{o}^{n} \frac{1}{Z_j}$$

Because of the large negative feedback, V_o is of the same magnitude as V_j , hence the right-hand side may be assumed zero if A is large It can be shown that the errors introduced by making this assumption are permissible within the accuracy of the instrument

$$\text{Finally } \sum_{o}^{n} \frac{V_j}{Z_j} = 0$$

If the impedances Z_1, Z_2, etc , are resistances, the nodal equation becomes

$$\sum_{o}^{n} \frac{V_j}{R_j} = 0 \quad \text{or} \quad \sum_{1}^{n} \frac{V_j}{R_j} = -\frac{V_o}{R_o}$$

There two forms of the same equation represent two different ways of looking at the summing device The first is the method adopted when deriving the block diagram of the analogue computer The second is useful

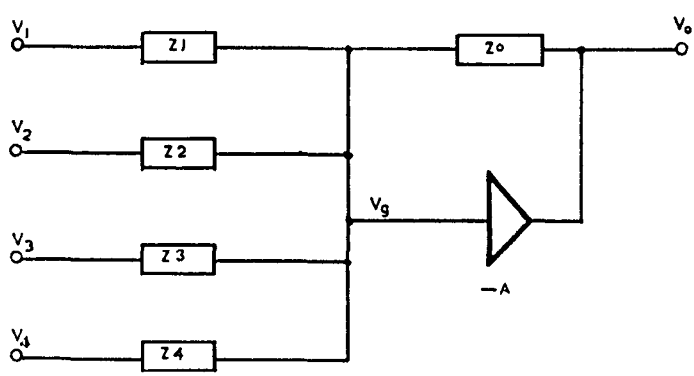
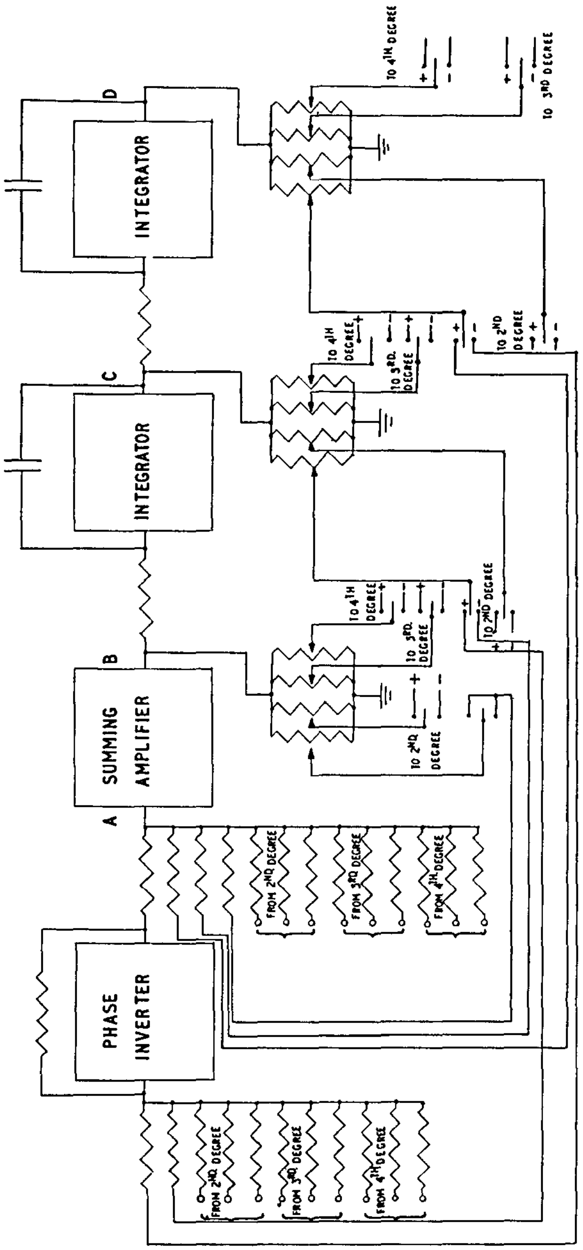


Fig 9

https://doi.org/10.1017/S2753447200003577 Published online by Cambridge University Press



SCHEMATIC OF ONE DEGREE OF FREEDOM

Fig 10

https://doi.org/10.1017/S2753447200003577 Published online by Cambridge University Press

if addition, as such, is required for some particular operation

Z_o is now replaced by a capacitor The generalised impedance of a capacitor is $\frac{1}{pC}$ where p is a complex frequency and at the same time an operator defined as $p \equiv \frac{d}{dt}, \frac{1}{p} \equiv \int dt$

$$\text{Therefore} \quad \sum_1^n \frac{V_j}{R_j} = - V_o \, pC$$

$$\text{or} \qquad V_o = - \sum_1^n \frac{V_j}{pCR_j}$$

Hence each input voltage is integrated and multiplied by the constant $-\frac{1}{CR_j}$

With these two elements and potentiometers as attenuators it is possible to build up a computer for solving the equations derived in the last chapter

Before doing so, a word must be said on accuracy It can be shown that the whole design of the amplifiers is intimately associated with the accuracy to be achieved, but some points may be worth noting here First, neither the bandwidth nor the gain of the amplifiers are infinite This restricts the signal frequencies which the amplifier will handle, and limits the accuracy with which the amplifier performs the operation defined by its nodal equation In addition, the operational impedances are not known exactly This limits the accuracy achieved in any operation

The equations which it is required to simulate are four simultaneous second order linear differential equations with constant coefficients The set-up shown in Fig 10 will simulate a second order equation A variable y_1 at B is integrated twice, and then via the potentiometers fed back to the summing amplifier At A, therefore, the equation

$$Ay_1 + B \int y_1 \, dt + C \iint y_1 \, dt = 0$$

is obeyed within the accuracy of the instrument

Replacing y_1 by $\frac{d^2x_1}{dt_2}$ the equation becomes

$$A \frac{d^2x_1}{dt^2} + B \frac{dx_1}{dt} + Cx_1 = 0$$

If now variables x_2, $\frac{dx_2}{dt}$ etc are added to the summing device, the system will solve the new equation denoted by

$$A \frac{d^2x_1}{dt^2} + B \frac{dx_1}{dt} + Cx_1 + S = 0$$

where S is the sum of all additional terms added

It is obvious that if the additional variables are available from some other part of the computer (i e, other second order set-ups) the whole set of simultaneous equations may be solved

The final set-up will consist of four " degrees of freedom," with the outputs of all the integrators and summing amplifiers cross connected as shown in Fig 11

Each amplifier has a negative gain This is required in order that the feedback is negative and the amplifier stable Hence each time an operation is performed on the variable the sign is changed This fact can be utilized since it makes available a sign reverser in the case of a unity gain summing amplifier This is added to the beginning of each degree of freedom If a positive sign is required for any variable the voltage is fed back in a negative

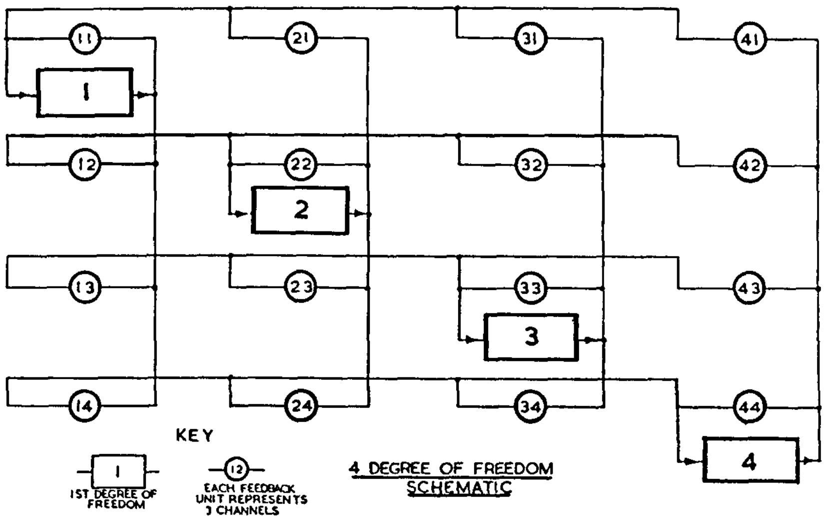


Fig 11

feedback sense (an odd number of amplifiers in the feedback loop) If a negative sign is required, the feedback is positive and there is an even number of amplifiers in the loop

Several methods are available for presenting the solution obtained from the computer Any of the variables or differentials of the variables may be plotted as functions of time on a pen recorder or similar device The paper moving at constant speed provides the independent variable (time) and the pen writes the function on the paper

It is also possible to plot two variables against each other if a pen having two degrees of freedom at right angles to each other is available

In the present computor only the behaviour of one variable is of interest, namely, the undercarriage displacement This can be plotted directly on a high speed recorder It is necessary to ensure that the frequency response of the recorder is much greater than the highest signal frequencies extant in the solution

To produce the solution, it is merely necessary to disturb the computer by adding a d c voltage for a short time to one of the degrees of freedom On removing this voltage, the computer provides the transient solution of the equations (*i e*, the solution of the complementary function)

The attenuators required to set up the coefficients shown in Fig 9 are ten-turn helically wound potentiometers By means of a suitable dial it is possible to set up the coefficients to an accuracy of about 2% of full scale For more accurate work a bridge calibration is provided which will set the coefficients up against a standard potentiometer to about 0 1% of the full scale value

Since d c amplifiers are used as the operational amplifiers care must be taken to avoid the errors caused by drift The power supplies are designed both to be stable and, also, to have a low output impedance This latter is required to avoid intercoupling between amplifiers connected to the same power supply

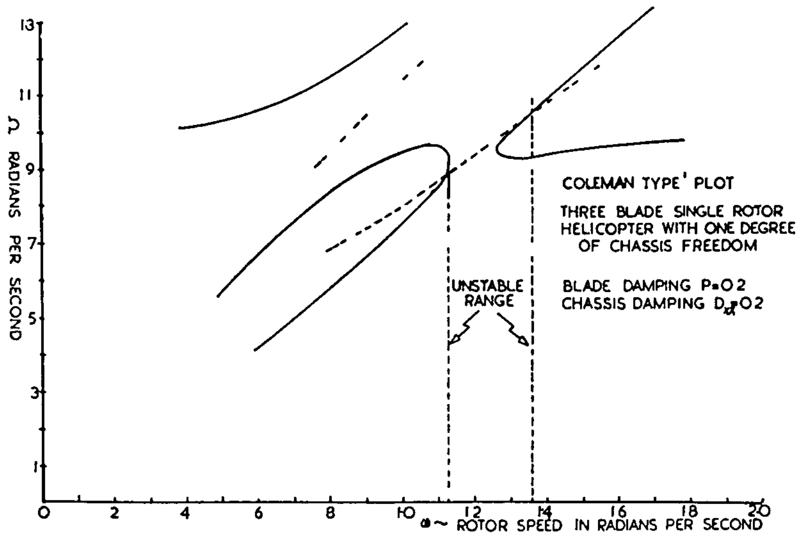


Fig 12

(III) A TYPICAL SOLUTION

Method of Presentation

The method of solution of the equations of ground resonance will depend upon the information which the designer is seeking

If the information required is merely the range of rotor speed at which the helicopter is unstable, the procedure is as follows The equations are written in matrix form as in Table I One table is made out for each value of rotor speed and then scaled for frequency and numerical size of coefficients Assuming the rotor speed lies in the range 0 8 radian per second to 1 6 radians per second, one matrix is written for each rotor speed at intervals of 0 1 radian per second Table II shows the complete set of equations Each equation is set up on the computer and the computer disturbed with a suitable amplitude in the undercarriage degree of freedom The solution

TABLE I

1	0 133	—0 967 ω^2	0	2 ω	0 133 ω	0 333	0	0
0	—2 ω	—0 133 ω	1	0 133	—0 967 ω^2	0	0	0
0 050	0	0	0	0	0	1	D_{xx}	1

TABLE II

1000	133	—242	0	1000	67	333	0	0	ω = 0 5
0	—1000	—67	1000	133	—242	0	0	0	
50	0	0	0	0	0	1000	250	1000	
833	112	—291	0	1000	67	278	0	0	ω = 0 6
0	—1000	—67	833	112	—291	0	0	0	
50	0	0	0	0	0	1000	250	1000	
715	96	—339	0	1000	67	238	0	0	ω = 0 7
0	—1000	—67	715	96	—339	0	0	0	
50	0	0	0	0	0	1000	250	1000	
525	84	—388	0	1000	67	208	0	0	ω = 0 8
0	—1000	—67	625	84	—388	0	0	0	
50	0	0	0	0	0	1000	250	1000	
556	74	—435	0	1000	67	185	0	0	ω = 0 9
0	—1000	—67	556	74	—435	0	0	0	
50	0	0	0	0	0	1000	250	1000	
500	67	—484	0	1000	67	167	0	0	ω = 1 0
0	—1000	—67	500	67	—484	0	0	0	
50	0	0	0	0	0	1000	250	1000	
455	60	—532	0	1000	67	150	0	0	ω = 1 1
0	—1000	67	455	60	—582	0	0	0	
50	0	0	0	0	0	1000	250	1000	
417	55	—581	0	1000	67	138	0	0	ω = 1 2
0	—1000	—67	417	55	—581	0	0	0	
50	0	0	0	0	0	1000	250	1000	
384	51	—629	0	1000	67	127	0	0	ω = 1 3
0	—1000	—67	384	51	—629	0	0	0	
50	0	0	0	0	0	1000	250	1000	
357	48	—678	0	1000	67	119	0	0	ω = 1 4
0	—1000	—67	357	48	—678	0	0	0	
50	0	0	0	0	0	1000	250	1000	
333	45	—726	0	1000	67	111	0	0	ω = 1 5
0	—1000	—67	333	45	—726	0	0	0	
50	0	0	0	0	0	1000	250	1000	
313	42	—774	0	1000	67	103	0	0	ω = 1 6
0	—1000	—67	313	42	—774	0	0	0	
50	0	0	0	0	0	1000	250	1000	

https://doi.org/10.1017/S2753447200003577 Published online by Cambridge University Press

of the equations is plotted on the pen-recorder

After a complete run has been made, the damping factors (the indices of $e^{\alpha t}$) causing the solution to increase or decrease in amplitude, are calculated from the graph For this purpose a set of exponential curves are drawn for various values of α on transparent paper, which may be superimposed over the graphs obtained from the computer Then a graph of α against rotor speed is plotted as shown in Fig 16 Where this curve lies below the $\alpha = 0$ axis, instability exists

The advantages of this method are

(1) Since an analogue computer is most inaccurate near an unstable range, the position of the curve crossing the $\alpha = 0$ line is obtained by extrapolation and does not depend upon an actual measurement of ω at this value of α

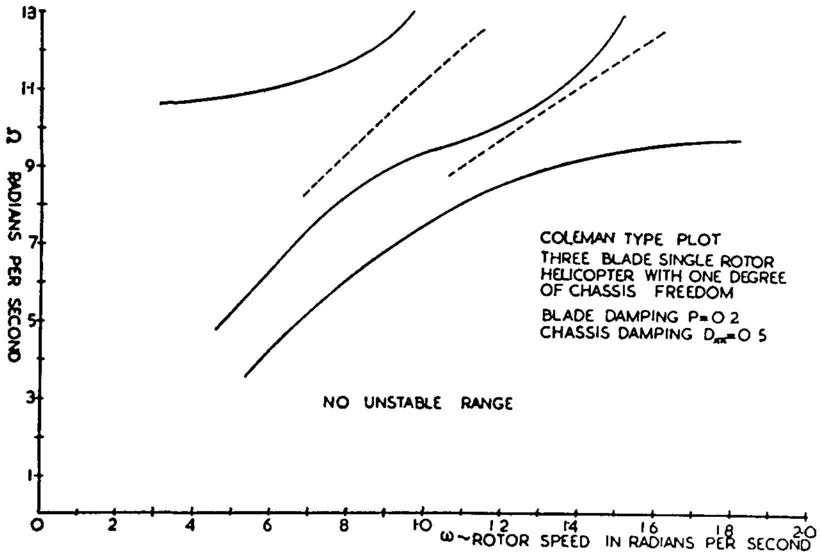


Fig 13

(2) The solution presents the 'resonant sensitivity' of the helicopter directly as a function of rotor speed

If the information required is the amount of damping needed to close the unstable range completely, the procedure is as follows The solution is plotted as before and the rotor speed found, at which α has its largest negative value (*i e*, the system is most unstable) The computer is then set up to solve the equation at this value of ω and the damping in the undercarriage and blade system are increased until the computer provides a stable solution (*i e*, $\alpha = 0$)

Exploration of the influence of any other parameter upon the stability of the helicopter may also profitably be carried out The equations are set up at a rotor speed near the unstable range and the selected parameter varied to see how the solution of the equation is affected In many cases it will be found that, for even small changes, some parameters have a great influence upon the stability

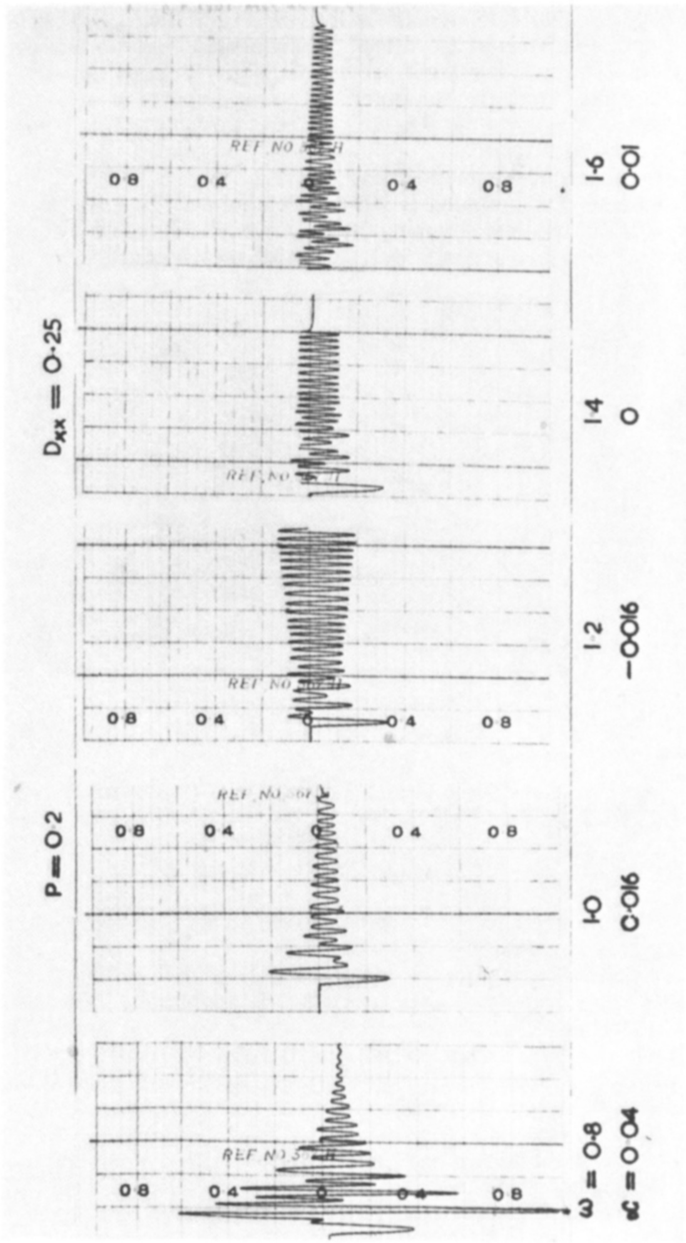


Fig 14

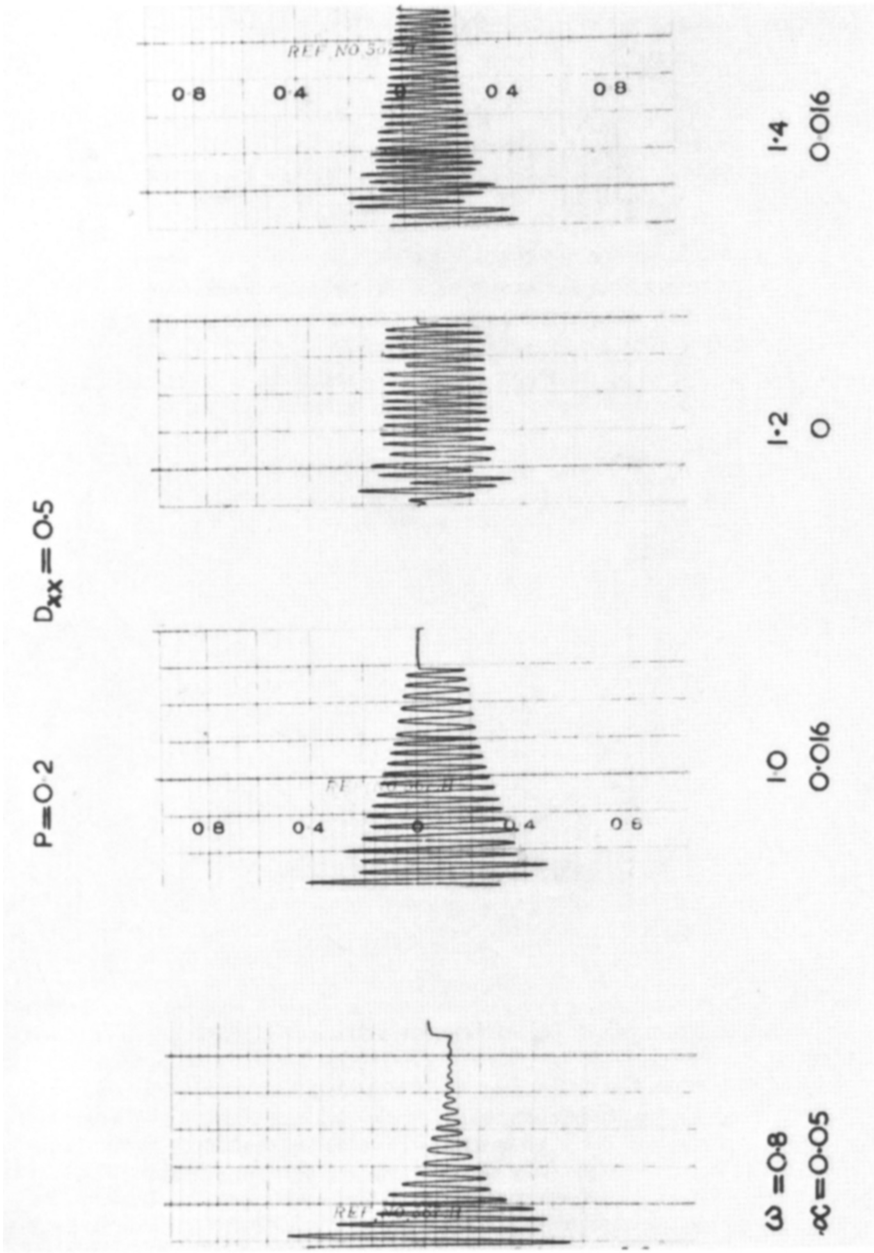


Fig 15

https://doi.org/10.1017/S2753447200003577 Published online by Cambridge University Press

The Skeeter

In order to illustrate the use of the computer more fully, a typical solution is set out below, using the Saunders-Roe Skeeter as an example

The equations in the range $\omega = 0\ 8 — 1\ 6$ radians per second are shown in Table II It will be seen that re-scaling was necessary as the rotor speed increased in this range The frequency scale remains unity

The solutions of the equations are shown in Figs 14 and 15 From these graphs α is calculated by means of the masks This results in a graph of α against ω shown in Figs 16 and 17 The relevant Coleman Plots are shown in Figs 12 and 13

It is seen from this set of solutions that

(1) The computer solution agrees with the Coleman Plots

(2) The unstable range is completely closed by increasing the chassis damping to 0 5 (blade damping P = 0 2)

(3) The helicopter is then only marginally stable at $\omega = 1\ 2$ radians per second and will still oscillate if D_{xx} is reduced due to, say, wear

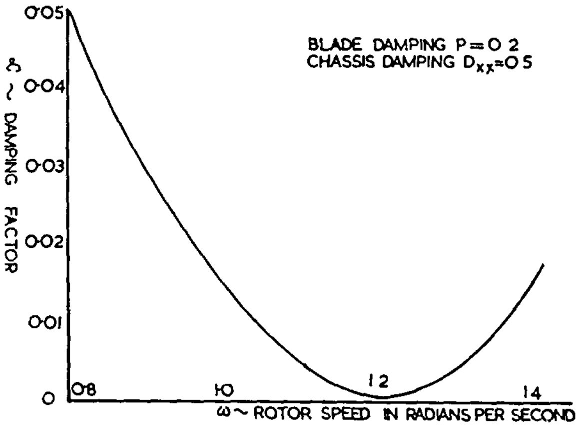


Fig 16

(IV) THE NON-LINEAR PROBLEM

Introduction

As will be seen from the introduction to ground resonance, Coleman made certain assumptions in deriving his equations of motion The main point of his assumptions was that the problems could be reduced to a set of linear equations and hence be solved analytically

On examination, Coleman's assumptions that the springs are linear and the dampers viscous are a good approximation in practice Even if this ideal is not realised completely the ideal spring or damper may always be used in the equations if the departure from linearity is small If, however, there is a sudden large change in the spring stiffness or in the damping, at a certain amplitude, no equivalent constant values can be used This problem can only be represented by a set of non-linear equations which must be solved *in toto* to obtain the solution with any degree of accuracy

Bouncing

It has been noticed experimentally that certain helicopters, particularly the Bristol 173, when ground resonating, bounce from wheel to wheel This occurs most readily when the rotor is taking a considerable proportion of the weight of the machine

Under these conditions the equations obeyed when the helicopter is sitting on both its wheels are different from those obeyed when one wheel is off the ground The following, it is thought by observers, may take place

(1) The wheel remaining on the ground continues to act linearly, *i e*, the undercarriage leg in contact with the ground continues to act as a conventional spring and damper

(2) It is possible that the oleo leg (which provides stiffness and damping) in contact with the ground is " bottomed " (completely compressed), in which case the only stiffness is provided by the tyre and there is practically no damping

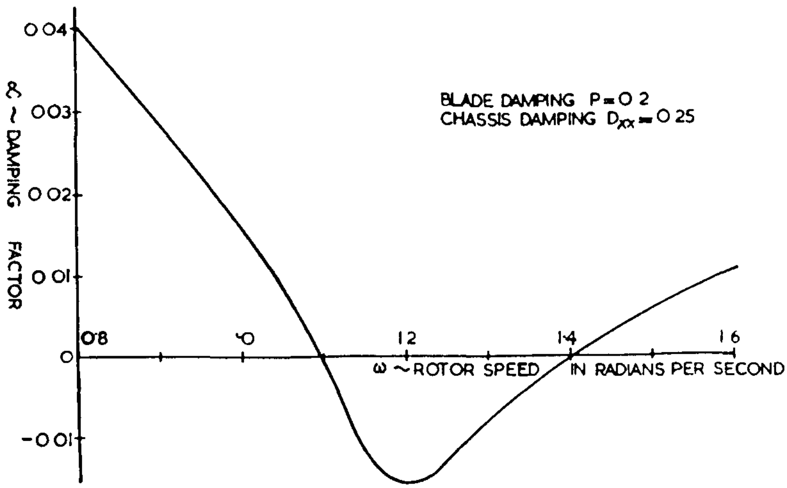


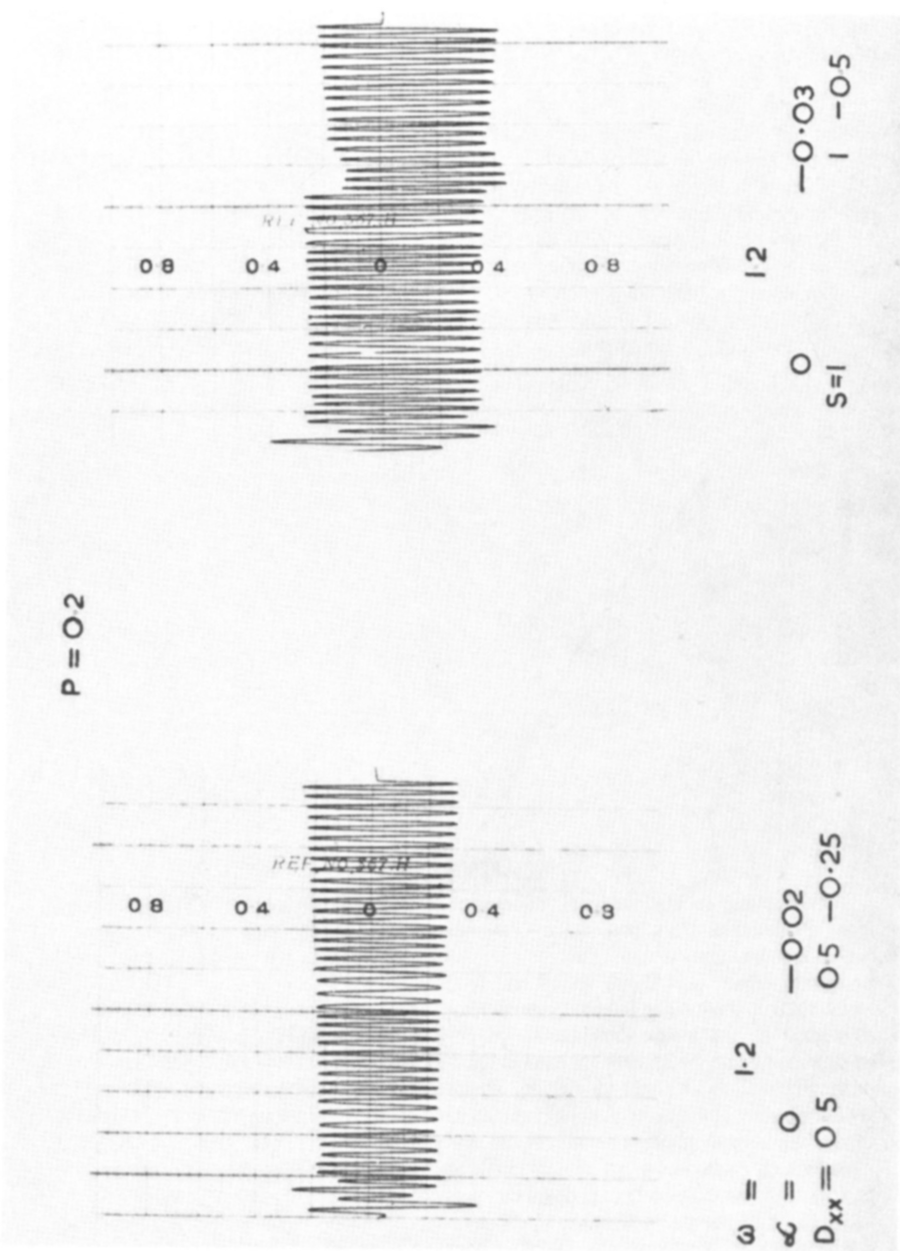
Fig 17

However, in the normal equation of ground resonance it is agreed by most observers that the main transverse stiffness in the undercarriage is provided by the tyres Hence, if one tyre leaves the ground the most probable effect will be to halve the transverse stiffness

As for damping in the transverse mode, it is obvious that the main damping in the undercarriage is provided by the extension or contraction of the oleo legs If one of these legs is removed from the operation, the damping will be halved Hence, it seems reasonable to assume that as the wheel leaves the ground, both the stiffness and damping in the transverse undercarriage degree of freedom are halved

For the sake of simplicity, it is intended to confine the analysis of the effect of bouncing to three degrees of freedom

Thus in the computer a switch is required which will change the relevant coefficients in the undercarriage degree of freedom to their new value at some predetermined value of the amplitude of oscillation When the amplitude again decreases below this value, the coefficients must change



(a) Fig 18 (b)

https://doi.org/10.1017/S2753447200003577 Published online by Cambridge University Press

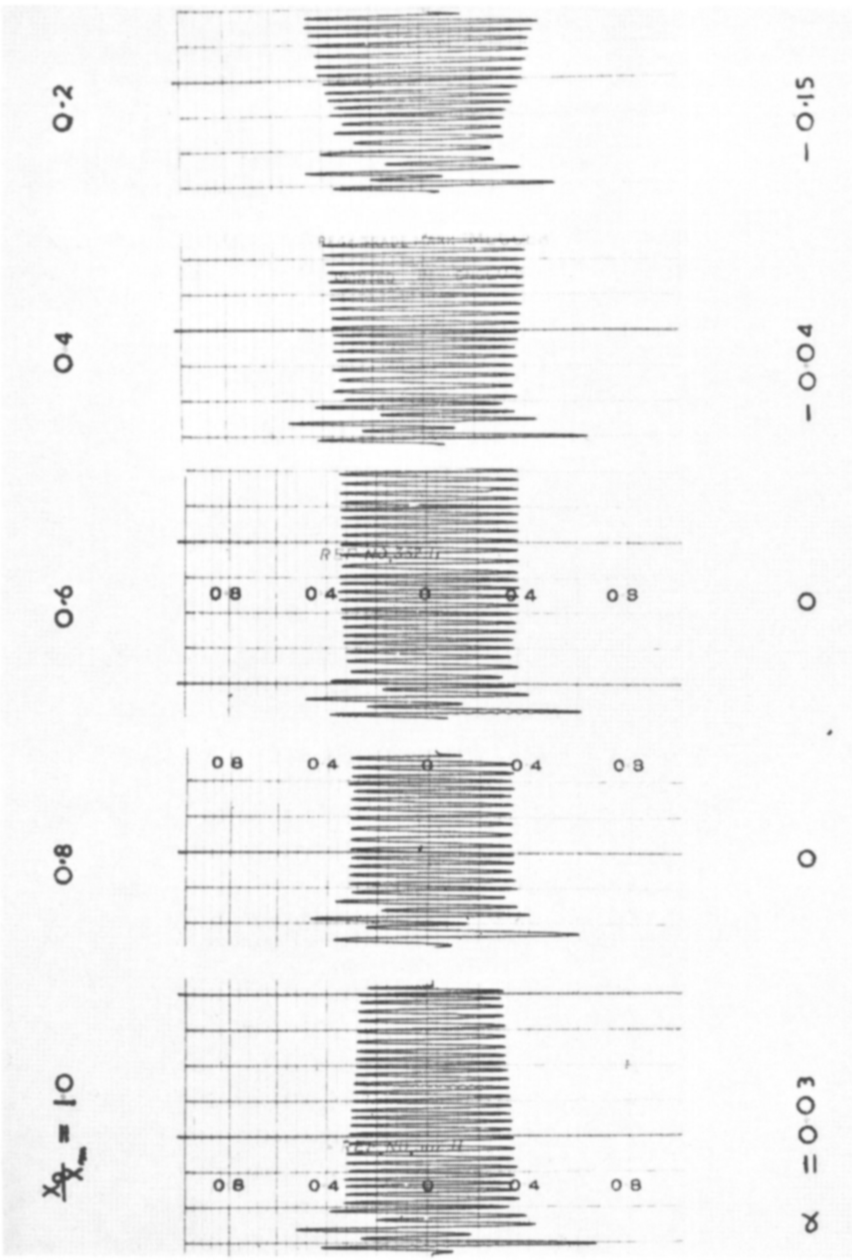


Fig 19

https://doi.org/10.1017/S2753447200003577 Published online by Cambridge University Press

back to their original value for small amplitudes Hence the computer will solve the complete non-linear equations

To investigate the effect of bouncing upon the stability of the helicopter, the computer was set up to solve the equations of ground resonance for a marginally stable case This was the equation for the Skeeter at a rotor speed of 1 2 radians per second and $D_{xx} = 0\ 5$ From Fig 17 it is seen that the equations give a just damped solution

The switch was adjusted such that the coefficient of damping would be reduced by a half at the peaks of the oscillation (*i e*, to $D_{xx} = 0\ 25$)

The computer was set oscillating with the linear equation set up and after about 30 seconds the switch was introduced by reducing x_0, the critical amplitude (by means of the dial on the front panel of the switch chassis), to about half the amplitude of the oscillation existing in the undercarriage degree of freedom The result is shown in Fig 18a

The same procedure was then adopted for the stiffness in the undercarriage degree of freedom The result is shown in Fig 18b

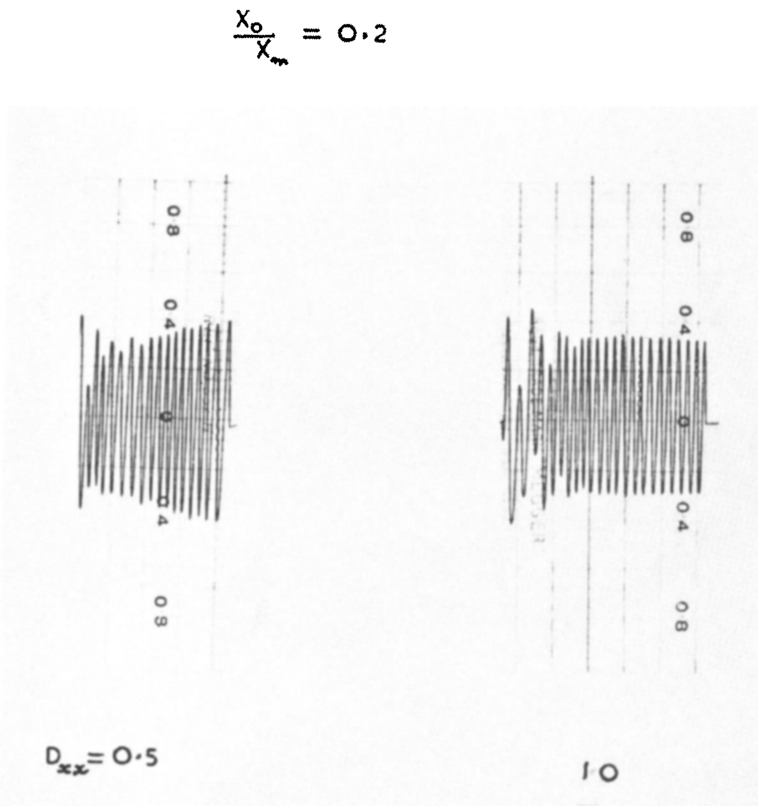


Fig 20

https://doi.org/10.1017/S2753447200003577 Published online by Cambridge University Press

From these results it is evident that a helicopter which is "cured" of ground resonance on Coleman's simple theory, may nevertheless ground resonate due to bouncing from wheel to wheel

Having established that bouncing could cause ground resonance to occur in stable helicopters, it was next decided to investigate how the ratio of x_o, the critical amplitude to x_m, the maximum amplitude, affected the stability The computer was set up to solve the equations of ground resonance at a rotor speed giving a stable (damped) solution The computer was then disturbed with constant amplitudes but the value of x_o progressively reduced The solutions for the ratio $\frac{X_o}{X_m} = 1\ 0,\ 0\ 8,\ 0\ 6,\ 0\ 4$ and $0\ 2$ are shown in Fig 19

Firstly it is seen that a stable helicopter becomes less stable as the ratio $\frac{X_o}{X_m}$ decreases In the case chosen as an example (and one which in practice could be a real state) the helicopter becomes completely unstable at a ratio of $\frac{X_o}{X_m} = 0\ 6$ At a ratio of 0 2 the helicopter is seriously ground resonant and would be extremely unsafe to use

It may be stated that in practice ratios as large as 0 2 can be encountered

Finally, it was decided to measure the amount of damping required to avoid instability in the case of bouncing, up to a ratio of $\frac{X_o}{X_m} = 0\ 2$, this being about the worst case likely to occur in practice

The computer was set up to solve the equations at the rotor speed giving the most unstable solution, *i e*, 1 2 radians per second At this point the helicopter was just stable with a damping $D_{xx} = 0\ 5$

The results are shown in Fig 20 It is seen that at $D_{xx} = 0\ 5$ and $\frac{X_o}{X_m} = 0\ 2$ the helicopter is wildly unstable When the damping is increased to $D_{xx} = 1\ 0$ the helicopter becomes just stable

CONCLUSION

An analogue computer has been built which may be used as a tool in the design of a helicopter, to ensure that no ground resonance will occur under normal operating conditions Both the simple Coleman theory and the case of bouncing have been considered

The computer reduces the work involved in the solution of the former case and makes possible the solution of the latter case with some accuracy

The results show

(1) The solution obtained from the computer, in the simple case, agrees with the analytical solution within about 1%

(2) The stability of the helicopter is seen at a glance from the graph of damping v rotor speed and gives a clear picture of the 'resonant sensitivity' of the machine at all rotor speeds

(3) In the case of bouncing the helicopter becomes more unstable due to two causes These are (*a*) reduction of the damping in the undercarriage degree of freedom at the peaks of the oscillation and (*b*) reduction of the stiffness in the undercarriage degree of freedom at the peaks of the oscillation

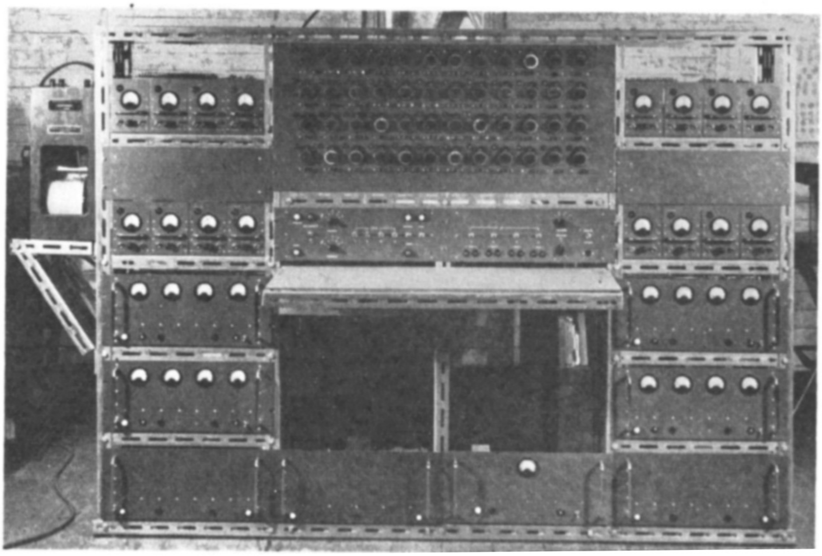


Fig 20a The Linear Computer

This latter phenomenon may account for certain anomalous ground resonance occurrences and will certainly play a significant part in cases of

(1) Taxying over rough terrain
(2) Standing on a rolling ship's deck
(3) One-wheel landings
(4) Take-offs in a high wind (particularly if it is gusty)
(5) Punctures at landing or take-off

It must be stated that at all rotor speeds corresponding to operating conditions, the helicopter is only marginally stable by conventional aircraft standards This fact contributes materially to the occurrences of ground resonance in apparently stable machines

Bibliography

(1) Coleman, R P (1943)
"Theory of self-excited mechanical oscillations of hinged rotor blades"
N A C A A R P 3929

(2) Coleman, R P and Feingold, A M (1947)
" Theory of ground vibrations in a two-bladed helicopter rotor on anisotropic flexible supports "
N A C A T N 1184

(3) Feingold, A M (1943)
" Theory of mechanical oscillations of rotors with two hinged blades "
N A C A A R P 3113

(4) Deutsch, M L (1946)
" Ground vibrations of helicopters "
Journ Aero Sci, 13, 5, 223

(5) Stewart, W Tech Memo No Aero 274

(6) Buchan, A L Helicopter Ground Vibrations Interim Report
Saunders-Roe N SR (H) A22