CORRESPONDENCE

(*To the Editors of the* Journal of the Institute of Actuaries)

SIRS,

During the discussion of Mr Phillips's paper on "The Curve of Deaths," I mentioned the application of the author's method of approximate integration to Contingent Assurances.

Having now had an opportunity of testing the method further, I think that the following numerical results may be of interest to members.

Calculation of $A^{1}_{x:y}$ – O[NM] and O[af] – $3\frac{1}{2}\%$

Age of [y]	Age of [x]	Approximate value	Published value *J.I.A.* Vol. XLV, p. 637	Percentage error
20	20	·20709	·20496	1·0
	40	·35352	·35153	·6
	60	·58447	·58134	·5
	75	·76150	·75999	·2
40	20	·14709	·14741	–·2
	40	·28826	·28691	·5
	60	·54416	·54077	·6
	75	·74252	·73918	·5
60	20	·08901	·08851	·6
	40	·16779	·16651	·8
	60	·42263	·42254	·02
	75	·67536	·67975	–·6
75	20	·04599	·04594	·1
	40	·07768	·07749	·2
	60	·22738	·22752	–·1
	75	·48990	·49087	–·2

Integral ages were used throughout as, although fractional ages would give greater accuracy, the figures given above seemed good enough for most practical purposes, and the saving in labour is considerable.

In all cases where the age of the Assured Life was not less than that of the Counter Life, Mr Phillips's 20-term formula was used without adjustment. Where the Counter Life was the older the 20-term formula was used, with an adjustment at the end which was the result of a good deal of experiment.

It will be realized that in these cases there is a limiting point in the $(l\mu)$ curve, beyond which the corresponding value of $\frac{D_{[y]+t}}{D_{[y]}}$ is nil. The part

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of the curve which lies between this limiting point and the last point of the 20-term formula was divided by ordinates at integral ages, so selected that the intervals between them more or less continued the run formed by the last few terms of the formula. With the areas so formed were associated the corresponding average values of $\frac{D_{[y]+t}}{D_{[y]}}$. An example will make the method clearer.

$x=[20]\quad y=[75]$

No. of Term	$\frac{D_{[y]+t}}{D_{[y]}}$	t	$l_{[x]+t}$	
0	1·00	0	100,000	$\bar{A}^1_{[x][y]}$
1	·95	1	99,580	$\doteq [1\cdot025 l_0 - \cdot05 (l_0+l_1+\ldots+l_{19}) - \cdot025 l_{19}$
2	·90	1	99,580	$+\cdot0333 (l_{19}-l_{20}) + \cdot0102 (l_{20}-l_{21})$
3	·85	2	99,003	$+\cdot0021 (l_{21}-l_{22}) + \cdot00015 (l_{22}-l_{23})] \div l_0$
4	·80	3	98,333	
5	·75	3	98,333	
6	·70	4	97,616	$1\cdot025 l_0 = 102,500$
7	·65	4	97,616	$\cdot0333 \Delta l_{19} = 85$
8	·60	5	96,879	$\cdot0102 \Delta l_{20} = 28$
9	·55	5	96,879	$\cdot0021 \Delta l_{21} = 8$
10	·50	6	96,137	$\cdot00015 \Delta l_{22} = 1$
11	·45	7	95,392	102,622
12	·40	7	95,392	
13	·35	8	94,641	$\cdot05 \Sigma_0^{19} l = 95,734$
14	·30	9	93,886	$\cdot025 l_{19} = 2,209$
15	·25	10	93,124	97,943
16	·20	11	92,355	
17	·15	12	91,578	$\bar{A}^1_{[x][y]} \doteq \frac{102,622-97,943}{100,000}$
18	·10	14	89,995	
19	·05	16	88,365	
20	·0165	19	85,810	$=\cdot04679$
21	·0039	22	83,085	$v^{1/2} = \cdot98295$
22	·0003	26	79,095	$A^1_{[x][y]} \doteq \cdot04599$
23	·0000	30	74,557	Published value $= \cdot04594$

Although the above example looks complicated, the work is really quite short.

The last few values for t in the 20-term formula were ... 11, 12, 14, 16. To continue this series, the further values 19, 22, 26, 30 were inserted (30 being the limiting value). The corresponding values of $\frac{D_{[y]+t}}{D_{[y]}}$ and $l_{[x]+t}$ were taken from the tables, and the formula completed by the same processes as those used by Mr Phillips. Thus the formula

$$\cdot975 (l_0 - l_1) + \cdot925 (l_1 - l_2) + \ldots + \cdot075 (l_{18} - l_{19})$$

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is completed by adding

$$\left(\frac{\cdot 05+\cdot 0165}{2}\right)\times(l_{19}-l_{20})$$

$$+\left(\frac{\cdot 0165+\cdot 0039}{2}\right)\times(l_{20}-l_{21})$$

$$+\left(\frac{\cdot 0039+\cdot 0003}{2}\right)\times(l_{21}-l_{22})$$

$$+\left(\frac{\cdot 0003+\cdot 0000}{2}\right)\times(l_{22}-l_{23}).$$

The final simplified form is as set out in the working.

I am, Sirs, etc.

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