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EVENTUALLY REGULAR SEMIGROUPS

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A semigroup is called eventually regular if each of its elements has some power that is regular. Thus the class of all eventually regular semigroups includes both the class of all regular semigroups and the class of all group-bound semigroups and so in particular includes the class of all finite semigroups [1].

Many results that hold for all regular semigroups also hold for all finite semigroups; often this occurrence is not just a coincidence but is necessarily the case since the results concerned hold for eventually regular semigroups. We show that many results may be generalized from regular semigroups to eventually regular semigroups. In particular Lallement's Lemma that for every congruence ρ on a regular semigroup S, every idempotent ρ-class contains an idempotent is shown to hold for eventually regular semigroups [1].

We define a relation that we denote by $\mu = \mu(S)$ on an arbitrary semigroup S and show that μ is an idempotent-separating congruence on S. For an eventually regular semigroup S it is shown that μ is the maximum idempotent-separating congruence on S. Let S be an arbitrary semigroup. We show that the semigroup S/μ is finite if and only if the set of idempotents of S, $E(S)$ is finite [1]. The semigroup S is called fundamental if the only idempotent-separating congruence on S is ι_S. It is shown that μ is the identity congruence on $S/\mu(S)$ [2]. (Using the previous result David Easdown has shown that for any semigroup

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S , S/μ is fundamental.)

From now on S denotes an arbitrary eventually regular semigroup and ρ is an arbitrary congruence on S . The question of when the biordered set of idempotents $E(S)$ of S is isomorphic to the biordered set of idempotents $E(S/\rho)$ of S/ρ is investigated and it is shown that $E(S) \simeq E(S/\mu)$ [1].

Sufficient conditions (some necessary) are given for S to be group-bound. It is shown that if $K = L, R$ or D and A and B are regular elements of S/ρ that are K-related in S/ρ then there exist $a \in A$, $b \in B$ such that a and b are K-related in S [4].

The lattice of congruences $\Lambda(S)$ on S is investigated *via* the equivalence θ on $\Lambda(S)$ of Reilly and Scheiblich and it is shown that θ is a congruence on $\Lambda(S)$ and that each θ-class is a complete sublattice of $\Lambda(S)$. The maximum element in each θ-class is determined using μ [3].

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