Chapters 8 to 10 and 12 are designed for a subsequent one-semester course of probability theory. The first half consists of the standard topics up to the central limit theorem, to be followed by fundamental martingale theory, the sub/superadditive ergodic theorem and finally stochastic processes in Chapter 12, mainly concentrating on a detailed account of Brownian motion. Chapter 11 and the final Chapter 13 contain additional material on convergence of laws on separable metric spaces, on standard Borel spaces and analytic sets, respectively.

The appendices include among other things a section on axiomatic set theory and in particular a very interesting discussion of measure-theoretic pathologies of compact non-metric spaces.

This book can be used as an excellent base for a one-year course in probability theory (including prerequisites), well preparing a student to proceed to research in many different possible directions. The author (an expert who has worked in the area for many years) has taken great care to give a complete and pedagogically perfect presentation of both the necessary preparatory material of real analysis and the proofs throughout the text. Some of the topics and proofs are rarely found in other text books, for example Strassen's "nearby variables/nearby laws" theorem or Kindler's proof of the Daniell–Stone theorem. The well-organized references at the end of each chapter provide a very good guide to the related literature. Furthermore, extensive notes on the history of important results and theorems (obviously thoroughly researched by the author) contain interesting and sometimes surprising additional information.

I think that this book is a marvellous work which can be highly recommended and will soon become a standard text in the field both for teaching and reference.

M. G. RÖCKNER

ATIYAH, MICHAEL *Collected works*, Vol. 1: Early papers, general papers (Clarendon Press, Oxford, 1988), 386 pp., 0 19 853275 X, £30.

This is the first of the five volumes of Michael Atiyah's works. The second volume is on *K*-theory, two are on index theory and the fifth is on gauge theories. They include everything published by him up to 1985 other than the text book written with Ian Macdonald; a set of lecture notes on representation theory appears for the first time. Each main section starts with a commentary by Atiyah giving some of the historical background to the papers. The decision to arrange the papers chronologically within each subject brings out the mathematical development clearly: classical algebraic geometry gradually influenced by modern methods; algebraic topology leading to *K*-theory; the index theorem in its many forms and finally the mathematical aspects of gauge theories. There are numerous papers applying these topics to solve a variety of problems. The author is as imaginative and quick to see connections between different parts of mathematics as he ever was and is still very prolific. Several more volumes will be needed to cover the period from 1985.

The volume under review has two sections. The "Early Papers" are on algebraic geometry, published between 1952 and 1958. This was a very exciting period in algebraic geometry. Jean Leray had introduced both sheaves and spectral sequences and these techniques were ripe for exploitation. Homology had been clarified and put into its present day form. Kunihiko Kodaira had proved his basic vanishing theorem yielding the finite dimensionality of certain cohomology groups. Atiyah was well aware of these developments and of their potential; he met the leading exponents during his year in Princeton in 1955–56 and several of them became his lifelong friends and collaborators.

His first paper, written whilst he was an undergraduate, studies the tangents to the twisted cubic via the Klein representation. This fascinating technique, which captured Atiyah's imagination as well as Roger Penrose's at that time, was to prove extremely important to both of them more than twenty years later through twistors and gauge theory. As a graduate student Atiyah became interested in bundles and this led to three important papers: the first applies the method to the study of ruled surfaces, the second studies connections on complex analytic bundles, and the third classifies analytic vector bundles over an elliptic curve. The straightforward parts of topological bundle theory do not extend to algebraic geometry and these papers explore the new methods needed to overcome this problem. Topics first studied in these papers have been developed and extended by many authors until today.

Multiple integrals with singular integrands were first studied seriously by Emile Picard and developed much further by Solomon Lefschetz. However, major technical difficulties had arisen. Professor Hodge saw that sheaf theory should overcome these and collaborated with his student Atiyah in writing a very famous paper on integrals of the second kind. During his year at Princeton, Atiyah wrote what is probably his most abstract paper. This was an account of the Krull–Schmidt theorem in the context of abstract category theory but he had applications to algebraic-geometric sheaves firmly in mind. The influence of the French school through Jean-Pierre Serre is very clear.

His next paper is more topological; it solves an interesting question arising in algebraic geometry. If V is a singular variety one can attempt to get rid of the singularities by either deforming V to Y or by resolving the singularities $X \to V$. If $\dim V \neq 2$ then it is relatively easy to see that X and Y cannot be diffeomorphic (or even homotopy equivalent). In this paper it is proved that if V is a hypersurface in $\mathbb{P}_3\mathbb{C}$ with only double points then Y is diffeomorphic to X; this was unknown even for surfaces of degree 4. The study of the topology of algebraic surfaces has continued to be of interest and has reached a new peak over the last few years as a result of Donaldson's work on gauge theories which was in turn inspired by Atiyah's work in that area.

The first section ends with an expository lecture on complex manifolds that appeared in 1958 in the series "Bonner Mathematische Schriften" and which has been unavailable for some time. This lecture was one of the first given by Atiyah at the Arbeitstagung in Bonn.

The section "General Papers" consists mainly of lectures given by Atiyah since 1966 on various special occasions and some of them are rather informal. A number expound his general philosophy about mathematics, some convey his enthusiasms about mathematics whilst others, given to very general audiences, attempt to explain how mathematics relates to other disciplines. Coming from one with such enthusiasm and optimism, it seems a little strange that streaks of pessimism appear when computer science is discussed. He is afraid that talent may be attracted away from mathematics but he also admits that different mathematical challenges continually emerge from a variety of directions.

Atiyah admired his supervisor Professor Hodge and this shines through his very readable and scholarly obituary for the Royal Society. He also admired J. E. Littlewood's enthusiasm for mathematics and my favourite paper in this section is the one based on a tape recording of his lecture at Littlewood's 90th birthday celebration. The lecture was entitled "Singularities of Functions" and dealt with the behaviour of an integral $\int \phi(\mathbf{x}) f(\mathbf{x})^s \, d\mathbf{x}$ (where ϕ is smooth, f is a polynomial and $s<0$) at points where f vanishes. This topic is, of course, reminiscent of his early joint work with Hodge. Resolution of singularities plays an important role in understanding the problem, and the polynomials introduced by Bernstein, which enable an integration by parts to be performed, have led to important developments in algebra. There are further relationships with a number of interesting topics and this lecture epitomizes Atiyah's interests—a simple problem that has widespead ramifications in many areas of mathematics. He is a master at this kind of lecture and it is a pity that so few of them have been recorded in this way. I have been very fortunate to have had the opportunity of attending many of his lectures; they never fail to entertain and educate.

E. G. REES