Robotica (2008) volume 26, pp. 415–416.
doi:10.1017/S0263574708004785 Printed in the United Kingdom

Editorial

Robotic sensing is a relatively new field of activity compared with the design and control of robot mechanisms. In both areas the role of geometry is natural and necessary for the development of devices, their control and use in challenging environments. At the very beginning odometry, tactile and touch sensors dominated robot sensing. More recently, due to the fall in the price of laser devices, they have become more attractive to the community. On the other hand, progress in photogrametry, particularly during the nineties as the n-view geometry in projective geometry matured, bootstrapped the use of computer vision as an extra powerful sensor technique for robot guidance. Cameras were used in monocular or stereoscopic fashion, catadioptric systems for ominidirectional vision, fish-eye cameras and camera networks made the use of computer vision even more diverse. Researchers started to combine sensors for 2D and 3D sensing by fusing sensor data in a projective framework. Thanks to the continuous progress in mechatronics, the low prices of fast computers and increasing accuracy of sensor systems, one can build a robot to perceive its surroundings, reconstruct, plan and ultimately act intelligently. In these perception-action systems there is of course, the urgent need for a geometric stochastic framework to deal with uncertainty in the sensing, planning and action in a robust manner. Here geometry can play a central role for the representation and computing in higher dimensions using projective geometry and differential geometry on Lie groups manifolds with a pseudo Euclidean metric. Let us review briefly the developments towards modern geometry that have been often overlooked by the robotic researchers and practitioners.

Since Euclid, nearly two thousand years ago, the basic understanding of space remained essentially the same. At the end of the enlightenment period, the philosopher Immanuel Kant maintained that there is only one, absolute, geometry known a priori by an inner faculty of mind, according to this claim Euclidean geometry was synthetic a priori. This dominant view was overthrown by the revolutionary discovery of non-Euclidean geometry in the works of Gauss, Bolyai, and Lobachevsky, who demonstrated that ordinary Euclidean space is *only one possibility for development of geometry*. Riemann provided us with a much broader vision of the subject of geometry. His novel view of space proved crucial in Einstein's general relativity theory. Broadly speaking, Riemann working primarily with tools from mathematical analysis, introduced the Riemann surface and achieved the synthesis of diverse results concerning the geometry of surfaces and the behavior of geodesics on them, with techniques that can be applied to the study of differentiable manifolds with pseudo-Euclidean metric in higher dimensions. Why in robotics are we often obsessed with a Euclidean geometric perspective? Since the nineteenth century the crucial discovery of non-Euclidean geometry, the concept of space has undergone a spectacular transformation, leaving behind for ever the Euclidean geometry view. This is the goal of this special issue, to insist on the promising role of modern geometry for robotics. Modern geometry considers manifolds, spaces that are considerably more abstract than the familiar Euclidean space, which they only approximately resemble at small scales. One can see that the modern period in geometry begins with the formulations of projective geometry by J. V. Poncelet (1822); and the formulation of symmetry as the central consideration in the Erlangen Programme of Felix Klein (which generalized the Euclidean and non Euclidean geometries) and that of non-Euclidean geometry by N. I. Lobachevsky (1826) and János Bolyai (1832). Similar to Riemann, another exceptional geometer of that time was Henri Poincaré, the founder of algebraic topology and the geometric theory of dynamical systems.

Despite this enormous legacy, some computer scientist, engineers and roboticists are still reluctant to take advantage of modern geometry. Unfortunately, it is still difficult to bridge the gap between mathematicians, computer scientist and robotic practitioners, so that fruitful ideas of modern geometry like Lie groups, Riemman differential geometry, harmonic analysis and conformal geometry start to play a key role in robot sensing. Together with Jon Selig we conceived a special issue devoted to geometry in mechanics and robot sensing. Surprisingly, we received a good number of high quality papers which required us to split the papers into two special issues, one devoted to geometry for robotic mechanics (published in November 2007) and a second one on geometry for robot sensing. With these two special issues, we hope to promote further the understanding and use of modern geometry for the benefit of the large community working intensively developing sophisticated structures like humanoids and unmanned aircrafts. We believe that geometry, in its broad sense, it should not be forgotten and left to theoreticians. The incorporation of these relatively old mathematical concepts will undoubtedly help to reduce the computational complexity of real time perception-action systems. Next we describe the contributions sorted in a thematic order. The first group of contributions makes use, in a creative manner, of modern geometries like Lie algebra, projective geometry and conformal geometric algebra.

The first contribution by Wooram Park, Yan Liu, Yu Zhou1, Matthew Moses and Gregory S. Chirikjian shows a first attempt in a relatively new area of *robot stochastic planning*. The authors focus on kinematics state estimation and motion planning for stochastic nonholonomic systems. In this context the ensemble of robot trajectories is equivalent to the solution of a Fokker-Planck equation which can be solved using the operational properties of Fourier transforms on groups. The authors use effectively the exponential mapping from the Lie algebras of these groups, and takes advantage of the sparse nature of the Lie algebra representation matrices.

Sigal Berman, Dario G. Liebermann and Tamar Flash offer a rigorous yet simple representation of the 3D velocity of a rigid body in terms of motor algebra (dual quaternions). They examine the decomposition of 3D extended arm pointing and reaching movements into sub-movements and reconstruct the motion by assuming superposition of the velocity profiles of the underlying sub-movements. The reconstructed velocity profiles and final orientations are relatively close to the natural values, indicating that single axis sub-movements based on rotors and translators may be the basic building blocks underlying 3D movement construction.

It is a very interesting idea to describe the colour variation under illumination changes using the Lie algebra manifold of the Lorenz transformation. Reiner Lenz introduces the theory of the Lorentz group SU(1, 1) as a natural tool for analyzing colour image processing problems and derives some descriptions and algorithms for machine vision that are useful in the investigation of dynamical colour changes and compensate image sequences generated by dynamical colour changes.

The authors Carlos López-Franco and Eduardo Bayro-Corrochano use spherical projective geometry with the Lorentz metric in the conformal geometric algebra framework. Here the modelling and analysis of catadioptric systems in terms of the algebra of spheres is natural. In this framework, the authors tackle challenging the problems of multiple-sensors body calibration and reconstruction of 3D maps for robot navigation.

The next set of papers involve studies of realizations, coverage of space and rendering curves depending of certain number of degrees of freedom. These approaches do not follow the traditional track of Euclidean geometries.

Hongbo Li, Lina Zhao and Ying Chen present a symbolic approach to polyhedral scene analysis by parametric Calotte propagation. The paper analyses whether a 2D line drawing of a 3D polyhedron is realizable in space, and if so, if it is able to parameterize the space of all possible realizations. For generic 2D data, symbolic computation with Grassmann-Cayley algebra is needed in the analysis. In applications, it can lead to linear construction sequences for non-spherical polyhedron whose resolvable sequences do not exist.

The work of Subramanian Ramamoorthy, Ram Rajagopal and Lothar Wenzel is concerned with algorithmic techniques for the incremental generation of continuous curves that can efficiently cover an abstract surface. They introduce the notion of low-discrepancy sequences and a procedure for lifting these curves that efficiently cover abstract surfaces such as nonlinear manifolds. Applications of these ideas are of great interest in modern robotic object recognition and manipulation.

The work of Maxim Kolesnikov and Milos Zefran extends the existing penalty-based haptic rendering approaches which are based on penetration depth estimation in the strictly translational sense. In order to take object rotation into account, the authors propose a new 6-DOF haptic rendering algorithm which is based on determining the closest-point projection of the inadmissible configuration onto the set of admissible configurations. Their work offers a way to handle a major computational bottleneck by the control and simulation of robot dynamics.

The following contributions propose motion models exploiting the geometric structure of the problem.

The authors Elias K. Xidias and Nikos A. Aspragathos present a geometrical approach to generate simultaneously optimal/near-optimal smooth paths for a set of non-holonomic robots moving in a 2D environment cluttered with static and moving obstacles. The robots environment represented by a 3D bump-surface is embedded in a 4D Euclidean space. The optimization of multi-motion planning is resolved by finding simultaneously the paths for the set of robots represented by monoparametric smooth C2 curves onto the bump-surface satisfying certain optimization criteria and constraints.

By extending earlier work from 2D to 3D-parts Onno C. Goemans and A. Frank van der Stappen propose a new class of traps that remove a V-shaped portion of the track. The authors incorporate a more realistic part motion model into the design algorithm. They exploit the geometric structure of the design problem and build on concepts and techniques from computational geometry to obtain an efficient algorithm that reports the complete set of valid traps.

The final article is somewhat different in character, but still deals with a complex geometric scenario. The authors Yaniv Altshuler1, Vladimir Yanovsky1, Israel A. Wagner and Alfred M. Bruckstein re-examine the solution of the well-known Cooperative Hunters problem. By arranging the searchers into efficient geometric flight configurations, the UAVs optimize their integrated sensing capabilities, enabling the search of a maximal territory.

I am very thankful to everyone who has contributed to our endeavour to bring modern geometry into play in robotics, first of all to the authors for their valuable contributions, to the reviewers for their criticism and diligent hard work to improve the quality of the articles and finally to the chief editor, Greg Chirikjian, who encouraged us to push forward this project. My editorial work was possible thanks to a DFG-Merkator visiting professor program from fall 2008 to Summer 2008 at the robotics-humanoid laboratory of the TH Karlsruhe University in Germany and friendly hosted by Prof. Rudiger Dillman.

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