RADIATIVE DAMPING OF GRAVITY WAVES IN THE SOLAR ATMOSPHERE

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ABSTRACT

The nonlocal character of the radiation field significantly modifies the radiative damping of perturbations in the solar photosphere. Gravity waves are not usually considered to exist in the solar photosphere because the radiative damping time, when based on the Newtonian approximation, is too short. However, this restriction does not apply to low order gravity waves. In fact, with the inclusion of nonlocal effects, the radiative damping for low order gravity waves becomes negative for some region in the photosphere and thus acts as a driving mechanism for gravity waves there.

1. INTRODUCTION

Several of the features of the observational results reported by Stebbins et al. (1980) have been interpreted by Hill (1980) as evidence for the existence of gravity waves in the solar photosphere. Penetrative convection is an effective driving mechanism for gravity waves. This process should copiously produce gravity waves just above the convection zone in the solar atmosphere (Stein, 1967). However, the analyses of Souffrin (1966) and Stix (1970) using the Newtonian approximation for the radiative damping show that gravity waves cease to exist if the radiative damping time, τ_R, is less than the inverse of the Brunt-Väisälä frequency. The radiative damping time is quite small and less than the above critical inverse frequency when computed for the lower photosphere using the Newtonian approximation. As a result, it has been argued on theoretical grounds that gravity waves cannot exist in the lower photosphere, although an effective mechanism for their generation occurs there.

In examining the nonadiabatic term in the energy equation describing stellar oscillations, Hill and Logan (1980) discovered that the nonlocal character of the radiation field contributes significantly to radiative damping of wave phenomena in stellar atmospheres. For gravity waves

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this might resolve the discrepancy between Hill's (1980) interpretation of the observational work of Stebbins et al. (1980) and the previous theoretical result based on a local analysis.

2. THE BEHAVIOR OF RADIATIVE DAMPING FOR LOW ORDER GRAVITY WAVES

The nonadiabatic term in the energy equation contains a factor proportional to the mean intensity through which nonlocal effects are manifested. Let us consider the perturbation in the mean intensity at some point in the optically thin atmosphere of the sun. The perturbation in the mean intensity will be an integral of the perturbation in the source function over the entire atmosphere weighted by a transmission factor. If the spatial scale of the horizontal and vertical variation of the disturbance is larger than the local opacity scale height, distant regions of the atmosphere can influence this integral. The value of the perturbation in the mean intensity will then deviate significantly from that represented only by a local value of the perturbation in the source term.

The perturbation in the mean intensity, J, for an atmosphere stratified in the z direction is given by

$$J' = \frac{1}{4\pi} \int_0^{2\pi} d\phi \int_0^{\pi/2} d\theta \left[\left(\int_0^{\infty} + \int_0^{\bar{t}_0} \right) S' e^{-\bar{t}\sec\theta} \rho\bar{\kappa} \tan\theta \, dz \right] \tag{1}$$

where $\bar{\kappa}$ is the mean gray absorption opacity, $\bar{t}$ is the optical depth defined with the mean gray opacity, $\bar{t}_0$ is the optical depth of the atmosphere, S is the integrated (over frequency) source function, and the prime (') indicates the Eulerian perturbation. If we assume the horizontal variation in the source function is of the form $\exp(ik_h x)$, we can then express the variation in the source function as

$$S' = q' e^{ik_h x} e^{\alpha z} \tag{2}$$

where α is a complex number that can be used to model the z dependence of S'. The horizontal variation can then be immediately integrated and we can write

$$J' = \int_{-\infty}^{\infty} q' f(s) ds \tag{3}$$

where

$$f(s) = \frac{1}{2} \int_0^{\pi/2} J_0(k_h H_\kappa s \tan\theta) e^{-\bar{t}\sec\theta} e^{\alpha s H_\kappa} \rho\bar{\kappa} H_\kappa \tan\theta d\theta \tag{4}$$

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and J_0 is a zero order Bessel function, H_κ is the local opacity scale height which is a function of z, s is a dimensionless parameter defined by

$$s = z/H_\kappa \quad , \tag{5}$$

and the point of evaluation for J' is at z = 0. The relationship between the horizontal wavenumber for a wave in the solar atmosphere and the ℓ value of the spherical harmonic describing the oscillation is:

$$k_h = \frac{\sqrt{\ell(\ell+1)}}{R_\odot} \tag{6}$$

where $R_\odot$ is the solar radius.

Following the seminal work of Spiegel (1957), we write the local radiative damping time for a gray-absorbing atmosphere as

$$\frac{1}{\tau_R} = \frac{4\pi\bar{\kappa}}{c_p T'} (B' - J') \tag{7}$$

where c_p is the specific heat at constant pressure. If the horizontal and vertical wavenumbers multiplied by the local opacity scale height are both less than one, the integrand in equation (3) is not localized around the point of evaluation for J'. The dimensionless temperature perturbation can be associated with a complex vertical wavenumber β. To investigate the effect of the nonlocal aspect of the radiation field, we set α equal to the real part of β and expand J' as

$$J'(\bar{t}_0) = \sum_{r=0}^{\infty} \left(\frac{1}{r!} \frac{\partial^r q'}{\partial t^r} \right) \Bigg|_{\bar{t}_1} \sum_{p=0}^{r} \binom{r}{p} (-\bar{t}_1)^p \, I^{r-p} (\bar{t}_0, \ell, \alpha) \tag{8}$$

where $\bar{t}_1$ is the optical depth where the maximum contribution to the integral of equation (3) occurs,

$$I^r(\bar{t}_0, \ell, \alpha) = \frac{1}{2} \int_{-\infty}^{\infty} f(s) \bar{t}^r ds \quad , \tag{9}$$

and $\binom{r}{p}$ is a binomial coefficient. In order to integrate equation (9) one must, in general, have a relationship between $\bar{t}$ and s. However, when ℓ and α both equal zero, equation (9) can be integrated directly. In particular, the integral for r equal to zero is

$$I^0(\bar{t}_0, 0, 0) = 1 - \frac{E_2(\bar{t}_0)}{2} \tag{10}$$

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where E_2 is the exponential integral given by

$$E_2(t) = \int_1^\infty \frac{e^{-wt}dw}{w^2} \tag{11}$$

A finite optical depth for the atmosphere is easily seen from equation (10) to decrease the magnitude of J'.

Including only the first term in the series of equation (8), the damping time for a gray-absorbing atmosphere in L.T.E. in the low wave-number regime can be written as

$$\frac{1}{\tau_R} = \frac{16\sigma_B\bar{\kappa}}{c_p T_{\bar{t}_0}}\left[T_{\bar{t}_0}^4 - T_{\bar{t}_1}^4\, e^{(\beta-\alpha)z_1}\, I^0(\bar{t}_0,\ell,\alpha)\right] \tag{12}$$

where σ_B is the Boltzmann constant and z_1 is the difference in heights between $\bar{t}_1$ and $\bar{t}_0$. The typical value of the difference between $\bar{t}_1$ and $\bar{t}_0$ is 0.5 with $\bar{t}_1$ being the greater of the two (cf. Hill and Logan, 1980). Using equation (12) and assuming a constant opacity scale height of 90 km with $\beta - \alpha$ equal to zero, the damping times for the solar model of Bahcall et al. (1973) can be computed for different ℓ's and α's. Figures 1a through 1f show the computed damping times for αH_κ equal to 0, 0.18, and 0.45 respectively, with ℓ equal to 1 and 1000. The damping times computed in the Newtonian approximation are also shown for comparison (represented by the dotted curves). In all of the solid curves a prominent region of negative damping times appears. This corresponds to amplification of the disturbance.

3. IMPLICATIONS FOR GRAVITY WAVES IN THE SOLAR ATMOSPHERE

A gravity wave with a ℓ value less than or of the order of several thousand and with $|\beta H_\kappa|$ less than one is far from being overdamped in the photosphere and in fact should be driven as shown above. The growth rate of an oscillation will of course depend on its overall damping, but the wave will not be damped out of existence a priori. The change in the magnitude of the damping due to the inclusion of nonlocal effects not only allows gravity waves to overlap the region of penetrative convection, which can mechanically generate the waves, but also permits a coupling of the waves through radiative transfer to the penetrative convection and to turbulent motion in the convection zone.

The Newtonian damping time is independent of scale when the non-local aspect of the radiation field is included. However, the radiative damping time for small wavenumbers differs qualitatively from that for large wavenumbers. Letting the horizontal wavenumber go to infinity to derive an effective Brunt-Väisälä frequency from the dispersion relation therefore automatically excludes the proper consideration of the low wavenumber behavior.

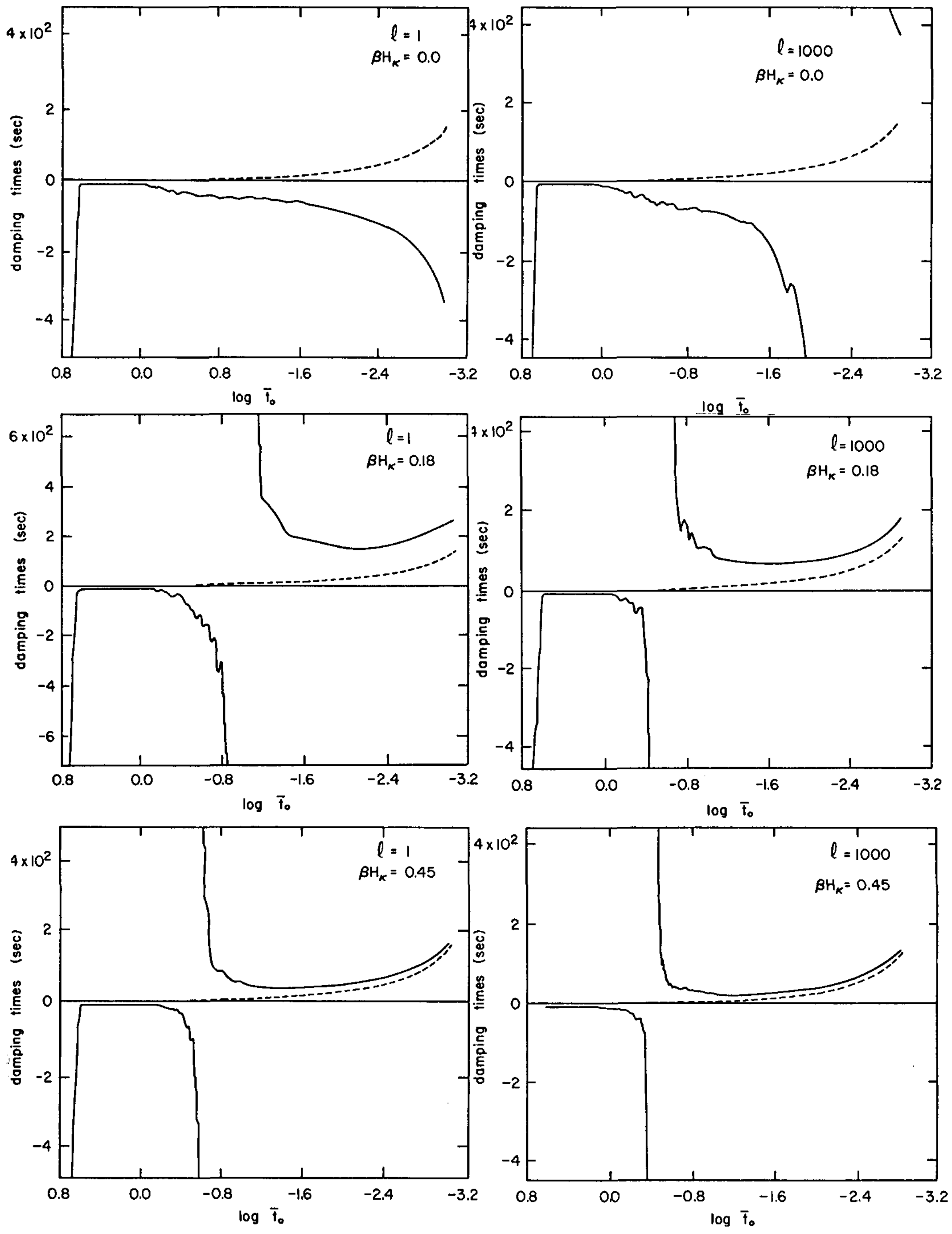

Figure 1. Radiative damping times computed from equation (12) as a function of optical depth for ℓ = 1 and 1000 and for βH_K = 0, 0.18 and 0.45. The dotted line represents the Newtonian radiative damping time.

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An examination of Figures 1a through 1f shows that the driving and therefore possibly the amplitude of an oscillation both depend on its vertical growth rate. This might explain the necessary "anomalous boundary conditions" discussed by Hill (1978) in the comparison of the long period solar oscillation observations and those of the 5 min modes.

Another consequence of driving is that the momentum and energy flux of an amplified wave must increase. This must show up in the momentum and energy budgets of the mean flow. Therefore, if waves which have been amplified through their interaction with the radiation field are present, we would expect radiative equilibrium to be established at a lower mean-field temperature than the local temperature for radiative equilibrium without the disturbances.

The above analysis shows that the Newtonian approximation fails to reproduce the qualitative behavior of radiative damping for low order gravity waves. One is led to conclude that a local analysis based on the Newtonian approximation cannot be used to dismiss the existence of gravity waves in the solar photosphere nor used against the interpretation of observations in terms of gravity waves.

In order to understand the observational properties of pulsations in stars other than the sun where radiative damping in the envelope is also important, it may be necessary to consider the nonlocal aspect of the radiation field.

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DISCUSSION

J. COX: How thick in kilometers is the driving region?

LOGAN: I don't know how τ and the depth in kilometers are related. We go to $\log \tau = -0.8$.

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