Introduction to proofs and proof strategies by Shay Fuchs, pp 349, £34.99 (paper), ISBN 978-1-00909-628-7, Cambridge University Press (2023)

This has a similar target readership to Cupillari's book (reviewed above), but with a wider range of mathematics topics, which probably makes it less useful to the sixth formers, but more useful to first year, and possibly also second year, undergraduates. It is strong on motivational remarks and commentaries on the proofs presented, and it draws attention to proofs which exhibit ingenuity or neatness, or prove unexpected results.

'Informal logic and proof strategies' is deferred until chapter 3, the first two chapters giving opportunities for readers to experience getting their hands dirty in trying to devise proofs.

Chapter 1 uses a proof of the quadratic formula as a case study to highlight issues such as why proof is necessary, what a proof achieves, and what prior knowledge it depends on. This leads to a discussion of inequalities, and in particular to the triangle and AM/GM inequalities. There are also geometric illustrations of the harmonic and geometric means, and comments on why proofs of inequalities often require more care than proofs of equalities.

Although logic in symbolic systematic form does not appear until chapter 3, the subtlety of logical implication does feature right from the start. For example, one of the problems of chapter 1 is to consider the following 'solution' to 'Find all real solutions of $x^2 + x + 1 = 0$, if there are any': 'Clearly $x \neq 0$ since $0^2 + 0 + 1 \neq 0$, so we can divide by x: $x + 1 + \frac{1}{x} = 0 \Rightarrow x = -1 = \frac{1}{x}$. So replace the x term in the original equation by $-x - \frac{1}{x}$;

$$x^2 + \left(-1 - \frac{1}{x}\right) + 1 = 0 \Rightarrow x^2 - \frac{1}{x} = 0 \Rightarrow x^3 = 1,$$

so $x = 1$ is the only real solution.'

A possible weakness, or at least a frustration, is that although the exercises embedded in the text all have full solutions, the end-of-chapter problems have neither solutions nor hints.

What I particularly like about this book is that it discusses the process of seeking a proof as well as understanding a finished one. Part of the process is the use of 'rough work', which is really mathematical experimentation to suggest a way forward. Although the rough work sometimes proves only the converse of what is required, it can still suggest a productive approach by highlighting a few significant connections.

Chapter 2 is a similarly thoughtful treatment of sets, functions and the field axioms, but although all are done thoroughly, the light touch prevents readers from feeling that they are just learning a load of pedantic jargon. The field axioms are introduced and used as an example of the sheer power of an axiom system, and as a peg on which to hang a discussion of the logistical status of such a system, including a statement that mathematics is not about *absolute* truth!

In chapter 3 the basics of propositional and quantifier logic get the same conversational yet careful treatment, with an emphasis on good mathematical proofs being a judicious mix of symbols and words. The words need to be there to tell the reader what you are doing (and possibly why); the symbols are abbreviations for what could be said in words but only by tripling the length of the proof and probably making it impenetrable.

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The final section discusses the specific strategies of direct proof, contrapositive and contradiction, with well-chosen examples.

Chapter 4 is on induction, the logic underpinning it, some of it variations, and several ways in which it can be abused, all with a good range of examples. The chapter ends with a section on the summation and product notations.

Part I concludes with three more chapters: bijections and cardinality, including the Schröder-Bernstein theorem; integers, divisibility and basic number theory up to unique prime factorisation; and relations, especially equivalence relations and including congruence arithmetic.

Part II is 'additional topics': combinatorics (permutations and combinations, the binomial theorem, Pascal's triangle, the pigeon-hole principle, inclusion and exclusion); a preview of real analysis (limits, continuity, differentiability); complex numbers up to de Moivre's theorem, roots and exponential form; and finally a preview of linear algebra ($\mathbb{R}^n$, vector spaces and isomorphism).

Definitely highly recommended.

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An introduction to infinite products by Charles H. C. Little, Kee L. Teo and Bruce van Brunt, pp 251, £29.99 (paper), ISBN 978-3-03090-645-0, Springer Verlag (2022)

Whereas infinite series are a canonical part of any course on mathematical analysis, infinite products are only sporadically treated. The same is with books. The authors aim to fill this gap in the literature and show that infinite products 'have their own lives and manifest themselves through beautiful and unexpected relations'. The six chapters of this book will undoubtedly convince the reader of this claim.

As infinite products are intimately linked to infinite series, the first chapter is devoted to the latter. It is a summary of known results, with no proofs and no exercises, except for double sequences and series which are seldom treated in standard curricula.

Chapter 2 is central in the book and presents the core results and examples on infinite products. The authors show with examples that there is no relationship between the convergence of $\sum_{n=1}^{\infty} z_n$ and that of $\prod_{n=1}^{\infty}(1 + z_n)$ and then give Cauchy's test for their simultaneous convergence. Highlights are Euler's infinite product expansion $\sin x = x \prod_{n=1}^{\infty}\left(1 - \frac{x^2}{\pi^2 n^2}\right)$ and the Weierstrass factorisation theorem. The first result is combined with the Maclaurin series for $\sin x$ and $\log(1 - x)$ to give a solution to the famous Basel problem from 1734 which asks for the exact value of the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$. Euler showed that the value is $\pi^2/6$ and then went on to determine the value of $\sum_{n=1}^{\infty} \frac{1}{n^{2k}}$ for $k = 1, 2, 3, \ldots$. The second result, from complex analysis, says that every nonzero entire function can be written as an infinite product involving Weierstrass primary factors. The chapter ends with Blaschke products and double infinite products.

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