vibrating strings. There is an intriguing (non-post Eulerian) account of the initial ideas on hyperbolic trigonometry due to Vincenzo Riccati (1757) and, more completely, by Johann Lambert (1768). The early nineteenth century witnessed the discovery of non-Euclidean geometries that invoked the application of trigonometry to surfaces other than the Euclidean plane.

In *The Mathematics of the heavens and the earth* Van Brummelen provided extensive treatment of trigonometry in India and the Islamic world. In this book, chapter 4 provides an account of the work of Chinese trigonometers. It also outlines the assimilation of Indian and Islamic ideas into Chinese mathematics, and it concludes with a description of the eventual importation of European trigonometry.

The depth and breadth of historical analysis and the extensive mathematical scope of this scholarly work are partially evidenced by its extensive bibliography (1000 references), yet Van Brummelen's entertaining narrative makes it compulsive reading. There is reference to about 280 different contributors to the field of trigonometry and its applications. Most of these are less well known, but emphasis is placed upon many of the 'greats', such as de Moivre, Newton, James Gregory, Leibniz, Euler and the Bernoullis. For many of these, biographical detail is supplied, often with portrayal of the surrounding cultural and political context.

This is yet another enjoyable mathematical gem from Glen Van Brummelen.

*References*

1. Glen Van Brummelen, *The Mathematics of the heavens and the earth*, Princeton University Press (2009), reviewed in *Math. Gaz*. **95** (March 2011), pp. 150-151.

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**Introduction to linear and matrix algebra** by Nathaniel Johnston, pp. 482, £49.99 (hard), ISBN 978-3-03052-810-2, Springer Verlag (2021)

This book starts with basic coordinate-system vectors as met in most of the UK at GCSE, and progresses to eigenvectors and eigenvalues of matrices. It is organised in three long chapters, each of which consists of theory followed by detailed explorations of some applications, called 'Extra Topics'. Chapter 1 introduces matrices and their use to describe linear transformations, largely in two dimensions, as well as the dot product, with extra topics on the cross product and paths in graphs. Chapter 2 looks at linear equations, elementary matrices and matrix inverses, vector spaces in a purely geometric context (and hence called just 'subspaces' here), bases and dimensions, with extra topics on finite fields, linear programming, rank decomposition and the LU decomposition. Only in Chapter 3 do we meet determinants formally, followed by eigenvectors and diagonalisation. The extra topics here give more on determinants, including permutations and an algebraic proof of $\det AB = \det A \det B$; power iteration to find the eigenvalue of largest magnitude, including the PageRank algorithm; complex eigenvalues; and linear recurrence relations. There are appendices of mathematical preliminaries, including complex numbers, polynomials and proof techniques, so although the blurb says that readers are assumed to have completed one or two university-level mathematics courses, the author seems to have aimed at students with much less background.

Each section of each chapter ends with a substantial collection of exercises, of which roughly half are given solutions at the back of the book. Further exercises are available online, and as many of these have randomised numerical inputs there will be no shortage of examples to use.

A particular strength of the book is its copious use of geometrical illustrations. Most learners will find their intuition greatly helped by this (*O si sic omnes!*), and it hardly matters that some of the diagrams are not very clear.

The writing is accessible, indeed very readable (the phrase 'as a sanity check' is used repeatedly), but it does not lack rigour. Although mention is made of the computational advantages of block matrices and sparse matrices, very little use is made of computational aids:

> There is actually an explicit formula for the inverse of a matrix of any size, which we derive in Section 3.A.1. However, it is hideous for matrices of size $3 \times 3$ and larger and thus the method of computing the inverse based on Gauss-Jordan elimination is typically much easier to use.

(I can't resist mentioning that when as an undergraduate I shared a room with a very much better mathematician, my mathematical contribution to the partnership was to invert his matrices.) The deferring of the definition of determinants allows a non-recursive definition to be given, while the more familiar definition met at school level emerges as a consequence. The frequent use of commentary and explanation in the margins allows the main thread of the arguments to be clearly seen while appropriate 'hand-holding' is available at once.

One might take issue with certain bits of the exposition—I could imagine a clearer introduction to matrix multiplication. On the other hand I particularly liked features such as the explanation of the simplex algorithm in the context of solving simultaneous equations, the discussion of Vandermonde matrices, the use of a 74-dimensional matrix to describe the 'look-and-say' sequence, and the extension of the result $(U\Lambda U^{-1})^n = U\Lambda^n U^{-1}$ to negative and fractional $n$.

It will be apparent that this book covers relatively limited and elementary ground in considerable detail. (There is a second volume, *Advanced linear and matrix algebra*, which I have not seen.) It seems to me an excellent first course for undergraduates without a strong mathematical background who are taking subjects where the technicalities of linear algebra are needed, for instance computer animation or some courses in economics or financial modelling, while it also gives splendid enrichment for those taking, say, further mathematics A level. International Centres such as those taking more ambitious CIE examinations would certainly find it useful. Above all, any schools where further mathematics is taught would be well advised to have a copy for departmental use; teachers who read it would feel more confident in their teaching, and I am sure they would enjoy the experience.

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**Counterexamples in measure and integration** by René L. Schilling and Franziska Kühn, pp. 399, £34.99 (paper), ISBN 978-1-00900-162-5, Cambridge University Press (2021)

A young mathematician's view of the role played by counterexamples might well take one of the two following forms. Firstly, we picture a PhD student or postdoc