1. THE COMPUTATION OF THE LUNAR EPHEMERIS, 1972–81

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The computation of the lunar ephemeris from Brown's theory for the years 1972–81 is being undertaken by H.M. Nautical Almanac Office and we consider that some of the techniques that are to be used are of general application. The principal task is the evaluation of the longitude, latitude and parallax from multiple-harmonic series, each of which contains several hundred terms, and we have endeavoured to find methods that are efficient for use with electronic computers. The secondary task is to prepare, from these fundamental values, ephemerides that are in the most suitable form for use by astronomers: in addition to the published hourly ephemeris of apparent right ascension and declination we are proposing to prepare a corresponding ephemeris at a wide interval that is designed for use in automatic computation.

The evaluation of the harmonic series

The major part of the summations of the terms in Brown's Tables (1) was carried out up to the year 2000 by the Office during 1929 using simple punched-card tabulating machines. Some of this work may be used to provide independent checks on the new calculations, but its accuracy is limited by the approximations and omissions of the Tables. The techniques that were used by Eckert and his collaborators to compute an improved lunar ephemeris for the years 1952–71, using the IBM Selective Sequence Electronic Calculator (S.S.E.C.) together with an IBM 604 (and later an IBM 701), have already been fully described (2) but more direct methods requiring simple programs are suitable for modern computers especially if positions are needed mainly for isolated instants. It still seems to be true, however, that when ephemerides are required for many years at a time considerable economies in computer time can be achieved with only a moderate increase in the complexity of the computer program, provided that the computer has adequate storage capacity. In particular we have developed a new technique for the evaluation of sums of terms of the form $K \sin (a + nb)$ in which K, a, b may be treated as constant over a wide range of values of n. We are also of the opinion that there is still a place for the technique of subtabulating the sums of groups of long-period terms; when an electronic computer is used subtabulation by bridging-difference techniques (3) can take place between the calculation of each pivotal value and so no time is wasted by output and re-input of the pivotal values. The use of electronic computers for subtabulation enables wider intervals to be used, and so the fundamental values of longitude, latitude and parallax may be calculated at an interval of 16 hours instead of 12 hours, although here the saving of one quarter of the computing time is perhaps not significant when very fast computers are used.

The secular and additive effects are negligible for the many small terms of the series and so for these it is only necessary to evaluate the sums of groups of terms of the form $K_r \sin (a_r + nb_r)$ for $n = 0, 1, 2, \ldots N$ and $r = 1, 2, \ldots R$, say. Instead of evaluating all the terms for one date at a time, partial sums $S_r(n)$ for all $N + 1$ dates are built up one term at a time by using

$$S_r(n) = S_{r-1}(n) + T_r(n), \text{ for } n = 0, 1, 2, \ldots N,$$

where the term values $T_r(n)$, apart from $T_r(0)$ and $T_r(1)$ which are calculated directly, are given by the second-order recurrence relation

$$T_r(n) \equiv K_r \sin (a_r + nb_r) = 2 \cos b_r \, T_r(n - 1) - T_r(n - 2).$$

This relation only requires one multiplication and one subtraction for each new term value and so leads to a very simple and efficient program. The errors in using this recurrence relation arise from the accumulation of rounding errors at each step and from the rounding error in the adopted value of the constant $2 \cos b_r$. The former error varies as $n^{1/2} \operatorname{cosec} b$ and the latter as $n \operatorname{cosec} b$, but provided that sufficient guarding figures are retained the relation can be

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used over very long ranges. In practice, the storage capacity of the computer limits the range since $4R + N + 1$ storage locations, in addition to those for the program itself, are required if $N + 1$ sums for R terms are to be formed. Further N may be limited by the necessity to provide reasonably frequent checks on the accurate working of the computer and to reduce the number of guarding figures required. The overall accuracy of the results is judged by comparing the first sum ($n = 0$) of a new cycle with the final sum ($n = N$) of the previous cycle.

Machine-readable ephemeris

As electronic computers are being more widely used in astronomy so the printed ephemerides are becoming of less value. For example, the comparison of observation with theory is now usually carried out on a computer and we have found it necessary to prepare a machine-readable ephemeris of the Moon for the reduction of observations of occultations of stars. At first this ephemeris will be available on punched cards, but later it could be transcribed on to magnetic tape.

To reduce the amount of storage space required, both physically and in the store of the computer, it seems to be desirable that the ephemeris be at the widest convenient interval. Instead of giving differences, or modified differences, or the coefficients of economised polynomials we are giving for each interval the coefficients in the expansion of the function in terms of Chebyshev polynomials (4). In *The Astronomical Ephemeris* the apparent positions of the Moon are tabulated at an interval of 1 hour, but by using a Chebyshev expansion of the sixth degree an interval of 2 days can be used—this is much greater than the interval at which the differences begin to diverge. Longer intervals require a disproportionately large increase in the number of terms but may be suitable for very fast computers.

These Chebyshev expansions have the advantages that they give the maximum amount of information in the minimum number of terms and digits, and that the series can be truncated at any point if less than maximum accuracy is required. The expansions are of the form

$$f(x) = \tfrac{1}{2}a_0 + a_1 T_1(x) + \ldots + a_r T_r(x) + \ldots + a_n T_n(x), \text{ for } -1 \leqslant x \leqslant +1,$$

where $T_r(x) = \cos(r \cos^{-1} x)$. These expansions are evaluated by the use of the recurrence relation

$$b_r = a_r + 2x\, b_{r+1} - b_{r+2},$$

for $r = n, n-1, \ldots, 1, 0$; it is subject to the initial conditions that

$$b_{n+1} = b_{n+2} = 0;$$

then

$$f(x) = \tfrac{1}{2}(b_0 - b_2).$$

This is only slightly more complicated than the usual 'nesting' algorithm for the evaluation of a power series, and the number of machine instructions and the time taken are only slightly greater.

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