Dynamics of Solar and Stellar Convection Zones and Atmospheres
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Levels of stabilization of velocity and magnetic induction in the convective zone of the Sun

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Abstract. The induction and momentum equations of solar dynamo are simplified to a dynamic system for the convective Root-Mean-Square (rms) velocity and the rms magnetic field in the solar convection zone. The study of stable stationary points of this system gives a minor excess of the critical level of the dynamo and, accordingly, moderate magnetic field typically about 1 T (10 kG). A significantly lower rms magnetic field may be possible at some parameters of the system. The stable rms velocity is about 100 m/sec, and the characteristic magnetic times are about the half-period of solar rotation or about an average lifetime of sunspots. Relative magnetic energy is of order 5 kJ/kg that is about the kinetic energy. The unstable stationary points could be near zero magnetic fields as in periods of very lower solar activity similar to the Maunder minimum.

Keywords. convective velocity, convective zone, diffusion times, levels of stabilization, magnetic field value, solar dynamo

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1. Introduction

The complete system of solar magnetic dynamo equations [Gilman & Glatzmaier (1981), Lantz & Fan (1999)] is extremely complex, since it includes the induction equation for the magnetic field vector, the momentum equation for the velocity vector field of a conducting fluid, equation for entropy and equations of state. From the first decade of this century [Ghizaru *et al.* (2010)] to the present [Guerrero *et al.* (2019), Hotta *et al.* (2022)] some numerical models had been created that imitate the solar magnetic cycle and differential rotation, starting from an almost complete system. However, the key parameters (and, above all, the transport coefficients) of all such models differ by many orders of magnitude from their true values, and extremely far extrapolations to real values have to be made. Therefore, simplified models are very relevant, among which the mean field models seem to be the most reliable [Krause & Redler (1980), Moffatt & Dormy (2019), Charbonneau & Sokoloff (2023)]. But even they remain difficult enough for direct analysis and, at the same time, rely on unproven hypotheses.

The purpose of this work is to create a reliable dynamic system for the rms velocity and rms magnetic field in the interior of the solar or similar stellar convective zone with an active MHD dynamo based on first principles and on the dynamo integral equations. In this paper, I perform stability/instability analysis near the stationary points of this original system for the Sun. Many other potentially much more important manifestations of the obtained system for the Sun and other stars are supposed to be devoted to subsequent works.

In the next section, the required system of equations is derived based on the magnetic induction equation and the momentum equation. The energy and other equations of the dynamo are simply approximated by the fact that the integral power of the work of the Archimedes buoyancy force is given as, generally speaking, a function of time. The momentum equation is multiplied scalarly by the velocity vector and this product is integrated throughout the solar convection zone (CZ). This gives an evolutionary equation for the total kinetic energy, which, after dividing by the mass of CZ, sets the desired equation for the rms velocity, provided that each integral term appears to be proportional to the corresponding powers of the velocity and magnetic field. In a similar way, I multiply the induction equation by the magnetic field vector and integrate the product over the entire volume of CZ. This gives an evolutionary equation for the total magnetic energy, which, after dividing by the volume of CZ, sets the desired equation for the rms magnetic field, provided that each integral term appears to be proportional to the corresponding powers of the velocity and magnetic field.

The third, main section of this work is devoted to stable and unstable stationary points of the resulting simplified system for convective velocity and magnetic field. The stationary points themselves are defined in a standard way - as places where derivatives vanish. To assess the stability of these points, corresponding linear systems are constructed in their small neighborhood. Solutions of such systems that stabilize near these points are naturally considered stable, while those moving away (exponentially in time) from these points are considered unstable. Accordingly, we consider the location of the dynamo near stable points to be a more probable behavior. And placing dynamo near unstable points is less probable behavior. For dynamo, various estimates of physical quantities resulting from derived here equations are given. They are in agreement with known modern numerical, theoretical, and observational models of the magnetic and velocity fields. At the same time, new original relations were obtained.

A section with results is absent because this work intended to be short. One could find the major results in the abstract. For the same reason of the space shortage, I did not present proofs of stability and instability of the different stationary points here.

2. Derivation of equations

Here, I further derive the simplest dynamical system for the rms convective velocity and the rms magnetic field.

To obtain the equation for the evolution of the rms velocity $u(t)$, I integrate over the entire volume of CZ the scalar product of the velocity vector $\mathbf{U}$ and the momentum equation and, as a result of identical transformations and neglect of small terms (for details, see, for example, [Gilman & Glatzmaier (1981), Priest (2014), Starchenko (2019)]) I get

$$\frac{d}{dt}\left(\int_{r_i}^{r_o} \rho \frac{U^2}{2} dV\right) = \int_{r_i}^{r_o} \left(\rho \mathbf{A}\cdot\mathbf{U} - \frac{\mathbf{U}\times\mathbf{B}\cdot\nabla\times\mathbf{B}}{\mu_0} - \rho\nu|\nabla\times\mathbf{U}|^2\right) dV. \tag{1}$$

Here, within the integrals, r_i and r_o denote the radius of the inner and outer boundary of CZ, respectively, ρ is the average density, and ν is the coefficient of kinematic viscosity. I would especially like to note $\mathbf{A}$ - the acceleration caused by the buoyancy force of Archimedes. It generates convection, which, in turn, generates a magnetic field with vector $\mathbf{B}$.

Let us divide (1) by the mass of CZ M. We obtain on the left side of (1) directly by definition udu/dt. The first term one on the right is the specific integral power of the Archimedes buoyancy force a, which we consider being a given function of time. Next comes the specific magnetic force (or more precisely, power) of Lorentz, which, based on the degrees of velocity and magnetic field included in it, can naturally be estimated as $ub^2/(L\rho\mu_0)$. Here L is the characteristic external scale, divided, based on the corresponding vector product, by the typical sine of the angle between the velocity and magnetic field vectors. The end of (1)/M is the specific integral power of the diffusion force, which can naturally be estimated as u^2/T_u. Thus, within the framework of the developed approach, integrals are represented through their components, to which they are directly proportional. Time T_u is the diffusion time and b is the rms magnetic field. As a result, we obtain the evolutionary equation for velocity

$$udu/dt = a - ub^2/(L\rho\mu_0) - u^2/T_u. \tag{2}$$

In a similar way, I integrate over the volume the scalar product of the magnetic field vector and the induction equation and obtain (σ - conductivity):

$$\frac{d}{dt}\left(\int_{r_i}^{r_o} \frac{B^2}{2\mu_0} dV\right) = \int_{r_i}^{r_o} \left(\frac{\mathbf{U}\times\mathbf{B}\cdot\nabla\times\mathbf{B}}{\mu_0} - \frac{|\nabla\times\mathbf{B}|^2}{\sigma\mu_0^2}\right) dV. \tag{3}$$

Now dividing expression (3) by the volume of CZ and based on the considerations presented above, we finally obtain the evolutionary equation for the magnetic field (T_b is the diffusion time for the magnetic field):

$$db/dt = ub/L - b/T_b. \tag{4}$$

This equation (4) together with equation (2) constitutes the desired system.

3. Stationary points

The stationary points of the system obtained above from (2) and (4) are found by zeroing the time derivatives and then solving the corresponding algebraic systems. In this case, all the parameters considered below can, generally speaking, be time dependent, but for the order of magnitude estimates given in this section, it is natural to consider them constant.

Let's start with the stable stationary points of the system (2, 4), corresponding to a non-zero magnetic field (index - S):

$$u_S = L/T_b, \tag{5}$$

$$b_S = \pm\{\rho\mu_0[T_{ba} - L^2/(T_uT_b)]\}^{1/2}. \tag{6}$$

With a typical theoretical and practical value of the stationary velocity $u_S = 100$ m/sec [Gilman & Glatzmaier (1981), Hotta *et al.* (2022), Moffatt & Dormy (2019), Priest (2014)] and diffusion time $T_b = 1$ Msec [Moffatt & Dormy (2019), Priest (2014), Forgacs-Dajka *et al.* (2021), Fan (2021)], we have $L = 100$ Mm, which is an order of magnitude of the CZ half-depth, indicating a corresponding insignificant excess of the critical dynamo level. With the given power about $a = 10$ mW/kg (ratio of the well-known solar luminosity to mass of CZ), one obtains that for the generation, or rather, for the very existence of a stationary magnetic field, it is necessary, following from (6) and the above estimates, that the threshold condition be met

$$T_u > L^2/(T_b^2 a) = 1\text{Msec}. \tag{7}$$

The last time interval is approximately a half-month which is about an average solar spots lifetime or half-period of the solar rotation rate [Moffatt & Dormy (2019), Priest (2014), Forgacs-Dajka *et al.* (2021), Fan (2021)]. The corresponding kinetic scale or convective cell size is $u_S(1\text{Msec}) = L$. This is about the known estimates like u_ST_u for such a giant cell scale.

If condition (7) is satisfied with a margin, then the rms field b_S from (6) is about 1 T (10 kG), which corresponds to the largest magnetic field value possible in this system.

The relative magnetic energy $b^2/2\rho\mu_0$ is as well the largest about 5 kJ/kg, which is close to the relative kinetic energy value $u^2/2$. If condition (7) is satisfied almost without a margin, then the rms magnetic field could be much lower than 1 T. This could be actually the way to the following unstable stationary points.

Let's finally study unstable stationary points approaching zero magnetic fields:

$$B_0 = 0, u_0 = \pm(T_u a)^{1/2}. \tag{8}$$

Those unstable stationary points may be associated with periods of very lower solar activity similar to the Maunder minimum, and possibly (although unlikely) with an extremely rare catastrophic zeroing of the magnetic field that may produce as well catastrophic super-flare.

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