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A Simple Mean-Field Diagnostic from Stokes V Spectra

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Abstract. It is shown that the diagnostics from an observed circularly polarized line in a rapidly rotating star are *directly* interpretable, not in terms of the observed Stokes V profiles, but in terms of its antiderivative with respect to wavelength (in velocity units if preferred). This also leads to a new mean-field diagnostic that is just as easily obtained as the standard "center of gravity" approach, and is less susceptible to cancellation if the line-of-sight field changes sign over the face of the star.

Keywords. line:profiles, magnetic fields, techniques: polarimetric

1. Introduction

Circular polarization in magnetically sensitive lines is produced by a line-of-sight (LOS) magnetic field via the Zeeman shift, which has the opposite sign for left and right circular polarization. In first order, this leads to a circularly polarized absorption line profile that scales linearly with field strength, and can be used to characterize the LOS field. In a rapid rotator, Doppler shifts connect different locations Ω on the star to different points in the profile x, where x can be in wavelength, velocity, or fractional linewidth units. Rapid rotation also implies something less often appreciated: the polarization signal is now also sensitive to *gradients* in the LOS field at these points. Remarably, this added complexity is readily navigated by consideration of the antiderivative of the Stokes V profile, as shown below. Since toroidal fields, and magnetic spots, are important examples of LOS field gradients, their presence on rapid rotators is particularly important for the relevance of the results here.

2. The Antiderivative of the Stokes V

The observed Stokes V flux, $F_V(x)$, is given to first order in the field by

$$F_V(x) \;=\; \int \mathrm{d}\Omega \; h(\Omega) \frac{\partial}{\partial x} I(x,\Omega) \;=\; -\frac{\partial}{\partial x} \left\{ \int \mathrm{d}\Omega \; h(\Omega) \left[I_c(\Omega) \;-\; I(x,\Omega) \right] \right\} , \qquad (2.1)$$

where h is proportional to the LOS field at locations parametrized by the solid angle Ω, and I_c is the continuum level. Defining an effective mean field as a function of x via

$$\bar{H}(x) \;=\; \frac{\int \mathrm{d}\Omega \; h(\Omega) \left[I_c(\Omega) \;-\; I(x,\Omega) \right]}{\int \mathrm{d}\Omega \left[I_c(\Omega) \;-\; I(x,\Omega) \right]} \;=\; \frac{\int \mathrm{d}\Omega \; h(\Omega) \left[I_c(\Omega) \;-\; I(x,\Omega) \right]}{\left[F_c \;-\; F(x) \right]} \qquad (2.2)$$

where F_c and $F(x)$ are the observed continuum and line fluxes, and defining the negative antiderivative

$$A(x) \;=\; \int_x^\infty \mathrm{d}x' \; F_V(x'), \qquad (2.3)$$

we obtain from simple algebra that

$$\bar{H}(x) \;=\; \frac{A(x)}{[F_c \;-\; F(x)]} \;. \tag{2.4}$$

This is the fundamental expression that connects the observables to what can be known about the mean LOS field, and interestingly, it does not involve $F_V(x)$ directly, but rather its negative antiderivative $A(x)$. In the limit of extreme rotation, $\bar{H}(x)$ becomes the absorption-profile-weighted average of h over the resonant region associated with each x, which can then be inferred from the observables via eq. (2.4).

3. A Simple Alternative Mean-Field Diagnostic

This insight also allows us to use the observed antiderivative to infer a mean-field diagnostic over the whole stellar surface, yielding less opposite-polarity cancellation than the standard "center of gravity" approach when the star is rapidly rotating. The absolute value of $F_V(x)$ cannot be used when signal-to-noise is weak, but the absolute value of $A(x)$ can be used, because noise tends to cancel as signal accumulates, even before the absolute value is taken. This allows a new mean-field diagnostic $< h >$, which suffers less cancellation when the polarity changes across the stellar surface, given by

$$< h > = \int \mathrm{d}x \; |\bar{H}(x)| \;. \tag{3.1}$$

In particular, this new diagnostic yields a mean-field measure that gives a nonzero result for a toroidal field, or for a dipole field that is tilted with respect to the rotation axis and is seen from the magnetic equator, both of which would give a zero result in the center-of-gravity approach.