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BOOK REVIEWS

REES, D. *Lectures on the asymptotic theory of ideals* (London Mathematical Society Lecture Note Series 113, Cambridge University Press, Cambridge, 1989), 224 pp. 0 521 31127 6, £15.

For a thumbnail sketch of the basis of the classical theory underlying this book, consider the following. Given a commutative noetherian domain A with identity element, and an ideal I of A, the Rees ring $R=A[It,t^{-1}]$ is formed as a subring of the Laurent polynomial ring $A[t,t^{-1}]$. Thus R encodes information about each I^n, $n\geqq 1$. R can be viewed as an algebraic description of a blow-up, in the sense of algebraic geometry, in that we have moved from I, corresponding to a subvariety, to the principal ideal $t^{-1}R$, corresponding to a hypersurface. If, further, one normalises by forming the integral closure $\bar{R}$ of R, a Krull domain is obtained; so $\bar{R}$ comes equipped with its set V of essential valuations, which encode all important information about $\bar{R}$. The intersection of $\bar{R}$ with $A[t,t^{-1}]$ can be viewed as a type of generalized Rees ring, one which encodes information about the integral closure of each I^n, $n\geqq 1$. This process can also be considered in terms of the valuations comprising V. More recently, other types of generalised Rees rings have been investigated, often using the general language of filtrations, the filtration in the classical case being the set $\{I^n|n\geqq 1\}$.

The present set of lecture notes by D. Rees consists of a highly refined and polished account of filtrations, generalized Rees rings and related valuations, specializing after a while to the classical situation (essentially). Applications are made to the theory of analytically unramified rings and of quasi-unmixed rings, and to multiplicity theory. Great stress is given to associativity formulae for multiplicity and degree functions. Other highlights include an exposition of the classic theorems of Matijevic and Mori-Nagata, following ideas of Kiyek and Querré. The book closes with a discussion of the very intriguing theory of general extensions of local rings and of general elements, with applications to Teissier's mixed multiplicities.

There are many beautiful arguments and ideas in this book. As already mentioned, the exposition is highly polished (though, now and then, a little on the terse side). The material is an interesting blend of the classical and the modern. Undoubtedly, the book richly repays close study. My one cavil (leaving aside the occurrence of a few misprints) is that the presentation is almost too refined—a beginner or interested amateur would have to work hard before finding a foothold or perspective from which to view the subject matter of the book in the large. The contrast between this account and the one in the first part of the book *Equimultiplicity and blowing up*, by M. Herrmann, S. Ikeda and U. Orbanz (Springer-Verlag, 1988) is very interesting in an almost (one might say) meta-mathematical way.

L. O'CARROLL

DUDLEY, R. M. *Real analysis and probability* (Wadsworth and Brooks/Cole Mathematics Series, Pacific Grove, California, 1989), xii+436 pp. 0 534 10050 3, \$52.95.

This book is an introduction to probability theory including a substantial amount of real analysis and measure theory for the necessary background knowledge. It is a text book for students at beginning graduate level based on lectures given by the author, but a few parts also contain material for researchers in the field.

Chapters 1 to 5 provide a one-semester course in real analysis, in particular basic measure theory. Topics covered are elementary set theory, general topology, measure and integration, basic functional analysis, and L^p-spaces. Chapters 6 and 7 can be considered as supplements containing further material from functional analysis and measure theory on topological spaces, especially metric spaces.