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EXPECTED SNOW LOADS ON STRUCTURES FROM INCOMPLETE HYDROLOGICAL DATA

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Abstract. An assessment of snow loads in Switzerland was required for a revision of the building code. Settling curves of snow are used to compute water equivalents of snow if direct measurements are not available. Based on a frequency analysis, relations between the snow load and the altitude are given for various return periods. Problems of regional effects and of converting the snow-cover data to roof loads are outlined.

Résumé. *Estimation des surcharges dues à la neige sur les constructions à partir de données hydrologiques incomplètes.* Une étude de la pression de la neige en Suisse a été exigée en connexion avec une revision des normes du bâtiment. Les courbes de tassement de la neige sont utilisées pour calculer l'équivalent en eau en absence des mesures directes. Basées sur l'analyse fréquentielle, les relations entre la pression de la neige et l'altitude sont présentées pour les périodes de retour différentes. Les effets régionaux et la conversion des données sur la couche de neige en pression sur les toits sont traités.

Zusammenfassung. *Erwartungswerte für Schneedachlasten aus unvollständigen hydrologischen Daten.* Eine Auswertung der Schneelasten in der Schweiz wurde für eine Revision der Bauvorschriften verlangt. Setzungskurven der Schneedecke werden herangezogen, um den Schneewasserwert zu berechnen, falls direkte Messungen fehlen. Aufgrund einer Frequenzanalyse wurden Relationen zwischen der Schneelast und der Meereshöhe für verschiedene Wiederkehrperioden abgeleitet. Probleme der Regionalität und der Umwandlung von Schneedaten in Dachlasten werden behandelt.

Introduction

A snow load is given by the water equivalent of a snow-pack. Data from snow gauging networks are frequently insufficient for establishing the snow accumulation characteristics for different regions and altitudes. In particular in most countries the direct measurements of the water equivalent of snow are incomplete although their expediency and reliability is gradually improved by new instruments. On the other hand, daily observations of the total depth of snow cover are generally available and can be used for an indirect reconstruction of data.

Settling curves of snow

When the maximum load of a winter is reached, snow has a varying density depending on the preceding build-up of the snow-pack. At Weissfluhjoch, 2 540 m a.s.l., a density of 490 kg m^{-3} was measured at this point in the winter 1975 while in 1949 it was only 290 kg m^{-3} and in lower stations it can be less than 200 kg m^{-3}. These examples illustrate the errors to be expected if one converts the snow depth at the point of maximum accumulation into the water equivalent using a constant ratio.

The densification process is reflected in a gradual decrease of snow depth. A settling curve derived following Haefeli (Bader and others, 1939, p. 80–81)

$$H_{st} = H_{s0}\left[\left(1-\frac{\rho_s}{\rho_i}\right)\exp\left(-\beta t^{\alpha}\right)+\frac{\rho_s}{\rho_i}\right], \qquad (1)$$

where H_{s0} is the initial snow depth, H_{st} is the snow depth after the elapse of time t, ρ_s is the initial density of snow, ρ_i is the density of ice, α, β are parameters related to the temperature, vertical load and type of snow, leads logically to the final depth $H_{s0}(\rho_s/\rho_i)$ corresponding to ice. The settling curve can assume varying shapes which can be simulated by laboratory experiments and described by a more physical approach (de Quervain, 1945, 1946).

When there is an immediate practical need of estimating the settling of a natural snow cover, it is not possible to take into account the variation of temperature, wind velocity, load of superimposed snow layers and partly the intermittent melting or rain, especially if the pertinent data are not available. In order to meet the demands, a simplified settling equation with a time function can be derived from measurements of snow layers in conditions of a natural snow cover, such as

$$H_{sn} = H_{s0}(n+1)^{-0.3}, \quad (2)$$

where H_{s0} is the depth of new snow measured on the first morning after snowfall and H_{sn} is the depth of snow after n days.

The validity of such an equation is limited to snow conditions and to the time period in which it was derived. The theoretical merit of converging to a depth corresponding to ice after a very long time is not fulfilled (such extrapolation is anyhow questionable due to the increasing effect of evaporation and is of no practical interest for a seasonal snow cover). The aim is to obtain a good estimate of snow density attained at the point of the maximum snow accumulation (in terms of the water equivalent) of a winter.

Equation (2) was originally derived in the Krkonoše mountains in Central Europe (Martinec, 1956). In Figure 1 it is compared with an example of a settling curve obtained with regulated temperature conditions in a laboratory.

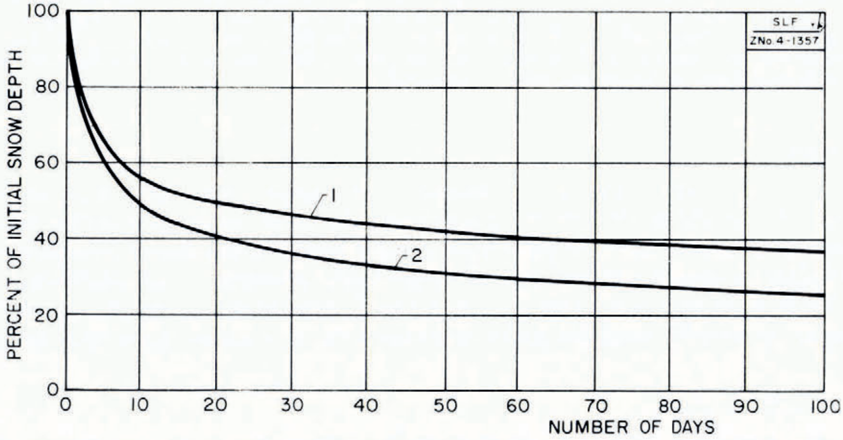

Fig. 1. Comparison of settling curves, 1. de Quervain (1945), $T = -2.5°C$, $\rho_0 = 115$ kg m^{-3}, 2. Martinec (1956), natural snow cover 1 410 m a.s.l.

Simulation of the water equivalent of snow

The mean density of snow is related to its water equivalent H_w and depth H_s by the equation

$$H_s = \frac{H_w \rho_w}{\rho},$$

where ρ_w is the density of water and, substituting in Equation (2),

$$\frac{H_{wn}}{\rho_n} = \frac{H_{w0}}{\rho_0}(n+1)^{-0.3}. \quad (3)$$

Neglecting the evaporation, H_{wn} equals H_{w0} and Equation (3) becomes

$$\rho_n = \rho_0(n+1)^{0.3}. \quad (4)$$

In order to calculate ρ_n and H_{wn} a reasonable value of ρ_0 is thus needed.

When an assessment of snow loads in Switzerland was requested for a revision of building standard specifications, Equation (2) was found also to approximate the average conditions in the Alps if $\rho_0 = 100 \text{ kg m}^{-3}$ was substituted equally as in previous applications. By a computerized procedure, the sequence of daily total depths was converted into daily water equivalents so that the maximum could be determined for each year. Figure 2 shows this simulation compared with direct measurements carried out twice a month in two Alpine stations.

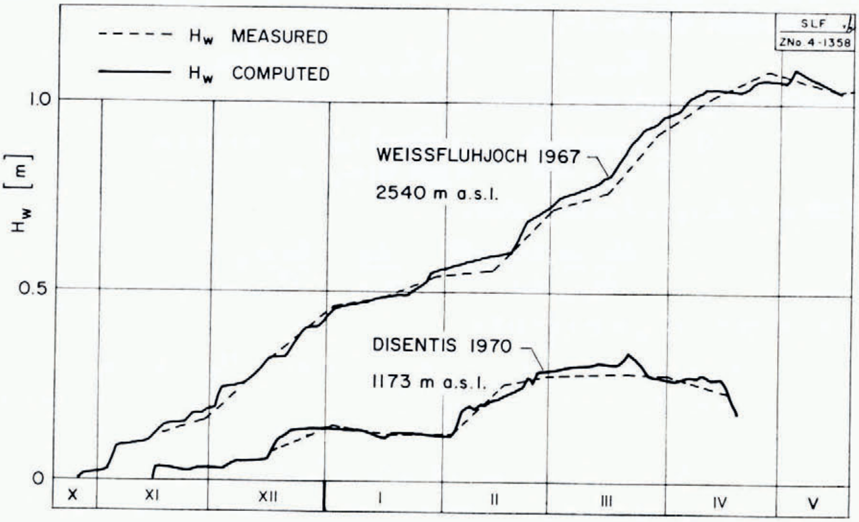

Fig. 2. Snow water-equivalents computed from snow depths compared with direct measurements.

An approximation which is satisfactory in different localities seems surprising, since the procedure depends on the substitution of an initial snow density which is variable. An effect of temperature was established by Diamond and Lowry (1953, p. 1–3) and the wind velocity as well as the type of the falling snow crystals are also involved. According to Equation (4) a range of density ρ_0 from 80 kg m^{-3} to 120 kg m^{-3} would cause an uncertainty of $\pm 20\%$ in computing ρ_n after n days. The actual scatter is far better probably for the following reason: Equation (2) indicates the initial rate of settling for the density of 100 kg m^{-3}. If the new snow has a greater density, it can be assumed that this stage, although caused by other effects, is equivalent to a certain time of deposition after which the rate of settling according to Equation (2) is already reduced. A snow density of 120 kg m^{-3} then corresponds to a fictitious time shift of 0.85 d and 80 kg m^{-3} to -0.5 d (backwards from the starting time of Equation (2)).

If the settling and density of snow are computed for example for $t = 60$ d the following deviations result:

$$
\begin{aligned}
H_{S0} &= 1 \text{ m}, \\
\rho_0 \text{ substituted as } &100 \text{ kg m}^{-3}. \\
H_{S\,60} &= 291.3 \text{ mm}, \\
\rho_{60} &= 343.2 \text{ kg m}^{-3}, \\
\text{actual } \rho_0 &= 120 \text{ kg m}^{-3}. \\
H_{S\,60} \rightarrow H_{S\,60.85} &= 290.1 \text{ mm}, \\
\rho_{60} \rightarrow \rho_{60.85} &= 344.7 \text{ kg m}^{-3}. \\
\text{actual } \rho_0 &= 80 \text{ kg m}^{-3}, \\
H_{S\,60} \rightarrow H_{S\,59.5} &= 292.1 \text{ mm}, \\
\rho_{60} \rightarrow \rho_{59.5} &= 342.4 \text{ kg m}^{-3}.
\end{aligned}
$$

Thus, when variations of the initial snow density ranging from 80 kg m^{-3} to 120 kg m^{-3} are ignored by assuming always $\rho_0 = 100$ kg m^{-3}, the resulting deviations after 60 d are within $\pm 0.5\%$. Recalling that the actual snow depths are available, $H_{s\ 60.85}$ is only used to determine $\rho_{60.85}$ (which can be also computed by Equation (4)). It is further understood that if the starting snow depth for a layer with $\rho_0 = 120$ kg m^{-3} is 1 m, H_s for $\rho = 100$ kg m^{-3} would have been 1.2 m. The actual snow depths are given by the settling curve starting with these values but taken up at the time $n = 0$, $\rho_0 = 120$ kg m^{-3}, $H_{s0} = 1$ m. Consequently the actual snow depth after 60 d is

$$H_{s\ 60}(\text{actual}) = 1.2 H_{s\ 60.85} = 0.348 \text{ m},$$

and the resulting water equivalent of the snow

$$H_{w\ 60}(\text{computed}) = (0.348 \text{ m})\rho_{60}/\rho_w = 0.119\,4 \text{ m},$$
$$H_{w\ 60}(\text{actual}) = (0.348 \text{ m})\rho_{60.85}/\rho_w = 0.12 \text{ m}.$$

The error for $\rho_0 = 80$ kg m^{-3} is by analogy

$$\frac{H_{w\ 60}(\text{computed})}{H_{w\ 60}(\text{actual})} = \frac{\rho_{60}}{\rho_{59.5}} = \frac{H_{s\ 59.5}}{H_{s\ 60}}.$$

In the given example, $H_{w\ 60}$ is computed as

$$1.2 H_{s\ 60.85}\rho_{60}/\rho_w = H_{s\ 60}\rho_{60}/\rho_w + (1.2 H_{s\ 60.85} - H_{s\ 60})\ \rho_{60}/\rho_w.$$

In a real case, the slower decrease of snow depths can also result from subsequent snowfalls. Therefore

$$H_{w\ 60} = H_{s\ 60}\rho_{60}/\rho_w + (1.2 H_{s\ 60.85} - H_{s\ 60})\ \rho_\tau/\rho_w,$$

where τ is a medium age of all new snow layers ($\tau < 60$ d). The diminution of error with time is due to the characteristic shape of settling curves.

Since it takes usually several weeks or months to reach the maximum snow accumulation of a winter, the simple Equation (2) can be applied to estimate the maximum water equivalent of snow although it is not able to take into account the variable starting conditions of each snow layer.

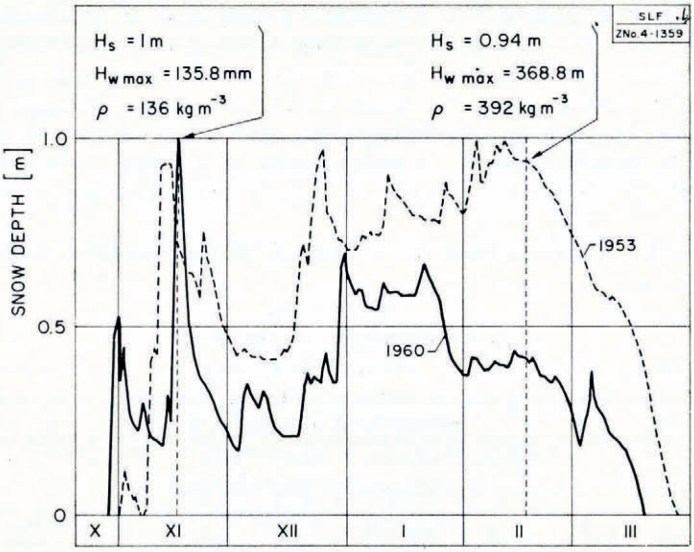

Fig. 3. Different patterns of snow accumulation, station Disentis, 1 173 m a.s.l.

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Figure 3 shows two different patterns of snow accumulation with the computed water equivalents and densities. In converting a sequence of daily snow depths to water equivalents, the decrease of snow depth was attributed to the increase of the snow density neglecting a possible small effect of evaporation. On the other hand, an intermittent melting of snow could have affected the results especially in lower stations. Therefore, whenever the snow depth decreased more than indicated by the settling equation, it was taken as a signal of snow melt and the corresponding amount of water was subtracted.

By using the snow depth instead of the elapsed time as an index of the snow density, as for example in the formula (Schriever and Otstavnov, 1968, p. 14)

$$\rho = 250+H_s+6T, \tag{5}$$

where ρ is in kg m^{-3}, H_s is in cm and T is the average temperature in °C for the three coldest months of the winter, results contradictory to the age of snow are obtained:

$\rho < 344$ kg m^{-3} for old snow in February 1953,
$\rho > 350$ kg m^{-3} for new snow in November 1959.

Frequency analysis of water equivalent of snow

The maximum water equivalent of each winter's snow was computed for 35 stations with daily observations of the total depth of snow cover only. Together with direct measurements, 76 stations ranging from 276 m a.s.l. to 2 540 m a.s.l. with a total of 1 482 years of record were available for assessing the expectancy of snow loads. The data agreed well with the Fisher–Tippett extremal distribution

$$P(X \leqslant x) = \exp[-\exp\{-(a+x)/c\}], \tag{6}$$

where P is the probability that an extreme value X is equal to or smaller than the variate x and a and c are parameters.

The parameters have been evaluated (Chow, 1964) as

$$a = \gamma c - \mu,$$

$$c = \frac{\sqrt{6}}{\pi}\sigma,$$

where $\gamma = 0.577\,21\ldots$ (Euler's constant), μ is the mean value and σ is the standard deviation.

In Figure 4, the maximum water equivalents of each winter's snow measured at Weissfluhjoch are plotted against their respective return periods T_r on the Gumbel–Powell probability paper designed for the Fisher–Tippett distribution, with (Weibull, 1939, p. 29)

$$T_r = \frac{N+1}{m}, \tag{7}$$

where N is the number of items, m is the order number of items arranged in descending magnitude, and

$$y = -\ln[-\ln\{(T_r-1)/T_r\}], \tag{8}$$

where $T_r = 1/(1-P)$.

The relation between the average return period and the expected yearly maximum of the water equivalent of snow at Weissfluhjoch was evaluated by the least-squares method as

$$H_w = (214.5 \text{ mm})\,(-\ln[-\ln\{(T_r-1)/T_r\}])+721.8 \text{ mm}. \tag{9}$$

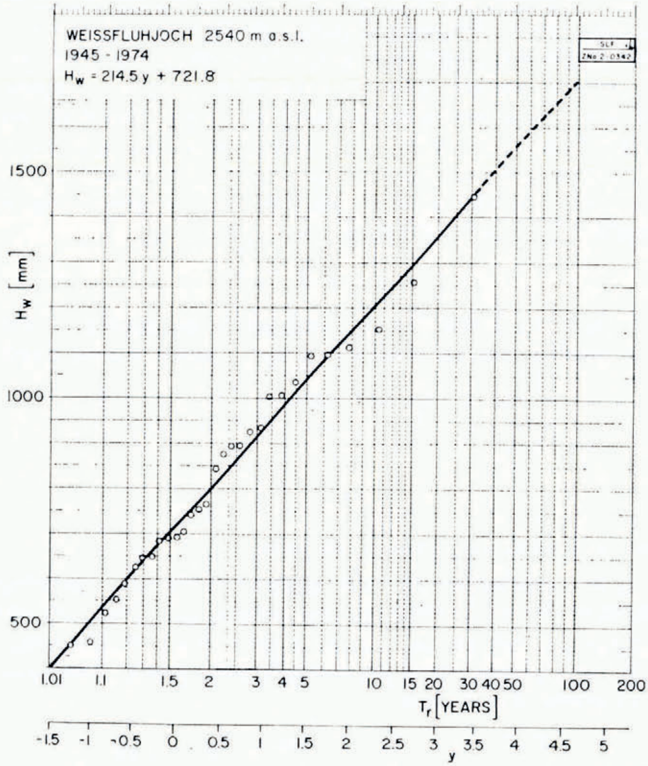

Fig. 4. Maximum yearly snow water-equivalents at Weissfluhjoch, 2 540 m a.s.l., related to return periods.

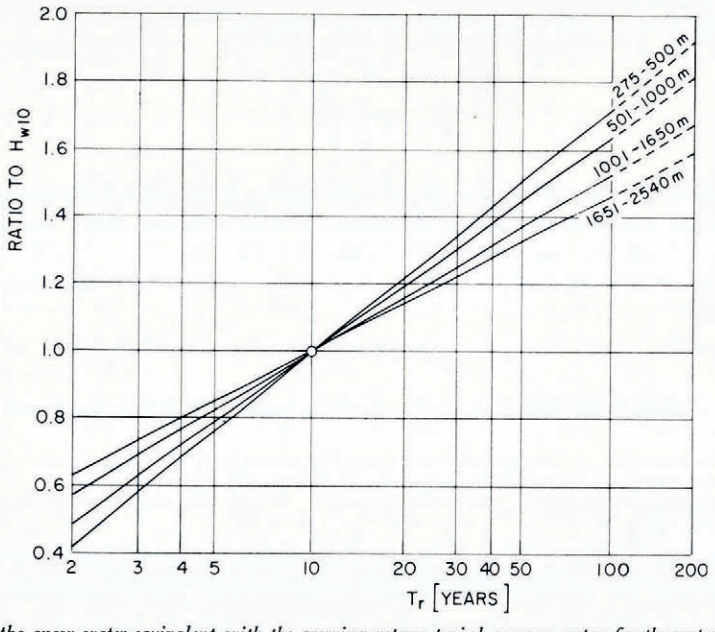

Fig. 5. Increase of the snow water-equivalent with the growing return period, average rates for the respective altitude ranges.

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Similar equations have been determined for all other stations. Exceptionally, the highest points departed upwards from the general path. This was considered as a sign of a recurrence interval greater than the length of record and such data were excluded from the fitting.

The satisfactory performance of the Fisher–Tippett distribution in this study does not prove its general validity for the frequency analysis of the water equivalent of snow. The Log-normal and the Log-Pearson Type 3 distributions, for example, have been found suitable in other cases (U.S. Weather Bureau, 1964; McAndrew, 1973).

As shown in Figure 5, the relative increase of the expected water equivalent with increasing return period generally decreases with the altitude. The frequency analysis enables the water equivalents to be determined for any desired return period within the limits of a reasonable extrapolation so that values of a comparable probability of occurrence become available for further evaluation.

Snow accumulation related to the altitude

In mountainous Switzerland the altitude is the dominant factor in snow accumulation. Figure 6 shows polynomial regressions

$$H_w = A_0 + A_1 h + A_2 h^2 + \ldots + A_m h^m, \tag{10}$$

where H_w is the water equivalent in mm for $T_r = 10$ years and h is the elevation in m a.s.l., of which the equation

$$H_w = 0.000\,368\,8h^2 - 0.278h + 153.4, \tag{11}$$

was found satisfactory for practical purposes. The curves in Figure 7 have been obtained for return periods ranging from 2 years to 100 years. All available data from 76 stations have been used. Curves in Figure 8 are based on selected stations with the biggest snow accumulation in the respective elevation zones.

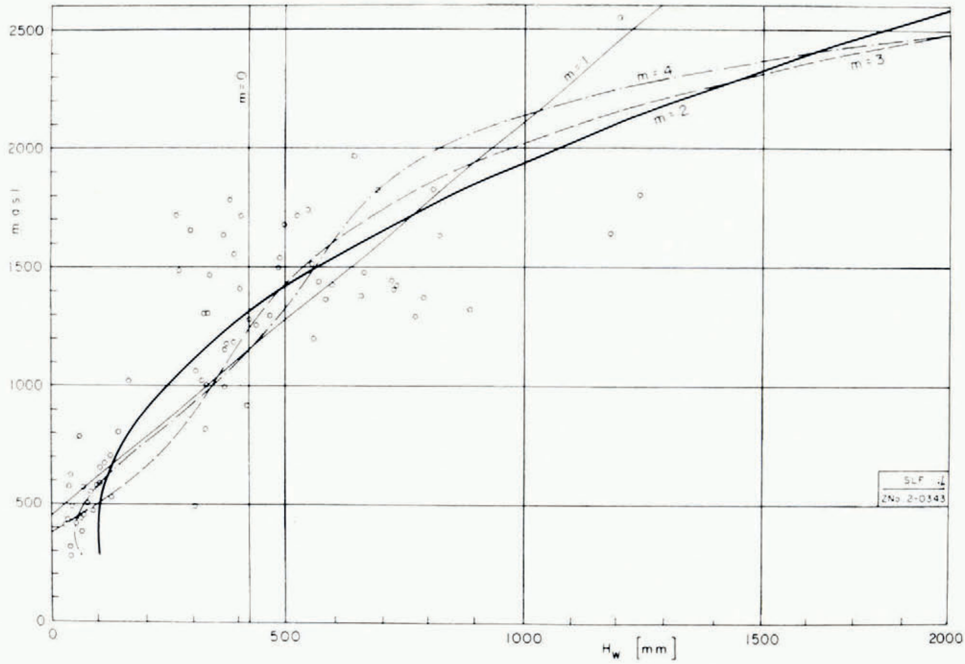

Fig. 6. Snow accumulation in Switzerland related to the altitude.

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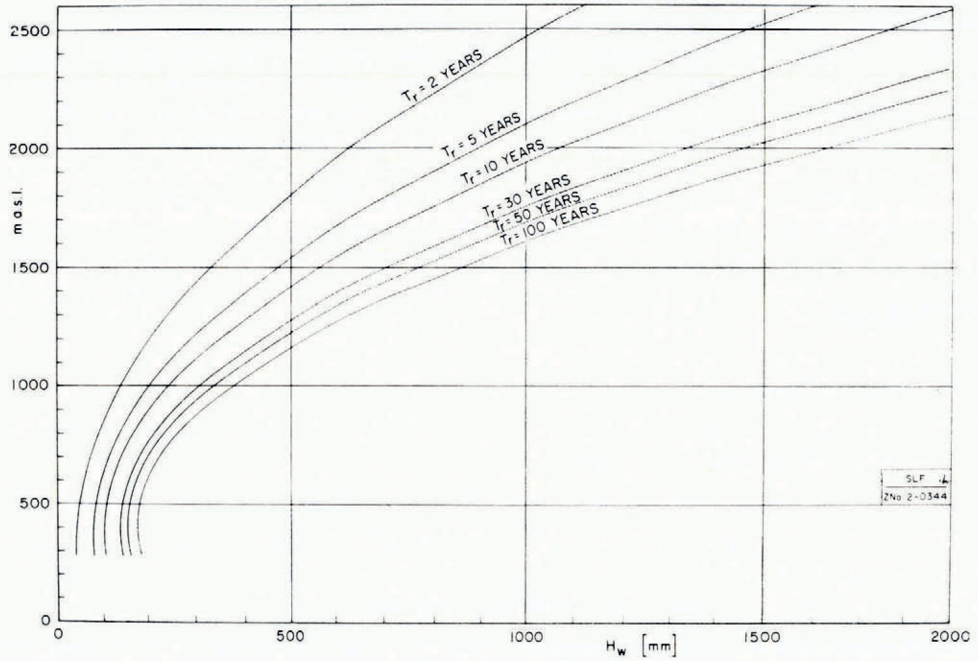

Fig. 7. Average relations between the maximum yearly snow water-equivalent and the altitude for various return periods.

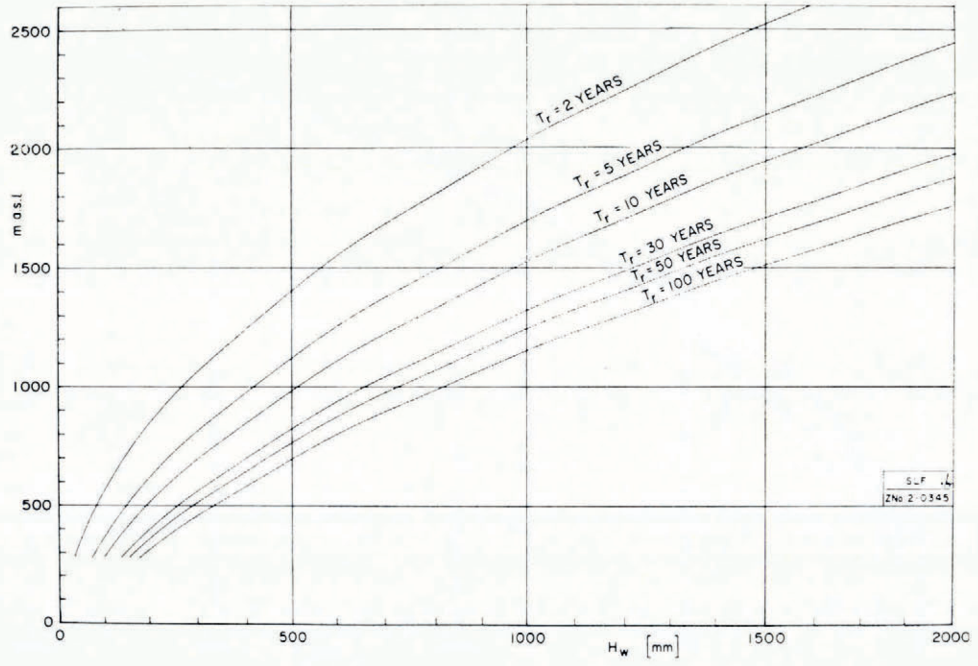

Fig. 8. Extreme curves of the maximum yearly snow water-equivalent related to the altitude for various return periods.

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With regard to the irregular distribution of rare events, the risk of a value characterized by a design return period T_r to be exceeded in a given period N (the desired lifetime of a structure) is obtained as

$$\sum_{n=1}^{N} p_{n,N} = 1-(1-1/T_r)^N, \tag{12}$$

where N is the number of years.

Probabilities for various alternatives are given in Figure 9. For $T_r = 10$ years and $N = 10$ years the risk is 65%. A value characterized by $T_r = 100$ years is needed to reduce this risk to 10%.

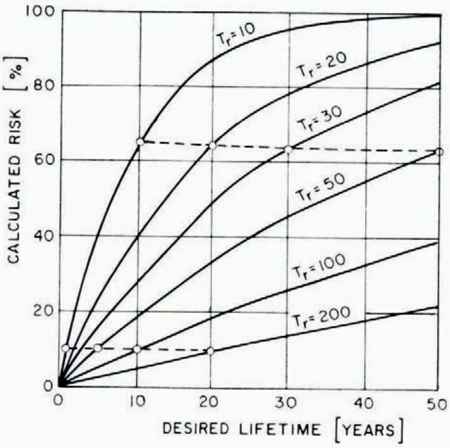

Fig. 9. Calculated-risk nomograph. (T_r in years.)

The scatter of points in Figure 6 reflects local and regional deviations in the accumulation of snow. According to an earlier study (Zingg, 1968), the regional effect in Switzerland is less important than that of the altitude but it can become a major factor in vast areas with a changing climate influenced for example by the Great Lakes in North America (U.S. Weather Bureau, 1964) or by the distance from the sea (Schaerer, 1971).

Conversion of snow-cover data to snow loads

By determining snow water-equivalents to be expected on the ground, the problem of snow loads on roofs is not yet solved. Factors involved in the different accumulation of snow on roofs have been summarized (Tobiasson and Redfield, unpublished, p. 15) as follows:

$$p_r = c_r c_e c_t p_g, \tag{13}$$

where p_r is the roof snow load, p_g is the ground snow load, c_r is the regional ground-to-roof conversion factor, c_e is the exposure of the structure, and c_t is the thermal characteristic of the roof.

A variety of effects can be considered, for example (Martinec, 1975)

$$p_r = (c_w c_f H_w + B - M + R + A)\, g, \tag{14}$$

where p_r is the roof snow load [in Pa = N m^{-2}], H_w is the snow water-equivalent on the ground [in mm of water = kg m^{-2}], c_w is the effect of wind, c_f is a coefficient of the roof form, B is the snow melt from the ground [in mm of water = kg m^{-2}], M is the snow melt by the heat transfer from the building [mm of water = kg m^{-2}], R is the incident rainwater [in mm of water = kg m^{-2}], A is the load from sliding snow or avalanche [in mm of water = kg m^{-2}], and $g = 9.81$ m s^{-2}.

The aerodynamic effects have been found very variable (Schriever and others, 1970). At the same time, the snow water-equivalent on the ground might have been underestimated by assuming a snow density of 192 kg m^{-3}. It is difficult to reconcile maximum snow loads on the ground in British Columbia corresponding to 600 mm water-equivalent (Peter and others, 1963) with ground load–altitude relations (Schaerer, 1971) or with snow surveys (*Snow Survey Bulletin*, 1972) whereby snow water-equivalents exceeding 2 000 mm or even 3 000 mm and snow densities around 500 kg m^{-3} have been measured.

The uncertainty in the ground-to-roof conversion is reflected in Building Codes valid in various countries (Schriever and Otstavnov, 1968, p. 14). In most cases the altitude serves as an index of snow load. Countries like Italy understandably emphasize local climatic conditions. The form factor is accounted for by the slope of the roof resulting in considerable reductions of the snow load.

Conclusion

In order to improve data for the structural design in Switzerland, the snow water-equivalent was indirectly determined in the absence of measurements and a frequency analysis was carried out to ensure a comparability of values and a choice of design return periods.

Foreign studies demonstrated great differences between the roof load and ground load depending on specific conditions. These effects, together with regional anomalies of the snow accumulation, have to be considered as further steps leading to more realistic indications of roof loads, especially in high areas and exposed localities.

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DISCUSSION

L. Lliboutry: Did you find that precipitation continues to increase with altitude? We found in the Western Alps that it goes through a maximum which depends upon the altitude of the surrounding summits and then declines with further increase in altitude.

J. Martinec: The highest station included in the relation between altitude and water equivalent of snow was 2 540 m a.s.l. For the practical use of this relation, it should be limited to 2 000 m a.s.l. I agree that at the highest altitudes the increase in precipitation with altitude can cease or even be reversed.