of choice, are connected with papers of Fraenkel on the same subject (2692, 26927). According to the author, his proofs contain the principal idea of Fraenkel, "but the way of carrying out this idea, which in Fraenkel's papers gave rise to serious doubts, has been entirely altered." The work dealing with this last question was done in collaboration with Lindenbaum, and a number of remarks on the same topic are contained in a paper of Lindenbaum and Mostowski, *Über die Unabhängigkeit des Auswahlaxioms und einiger seiner Folgerungen*, shortly forthcoming in the ***Comptes rendus des séances de la Société des Sciences et des Lettres de Varsovie.*** ALFRED TARSKI

ALFRED ERRERA. *Sur les démonstrations de non-contradiction.* ***Travaux du IX^e Congrès International de Philosophie,*** VI ***Logique et mathématiques,*** Actualités scientifiques et industrielles 535, Hermann et C^{ie}, Paris 1937, pp. 121–127.

These remarks concern in the main a method for establishing the consistency of a set of axioms, the underlying logic being assumed to be both consistent and sufficiently strong. Unfortunately the author proves no actual set to be consistent by the method in question, and the single example referred to concerns the relation of Euclidean to Lobachevskian geometry. Let p be Euclid's axioms without the axiom of parallels, let q be that axiom, and let the entire set be so chosen that the Lobachevskian parallel axiom can be written as $\sim q$. We require two rules: one such that, given an interpretation of $p.q$, we can specify within it an interpretation satisfying $p.\sim q$, and another such that, given an interpretation satisfying $p.\sim q$, we can specify within it an interpretation of $p.q$. If both rules can be found, then the two sets of axioms are equivalent with respect to consistency, and, moreover, from the consistency of p follows that of both sets. If a set of axioms $p_1, p_2, \cdots, p_n$ can be so arranged that each successive axiom is independent of the preceding axioms in this way, then given the consistency of p_1, that of the entire set is assured. (In one place Errera speaks as if the consistency of p_1 itself could be proved by his method and in another as if it could not.)

In order to establish the consistency of $p.q$ on the hypothesis that p is consistent, it is not necessary, as Errera seems to say it is, that we have two rules; indeed, it is neither necessary nor sufficient. If r implies $\sim q$ but is a stronger condition, and if we can represent $p.q$ in any interpretation of $p.r$ and conversely, then the two sets are equivalent with respect to consistency; but the consistency of neither set follows from that of p. If, however, r is identical with $\sim q$, then one rule, which provides a representation of $p.q$ within any system satisfying $p.r$, is enough.

However desirable it may be to have a method which enables us on occasion to show that if a set of assumptions is consistent, the addition of a certain further assumption will not introduce inconsistency, Errera's procedure seems to be subject to serious limitations. Let $p_1, p_2, \cdots, p_n$ be a set of axioms arranged in the manner required. In general, p_1 will admit of a finite representation; but in many important cases there comes a point at which the axioms are no longer finitely representable. Let q be the axiom whose introduction makes necessary an infinite representation, and let p be the preceding axioms. Then $p.q$ cannot be finitely represented, whereas $p.\sim q$ can be, since any finite system satisfying p must satisfy $\sim q$. And if we are to show that the consistency of $p.q$ follows from that of p, we must of course show that *every* system satisfying $p.\sim q$ provides a representation of $p.q$. C. H. LANGFORD

JACQUES PICARD. *Les normes formelles du raisonnement déductif.* ***Revue de métaphysique et de morale,*** vol. 45 (1938), pp. 213-254.

This paper attempts to classify ordinary deductive arguments on the basis of the formal logical principles involved in them. E.g., subsumptive syllogisms and relational arguments are shown to be special cases of the principle of relative product: $xRy.ySz\,.\supset.\,x\,R|S\,z$. A similar reduction and generalization of immediate inferences and of the opposition of propositions to, and in terms of, fundamental logical relations is attempted. The author explains the principles of simplification, composition, substitution, abstraction,and mathematical induction, indicating in regard to the latter Poincaré's objections and Russell's