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ON THE ESCAPING SET OF MEROMORPHIC FUNCTIONS WITH DIRECT TRACTS

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Abstract

Let f be a transcendental meromorphic function with at least one direct tract. In this note, we investigate the structure of the escaping set which is in the same direct tract. We also give a theorem about the slow escaping set.

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1. Introduction

There are many results on the dynamical system of transcendental entire functions (see, for example, [12]). The structures of the Fatou set and the Julia set are the main focus. Some of the results about entire functions have been carried over to meromorphic functions with finitely many poles. The dynamical systems for meromorphic functions with finitely many poles and entire functions have some similarities (see [6–10, 15–17]). Recently, Bergweiler *et al.* [2] discussed the more general class of meromorphic functions with direct tracts. This note will continue their work.

The paper consists of five sections. In Section 2, we collect together a number of results that will be used later and establish some notation and definitions. Section 3 provides the main results of this paper. In Section 4, the proofs are given. In Section 5, we give a result which is a generalisation of a result of Rippon and Stallard [10].

2. Wiman–Valiron–BRS theory

Based on the analysis of the definition of direct tracts, Bergweiler *et al.* [2] used subharmonic function theory to extend the Wiman–Valiron theory from its original

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setting of entire functions. (We call it the Wiman–Valiron–BRS theory.) They used this general approach in their discussion of complex dynamical systems.

In order to introduce this theory, we begin with the following definition.

Definition 2.1. Let D be an unbounded domain in $\mathbb{C}$ whose boundary consists of piecewise smooth curves. Suppose that the complement of D is unbounded. Let f be a complex-valued function whose domain of definition contains the closure $\overline{D}$ of D. Then D is called a direct tract of f if f is holomorphic in D and continuous in $\overline{D}$ and if there exists $R > 0$ such that $|f(z)| = R$ for $z \in \partial D$, the boundary of D, while $|f(z)| > R$ for $z \in D$. If, in addition, the restriction $f : D \to \{z \in \mathbb{C} : |z| > R\}$ is a universal covering, then D is called a logarithmic tract of f.

There are two conditions which can be used to determine the existence of direct tracts and logarithmic tracts.

Proposition 2.2 [14]. *Let f be a transcendental meromorphic function in the class B and suppose that there exists an $N \in \mathbb{N}$ such that the poles of f have multiplicity at most N. If $\delta(\infty, f) > 0$ or, more generally, if $m(r, f)$ is unbounded, then f has a logarithmic singularity over infinity.*

Proposition 2.3 [13]. *Let f be a transcendental meromorphic function. Suppose that there exist $R > 0$ and $N \in \mathbb{N}$ such that for each component U of $f^{-1}(\widehat{\mathbb{C}} \setminus \overline{D}(0, R))$ which contains a pole, the map $f : U \to \widehat{\mathbb{C}} \setminus \overline{D}(0, R)$ is a proper map of degree at most N. If the deficiency $\delta(\infty, f) > 0$ or, more generally, if $m(r, f)/\log r$ is unbounded, then f has a direct singularity over infinity.*

Remark 2.4. The deficiency, $\delta(a, f)$, was introduced by Nevanlinna in the value distribution theory of meromorphic functions: for $a \in \widehat{\mathbb{C}}$, define

$$\delta(a, f) = \liminf_{r\to\infty} \frac{m(r, 1/(f-a))}{T(r, f)} = 1 - \limsup_{r\to\infty} \frac{N(r, 1/(f-a))}{T(r, f)}.$$

Remark 2.5. Bergweiler *et al.* [2, Section 8] give many examples of meromorphic functions with direct tracts and logarithmic tracts.

Note that if f, D, R are as in the above definition, then the function $v : \mathbb{C} \to [0, \infty)$ defined by

$$v(z) = \begin{cases} \log \dfrac{|f(z)|}{R} & \text{if } z \in D, \\ 0 & \text{if } z \notin D \end{cases}$$

is subharmonic. Then

$$B(r, v) = \max_{|z|=r} v(z)$$

is increasing, convex in $\log r$ and tends to ∞ as r tends to ∞ (see [5, Section 2]). Hence,

$$a(r, v) = \frac{dB(r, v)}{d \log r} = rB'(r, v)$$

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exists except, perhaps, for a countable set of r-values and $a(r, v)$ is nondecreasing. We also put

$$M_D(r) = \max_{|z|=r, z\in D} |f(z)| = \exp B(r, v)$$

and write $M_D^n(r)$ to denote the nth iteration of $M_D(r)$ with respect to the variable r. Since $B(r, v) \to \infty$, we see that $M_D(\rho) > \rho$ for sufficiently large $\rho > R$. Thus, for $r > R$, $M_D^n(r) \to \infty$ for $n \to \infty$. For these r, define

$$A(f, D, r) = \{z \in D : f^n(z) \in D \ \forall n \in \mathbb{N} \text{ and } |f^n(z)| \geq M_D^n(r)\}. \tag{2.1}$$

We next introduce the key result of the Wiman–Valiron–BRS theory.

Theorem 2.6 [2]. *Let D be a direct tract of f and let $\tau > \frac{1}{2}$. Let $v(z)$ be defined as before and let z_r be a point satisfying $|z_r| = r$ and $v(z_r) = B(r, v)$. Then there exists a set $F \subset [1, +\infty)$ of finite logarithmic measure such that if $r \in [1, +\infty)\backslash F$, then $D(z_r, r/a(r, v)^\tau) \subset D$,*

$$f(z) \sim \left(\frac{z}{z_r}\right)^{a(r,v)} f(z_r), \quad z \in D\left(z_r, \frac{r}{a(r, v)^\tau}\right)$$

and

$$|f(z)| \sim M_D(|z|), \quad z \in D\left(z_r, \frac{r}{a(r, v)^\tau}\right)$$

as $r \to \infty$, $r \notin F$ and, if $k \in \mathbb{N}$, then

$$f^{(k)}(z) \sim \left(\frac{a(r, v)}{z}\right)^k \left(\frac{z}{z_r}\right)^{a(r,v)} f(z_r), \quad z \in D\left(z_r, \frac{r}{a(r, v)^\tau}\right).$$

Remark 2.7. We call $F \subset [1, \infty)$ a set of finite logarithmic measure if $\int_F dt/t < \infty$.

Remark 2.8. Recently, Bergweiler [1] estimated the radius in the above Wiman–Valiron–BRS-type theory.

Theorem 2.9 [2]. *For each $\beta > 1$, there exists $\alpha > 0$ such that if f, D, v, z_r and F are as in Theorem 2.6 and if $r \notin F$ is sufficiently large, then*

$$\left\{z \in \mathbb{C} : \frac{|f(z_r)|}{\beta} \leq |z| \leq \beta|f(z_r)|\right\} \subset f\left(D\left(z_r, \frac{\alpha r}{a(r, v)}\right)\right).$$

Remark 2.10. Bergweiler *et al.* [2] used the above two theorems to prove the existence of escaping points of meromorphic functions with direct tracts. Eremenko [4] earlier proved the existence of escaping points for entire functions by using Wiman–Valiron theory. The corresponding results for meromorphic functions with poles are due to Domínguez [3], who used the Ahlfors five islands theorem.

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3. Statement of the theorems

The importance of the escaping set $I(f)$, which was first studied by Eremenko [4] in transcendental dynamics, has increased significantly in recent years (see, for example, [1, 2, 6–10, 12]).

In order to describe the escaping speed, we introduce the following definitions (see [10]).

DEFINITION 3.1. Let f be a transcendental meromorphic function. The escaping speed is classified into various bands as follows:

(1) fast escaping set

$$Z(f) = \left\{ z \in I(f) : \limsup_{n\to\infty} \frac{1}{n} \log\log |f^n(z)| = +\infty \right\};$$

(2) slow escaping set

$$L(f) = \left\{ z \in I(f) : \limsup_{n\to\infty} \frac{1}{n} \log |f^n(z)| < +\infty \right\};$$

(3) moderately slow escaping set

$$M(f) = \left\{ z \in I(f) : \limsup_{n\to\infty} \frac{1}{n} \log\log |f^n(z)| < +\infty \right\};$$

(4) escaping set with special speed: for a positive sequence $a = (a_n)$ such that $a_n \to \infty$ as $n \to \infty$, we define

$$I^a(f) = \{ z \in I(f) : |f^n(z)| = O(a_n), n \to \infty \}.$$

We have the following result.

$$L(f) \subset M(f) \subset I(f) \setminus Z(f).$$

REMARK 3.2. Rippon and Stallard [10] proved that the sets of type (2)–(4) are nonempty and investigated in detail the structures of these sets. Rippon and Stallard [11] studied the fast escaping set of entire functions.

A natural question to ask is whether we can investigate the escaping speed for functions with at least one direct tract. Bergweiler *et al.* [2] considered this problem and Rippon and Stallard [10, 11] did further work.

We introduce the level sets of the fast escaping set, $A(f, D)$, corresponding to the direct tract D.

DEFINITION 3.3. Let f be a transcendental function meromorphic in the plane which has a direct tract D. Given R as in (2.1), define

$$\begin{aligned} A(f, D) &= \{ z \in \mathbb{C} : \exists K \in \mathbb{N} \text{ such that } f^K(z) \in A(f, D, R) \} \\ &= \{ z \in \mathbb{C} : \exists K \in \mathbb{N} \text{ such that } f^{n+K}(z) \in D \ \forall n \in \mathbb{N} \\ &\qquad \text{and } |f^{n+K}(z)| \geq M_D^n(R) \}. \end{aligned}$$

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Definition 3.4. Let $f(z)$ be a transcendental meromorphic function with a direct tract D. Let $L \in \mathbb{Z}$, $R > 0$ be such that $M_D(r) > r$ for $r \geq R$. The Lth level of $A(f, D)$ (with respect to R) is the set

$$A_R^L(f, D) = \{z : f^n(z) \in D,\ |f^n(z)| \geq M_D^{n+L}(R), n \in \mathbb{N}, n + L \geq 0\}.$$

Put

$$A_R(f, D) = A_R^0(f, D) = \{z : f^n(z) \in D,\ |f^n(z)| \geq M_D^n(R), n \in \mathbb{N}\}.$$

For any $n \geq 0$, $M_D^{n+1}(R) > M_D^n(R)$ and, for all $L \in \mathbb{Z}$, $A_R^L(f, D) \subset A_R^{L-1}(f, D)$. Hence,

$$A_R^{-L}(f, D) \subset A_R^{-(L+1)}(f, D), \quad L \in \mathbb{N}.$$

Note that the n in the definition should satisfy $n \geq L$ when $z \in A_R^{-L}(f, D)$.

We will use the idea of the level sets introduced by Rippon and Stallard [11] to handle the fast escaping set $A(f, D)$ for functions with direct tracts. We now state the main theorems of this paper.

Theorem 3.5. *Let $f(z)$ be a transcendental meromorphic function with a direct tract D and let $R > 0$ be such that $M_D(r) > r$ when $r \geq R$. Then $A(f, D) = \bigcup_{L \in \mathbb{N}} A_R^{-L}(f, D)$ and:*

(a) *$A(f, D)$ is completely invariant under f;*
(b) *$A(f, D)$ is independent of R;*
(c) *$A(f, D) \subset Z(f) = \{z \in I(f) : (1/n) \log \log |f^n(z)| \to \infty$ as $n \to \infty\}$.*

Remark 3.6. Parts (a) and (c) in Theorem 3.5 are due to Bergweiler *et al.* [2]. We give another proof here.

Theorem 3.7. *Let $f(z)$ be a transcendental meromorphic function with a direct tract D and let $R > 0$ be such that $M_D(r) > r$ for $r \geq R$. Then, for each $L \in \mathbb{Z}$, each component of $A_R^L(f, D)$ is closed and unbounded; in particular, each component of $A(f, D)$ is unbounded.*

Remark 3.8. Bergweiler *et al.* [2] proved that every component of $A_R^L(f, D)$ is an unbounded compact set.

Theorem 3.9. *Let $f(z)$ be a transcendental meromorphic function with a direct tract D. For $\varepsilon \in (0, 1)$ and $r > 0$, set $h(r) = \varepsilon M_D(r)$. Then*

$$A(f, D) = \{z : \exists\, L \in \mathbb{N} \textit{ such that for each } n \in \mathbb{N},\ \textit{we have } |f^{n+L}(z)| \geq h^n(R)\},$$

where $R > 0$ is sufficiently large so that $h(r) > r$ for $r \geq R$.

4. The proofs of Theorems 3.5–3.9

LEMMA 4.1. *If* $|z| < M_D^m(r)$ *for some* $m \in \mathbb{N}$, *then*

$$|f^n(z)| < M_D^{n+m}(r) \quad \text{for } n \in \mathbb{N} \quad \text{and} \quad \frac{\log M_D(r)}{\log r} \to \infty \quad \text{as } r \to \infty. \tag{4.1}$$

REMARK 4.2. We prove this in the same way as the similar result in [11].

PROOF OF THEOREM 3.5. It follows from (4.1) that, if $k > 1$, then

$$\frac{M_D(kr)}{M_D(r)} \to \infty \quad \text{as } r \to \infty. \tag{4.2}$$

Now let $R > 0$ be such that $M_D(r, f) > r$ for $r \geq R$. We have

$$M_D^n(R) \to \infty \quad \text{as } n \to \infty. \tag{4.3}$$

By Definition 3.4,

$$A_R^L(f, D) \subset \{z : |z| \geq M_D^L(R)\} \quad \text{for } L \geq 0; \tag{4.4}$$

$$f(A_R^L(f, D)) \subset A_R^{L+1}(f, D) \subset A_R^L(f, D) \quad \text{for } L \in \mathbb{Z}. \tag{4.5}$$

(a) The complete invariance of $A(f, D)$ under f follows directly from (4.4) and (4.5).

(b) Take $R' > R$. Clearly, $A_{R'}^L(f, D) \subset A_R^L(f, D)$ for $L \in \mathbb{Z}$ and so

$$\bigcup_{L \in \mathbb{Z}} A_{R'}^{-L}(f, D) \subset \bigcup_{L \in \mathbb{Z}} A_R^{-L}(f, D).$$

Now note that, by (4.3), there exists $m \in \mathbb{N}$ such that $M_D^m(R) > R'$ and so

$$\bigcup_{L \in \mathbb{Z}} A_{R'}^{-L}(f, D) \supset \bigcup_{L \in \mathbb{Z}} A_{M_D^m(R)}^{-L}(f, D) = \bigcup_{L \in \mathbb{Z}} A_R^{m-L}(f, D) \supset \bigcup_{L \in \mathbb{Z}} A_R^{-L}(f, D).$$

Together with (4.4), these relations show that $A(f, D)$ is independent of R.

(c) From (4.1) and (4.3), $\log M_D^{n+1}(R) / \log M_D^n(R) \to \infty$ as $n \to \infty$. Thus, for each $C > e$, there exists $N \in \mathbb{N}$ such that $\log M_D^{n+1}(R) > C \log M_D^n(R)$ for all $n \geq N$ and $\log M_D^N(R) \geq 1$. So, for $n > 2(N + L)$ and $L \in \mathbb{N}$,

$$\log M_D^{n-L}(R) > C^{n/2}$$

and hence, if $z \in A_R^{-L}(f, D)$, then

$$\frac{1}{n} \log \log |f^n(z)| \geq \frac{1}{n} \log \log M_D^{n-L}(R) > \frac{1}{2} \log C.$$

This gives (c), since we can choose C to be arbitrarily large. □

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Proof of Theorem 3.7. Let $z_0 \in A_R^L(f, D)$ for some $L \in \mathbb{Z}$. Then, for all $n \in \mathbb{N}$ with $n + L \geq 0$,

$$f^n(z_0) \in \{z : |z| \geq M_D^{n+L}(R)\} = E_n.$$

Now let L_n denote the component of $f^{-n}(E_n)$ that contains z_0. Since f^n is analytic, L_n is closed and also unbounded. Furthermore,

$$L_{n+1} \subset L_n \quad \text{for } n \in \mathbb{N}, n + L \geq 0.$$

In fact, $f^{n+1}(z) \in E_{n+1}$ implies that $f^n(z) \in E_n$, so $L_{n+1} \subset f^{-n}(E_n)$. Then

$$K = \bigcap_{n \in \mathbb{N}, n+L \geq 0} (L_n \cup \{\infty\})$$

is a closed connected subset of $\widehat{\mathbb{C}}$ which contains z_0 and ∞. Now let Γ be the component of $K \backslash \{\infty\}$ which contains z_0. Then Γ is closed in $\mathbb{C}$ and unbounded. Finally, we note that $f^n(z) \in E_n$. Indeed, if $z \in \Gamma$, then

$$|f^n(z)| \geq M_D^{n+L}(R) \quad \text{for } n \in \mathbb{N}, n + L \geq 0.$$

This completes the proof. □

Proof of Theorem 3.9. By assumption,

$$h^n(r) \to \infty \quad \text{as } n \to \infty \text{ for } r > R. \tag{4.6}$$

Moreover, by (4.2), there exists $R' \geq R$ such that for $r \geq R'$,

$$h(r) = \varepsilon M_D(r) \geq \frac{1}{\varepsilon} M_D(\varepsilon r) \geq r. \tag{4.7}$$

From (4.6),

$$h^n(r) \geq M_D^n(\varepsilon r) \quad \text{for } n \in \mathbb{N}.$$

Also, (4.6) implies that there exists $M \in \mathbb{N}$ such that $h^M(r) \geq R'/\varepsilon$ and, by (4.7),

$$h^{n+M}(R) \geq h^n\Big(\frac{R'}{\varepsilon}\Big) \geq M_D^n(R') \quad \text{for } n \in \mathbb{N}.$$

If there exists $L \in \mathbb{N}$ such that $|f^{n+L}(z)| \geq h^n(R)$ for $n \in \mathbb{N}$, then

$$|f^{n+M+L}(z)| \geq h^{n+M}(R) \geq M_D^n(R') \geq M_D^n(R) \quad \text{for } n \in \mathbb{N}.$$

Thus, $z \in A(f, D)$. □

5. The slow escaping set

In this section, we will prove the following result to illustrate the slow escaping speed.

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Theorem 5.1. *Let*

$$f(z) = kz + k + (1 - k)\log k - e^z,$$

where $k > 1$ is a constant. Then f has an invariant Baker domain U such that $\overline{U}\backslash\{z_0\} \subset L(f)$, where $z_0 \in \partial U$ is a fixed point of f, and a bounded wandering domain V such that $\overline{V} \subset L(f)$.

Proof. By a similar argument as in [10], we find the following properties of f:

(1) f has an invariant Baker domain U contained in $\{z : \operatorname{Re} z < 0\}$ such that the map $f : U \to U$ is univalent and ∂U is a Jordan curve through ∞;
(2) f has a bounded Fatou component V_0 containing the super-attracting fixed point $\log k$;
(3) f has bounded Fatou components of the form $V_l = \{z + 2\pi li : z \in V_0\}$ for $l \in \mathbb{Z}$ such that $f(V_l) = V_{l+1}$ for $l \in \mathbb{Z}$.

In particular, $V = V_1$ is a bounded wandering domain and $\overline{V} \subset L(f)$. Moreover, ∂U meets the real axis at a repelling fixed point z_0 of f. Since

$$\left(\frac{k+1}{2}\right)^n |z| \le |f^n(z)| \le (k+1)^n|z| \quad \text{for } z \in \overline{U} \cap \{z : |z| \ge k[(k+1) + \log k]\},$$

it follows that

$$\overline{U} \cap \{z : |z| \ge k[(k+1) + \log k]\} \subset L(f).$$

Since f is univalent on U, U is conjugate, via a Riemann map, to a Möbius transformation of the unit disc onto itself. Since ∂U is a Jordan curve, the Riemann map extends to a homeomorphism on the closed unit disc, so the conjugate Möbius transformation fixes two boundary points, one repelling and one attracting. The latter attracts all points of $\mathbb{C}$ except the repelling fixed point. It follows that $\overline{U}\backslash\{z_0\} \subset I(f)$. Hence, $\overline{U}\backslash\{z_0\} \subset L(f)$. □

Remark 5.2. The case $k = 2$ of the theorem was proved by Rippon and Stallard [10].

Remark 5.3. The function f is a transcendental entire function, so it has at least one direct tract.

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